



- Concept of square roots and square.
- Identify perfect square number; Relationship between factors of numbers and square roots.
- To find square root of a perfect square number by factorization method.
- To find square root of a perfect square number by division method.
- Number of digits relation of a square number and its square root.
- Square root of a decimal number by division method.
- To find square root of such type of numbers which are not perfect square.

2.1 Introduction

In previous classes we have studied about powers and exponents. If a number is having power “2” then we call it square of that number i.e. 2^2 is called, “2 to the power of 2” or square of 2.

Clearly 2^2 means 2×2 .

Now to know about a given number we have to find a number which when multiplied to itself product is given number. We have to find square root of the given number.

Similarly, if area of a square is known then to find side of the square. We have to find square root of the Area of square

In our daily life we more often require to find square or square root. In this unit we will study the methods to find square root of positive numbers.

2.2. Concept of square and square root

We know if a number is multiplied by itself then product found is called square of the said number, for example

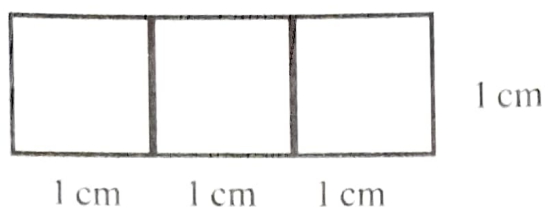
$$5 \times 5 = 25 = 5^2$$

$$6 \times 6 = 36 = 6^2$$

$$7 \times 7 = 49 = 7^2$$

From above statement it can be said that 25 is square of 5; 36 is square of 6; 49 is square of 7. In this way power two (2) to any number is called square of the number.

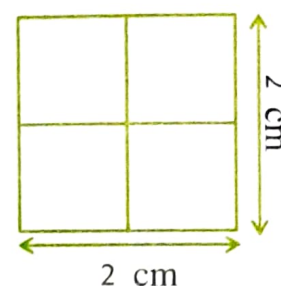
Look carefully to the figure given below and answer the questions.



- Above figure is divided in how many parts?
- How many sides does each part have?
- What is the relationship between length and breadth size of each part?
- What will be the name for each sub figure.

In this way by observation we find that :

- Given figure has three parts in all.
- Every part has four sides.
- Size of length and width of each part is equal
- Every sub-figure is a square.



In this way if size of side of square is 2cm, then there will be

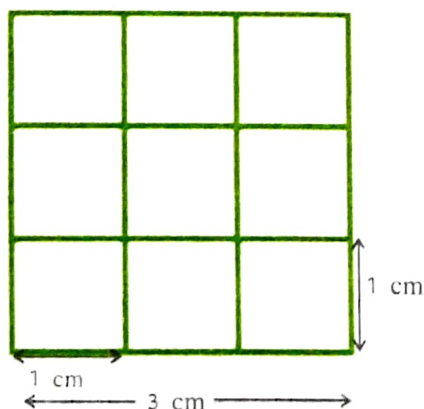
$$2 \times 2 = 2^2 = 4 \text{ squares in it with area } 1 \text{ cm}^2 \text{ each.}$$

Which is also true according to the figure i.e 4 is square of 2.

Example 1. Find number of squares of area 1 cm^2 in a square of side 3 cm by figure.

Solution: First of all we draw a square of side 3 cm and divide each side of it into three equal parts then join the points exactly opposite to each other. In this way total number of squares is 9.

Hence 9 is square of 3.



Example 2. As in example 1 find the number squares with area 1cm^2 in a square of side 4 cm.

Solution: Number of squares with area 1cm^2 in a squares of side $4\text{cm} = 4 \times 4 = 16$.

Example 3. What is the number which if multiplied by itself gives product 25?

Solution : Since $5 \times 5 = 5^2 = 25$

therefore, 5 is the required number

Similarly taking other integers we see:

$$6 \times 6 = 6^2 = 36$$

$$7 \times 7 = 7^2 = 49$$

As we know subtraction is reverse operation of addition; division is reverse operation of multiplication.

Finding square root is reverse operation of finding square of the number.

(i) 25 is square of 5 & 5 is square root of 25.

(ii) 36 is square of 6 & 6 is square root of 36.

(iii) 49 is square of 7 & 7 is square root of 49.

(iv) 0 is square of 0 & 0 is square root of 0.

We have learnt earlier

$$\text{Area of square} = \text{side} \times \text{side} = (\text{side})^2$$

In this way size of one side of a square is square root of the area of the square.

Positive Square root is denoted by “ $\sqrt{\quad}$ ”.

$\sqrt{25}$ means; square root of 25.

$\sqrt{36}$ means; square root of 36.

Example 4. Find square of negative integers -1, -2, -3 and -4.

Solution: $(-1)^2 = -1 \times -1 = 1$

$$(-2)^2 = -2 \times -2 = 4$$

$$(-3)^2 = -3 \times -3 = 9$$

$$(-4)^2 = -4 \times -4 = 16$$

Observe:

1. $\therefore$ Square root of $1\left(\sqrt{1}\right)$ is 1.

2. $\therefore$ Square root of $4\left(\sqrt{4}\right)$ is 2.

3. $\therefore$ Square root of $9\left(\sqrt{9}\right)$ is 3.

4. $\therefore$ Square root of $16\left(\sqrt{16}\right)$ is 4.

Note carefully: $\sqrt{25}$ means 5 and $-\sqrt{25}$ means -5. 5 and -5 both are square root of 25.

Conclusions:

1. Square of integers is positive integers or natural numbers.
2. Square root of positive perfect square integers is positive or negative integers Ex. $\sqrt{x^2} = \pm x$
3. Square of zero is zero.
4. Square root of zero is zero.

Example 5. Find square of rational numbers $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.

Solution: $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$$\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$\left(\frac{4}{5}\right)^2 = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$$

$$\left(\frac{5}{6}\right)^2 = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$$

In this way we can see again that

$$(i) \left(\frac{-2}{3}\right)^2 = \frac{4}{9}; \left(\frac{-3}{4}\right)^2 = \frac{9}{16}; \left(\frac{-4}{5}\right)^2 = \frac{16}{25}; \left(\frac{-5}{6}\right)^2 = \frac{25}{36}$$

(ii) Therefore, square root of $\frac{4}{9}$ is $\pm \frac{2}{3}$.

square root of $\frac{9}{16}$ is $\pm \frac{3}{4}$.

Conclusion:

1. Square of non-zero rational numbers are positive rational (fraction).
2. Square root of non-zero perfect square rational numbers are simultaneously positive and negative rational numbers.

Try:

Table based on square and square root of numbers 1 to 9.

x	x^2	$\sqrt{x^2}$	x
1	1	$\sqrt{1}$	1
2	4	$\sqrt{4}$	
3		$\sqrt{\quad}$	3
4	16	$\sqrt{16}$	
5		$\sqrt{\quad}$	5
6	36	$\sqrt{\quad}$	
7		$\sqrt{\quad}$	
8	64	$\sqrt{\quad}$	8
9	81	$\sqrt{\quad}$	

Fill the gaps.

Note:

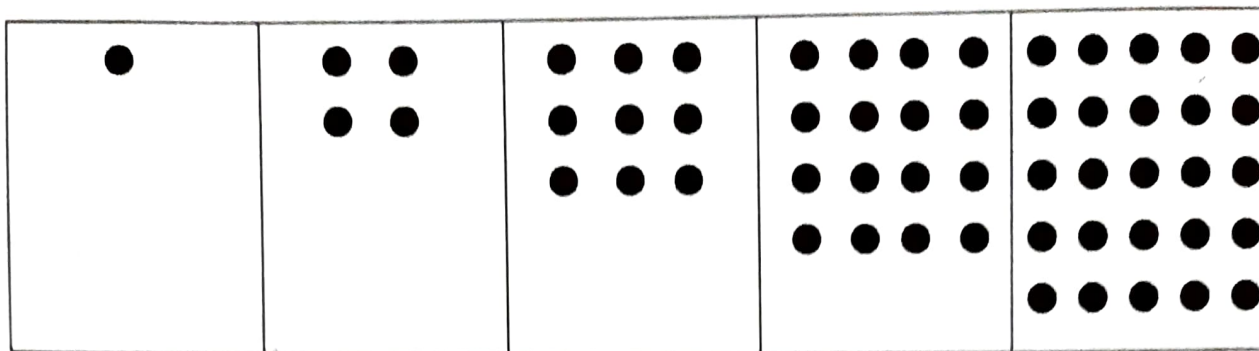
1. If x is a natural number the square x , x^2 is a natural number.
2. If x^2 is a natural number then square root of x^2 are $\pm x$ but $-x$ is not a natural number.

2.3. Identify perfect square number and relation of its square roots and factors.

Perfect square number:

Example 6. Show points 1, 4, 9, 16 on your copy so that number of points in horizontal and vertical are equal.

Solution : 1, 4, 9, 16, 25 points are geometrically expressed so that number of rows and columns are equal and joining the points will form a square as shown below.
Therefore, 1, 4, 9, 16, 25 are perfect square numbers or square numbers.



$$1 = 1 \times 1$$

$$= 1^2$$

$$4 = 2 \times 2$$

$$= 2^2$$

$$9 = 3 \times 3$$

$$= 3^2$$

$$16 = 4 \times 4$$

$$= 4^2$$

$$25 = 5 \times 5$$

$$= 5^2$$

therefore 1, 4, 9, 16, 25 numbers are called perfect square numbers.

Example 7. Make a perfect square number table by squaring the natural numbers 1 to 9.

Solution:

a	a^2	Perfect square number
1	1^2	1
2	2^2	4
3	3^2	9
4	4^2	16
5	5^2	25
6	6^2	36
7	7^2	49
8	8^2	64
9	9^2	81

Try:

Form a perfect square table by squaring natural numbers 10 to 20.

Note:

36, 49, 81, 100 etc. are the numbers that can be expressed as perfect squares.

2.3.1. Examining a perfect square number

Example 8. Find out whether 225 is a perfect square number?

Solution: Firstly we will find prime factorization of 225.

Here,

$$\begin{array}{r|l} 5 & 225 \\ \hline 5 & 45 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

Since, $225 = 3 \times 3 \times 5 \times 5$

therefore, $225 = \overline{3 \times 3} \times \overline{5 \times 5} = (3 \times 5) \times (3 \times 5) = (3 \times 5)^2$

Here, we get a pair 3×3 and another pair 5×5 and no other prime factor remained.

Hence 225 is a perfect square.

Example 9. Find whether 360 is a perfect square number?

Solution : First of all we find prime factorization of 360.

$$\begin{array}{r|l} 2 & 360 \\ \hline 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

therefore, $360 = \overline{2 \times 2} \times 2 \times \overline{3 \times 3} \times 5$

Here we get pair 2×2 and 3×3 while prime factors 2 and 5 are remained with no pair.

Hence, 360 is not a perfect square number.

Try:

Find which number among following is a perfect square number

64, 144, 81, 810

From above examples it is clear

After prime factorization of a number if no prime factor remained unpaired then the number is a perfect square number.

2.3.2 Characteristics of Even and Odd Square

Example 10. Find squares of even numbers 2, 4, 6 and 8 and tell whether the number found is even or odd.

Solution: Square of $2 = 2^2 = 4$ an even number
Square of $4 = 4^2 = 16$ an even number
Square of $6 = 6^2 = 36$ an even number
Square of $8 = 8^2 = 64$ an even number

Try:

Take some even numbers and form a table of their squares. Whether number found are even numbers.

Example 11. Find square of the odd numbers 1, 3, 5 and 7. Whether the number found is odd or even.

Solution: Square of $1 = 1^2 = 1$ an odd number
Square of $3 = 3^2 = 9$ an odd number
Square of $5 = 5^2 = 25$ an odd number
Square of $7 = 7^2 = 49$ an odd number

Similarly, take some other odd number and find their square and table whether number found are even or odd.

- (i) Square of an even number is an even number.**
- (ii) Square of an odd number is an odd number.**

Discuss in group

1. Which one of the following numbers is a perfect square.
64, 121, 144, 110, 81, 36
2. Which among the following is square of an even number.
121, 256, 1296, 225, 676
3. Which among the following is square of an odd number.
169, 144, 289, 256, 361

Exercise 2(a)

1. Find square of all odd numbers between 1 to 15.
2. Evaluate the following:
 - (i) 56^2
 - (ii) 82^2
 - (iii) 65^2
 - (iv) 75^2
3. Find squares of the following rational numbers:

- (i) -5
- (ii) $\frac{13}{17}$
- (iii) $\frac{-6}{7}$
- (iv) $\frac{15}{19}$
- (v) -125
- (vi) $\frac{9}{-23}$

2.4. Square root of a perfect square number

If y is a perfect square rational number then y can be written as x^2 or $(-x)^2$ where x is a rational number.

Therefore, $y = x^2$ or $y = (-x)^2$

Hence, square root of y , $\sqrt{y} = x$ and $\sqrt{y} = -x$ both.

Conclusion:

For any perfect square number of $y = x^2$ has square roots $\sqrt{y} = \pm x$ where x is a rational number.

Special feature:

- (1) Square root of a perfect square number are same number with opposite sign and they are called square roots.
- (2) For the sake of ease while finding square root of a number. We find one square root and by assigning opposite sign to it we get the other.

2.4.1. Evaluating square root of perfect square number by factorization method

Example 2. Find square root of the following perfect square number.

- (i) 36
- (ii) 144

Solution:

- (i) To find square root of 36
First find prime factorization of 36.

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$36 = 2 \times 2 \times 2 \times 3 \times 3$$

Make pairs of prime factors

$$36 = \overline{2 \times 2} \times \overline{3 \times 3}$$

from each pair take one prime factor and find product.

$$\text{Here, } 2 \times 3 = 6$$

$$\text{therefore, positive square root of } 36 = \sqrt{36} = 6$$

$$\text{and negative square root of } 36 = \sqrt{36} = -6$$

In this way square root of 36 are +6 and -6.

- (ii) Finding square root of 144

First, we find prime factor of 144.

$$\begin{array}{r|l} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Pairing the prime factors

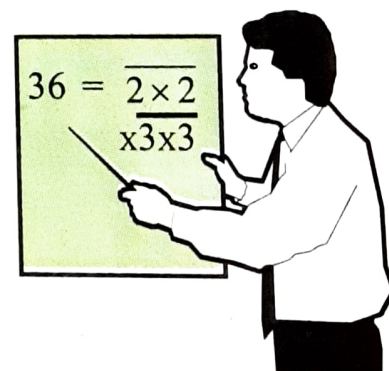
$$144 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3}$$

take one prime factor from each pair and multiply

$$\text{Here, } 2 \times 2 \times 3 = 12$$

$$\text{therefore } \sqrt{144} = 12$$

In this way square root of 144 are +12 and -12.



Example 13. Find positive square root of following perfect square numbers by factorization method?

Solution:

(i) 576 (ii) 2025

(i) Square root of $576 = \sqrt{576}$

$$\therefore 576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$= \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3}$$

$$\sqrt{576} = 2 \times 2 \times 2 \times 3$$

$$= 24$$

(ii) Square root of $2025 = \sqrt{2025}$

$$\therefore 2025 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$$

$$= \overline{5 \times 5} \times \overline{3 \times 3} \times \overline{3 \times 3}$$

$$\therefore \sqrt{2025} = 5 \times 3 \times 3$$

$$= 45$$

$$\begin{array}{r|l} 2 & 576 \\ \hline 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$$\begin{array}{r|l} 5 & 2025 \\ \hline 5 & 405 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

For evaluation of square root of a number:

1. Firstly find out prime factorization of the number.
2. Make pairs of prime factors.
3. Select one from each prime factor pair.
4. Product of all these selected prime factors is the positive square root of the number.
5. By changing the sign of evaluated square root we can find both square roots.

2.4.2. To find square root of product or division of two numbers by factorization method

We have evaluated square root of perfect square integers. Similarly, we can also find square root of perfect square rational numbers.

Learn following example carefully.

$$(a) \sqrt{9 \times 100} = \sqrt{(3 \times 3) \times (2 \times 2) \times (5 \times 5)}$$

$$= 3 \times 2 \times 5 = 3 \times 10 = \sqrt{9} \times \sqrt{100}$$

$$\text{therefore, } \sqrt{9 \times 100} = \sqrt{9} \times \sqrt{100}$$

$$(b) \sqrt{\frac{4}{9}} = \sqrt{\frac{2 \times 2}{3 \times 3}} = \sqrt{\frac{2}{3} \times \frac{2}{3}}$$

$$= \frac{2}{3} = \sqrt{\frac{4}{9}}$$

therefore, $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$

From above examples we see that

If a and b two perfect square numbers, then

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b} \quad \& \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{where } b \neq 0$$

Example 14. Find square root of rational numbers $\frac{256}{441}$.

Solution: Square root of $\frac{256}{441} = \sqrt{\frac{256}{441}} = \frac{\sqrt{256}}{\sqrt{441}}$

$$\sqrt{256} = 2 \times 2 \times 2 \times 2$$

$$\sqrt{441} = 3 \times 7$$

$$\therefore \sqrt{\frac{256}{441}} = \frac{2 \times 2 \times 2 \times 2}{3 \times 7}$$

therefore, square root of $\frac{256}{441}$ are $\pm \frac{16}{21}$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

$$\begin{array}{r|l} 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Example 15. Find square root of mixed fraction $2\frac{14}{25}$

Solution: $2\frac{14}{25} = \frac{64}{25}$ (changing to odd fraction)

Square root of $2\frac{14}{25} = \sqrt{2\frac{14}{25}} = \sqrt{\frac{64}{25}}$

$$= \frac{\sqrt{64}}{\sqrt{25}}$$

$$\begin{array}{r|l} 5 & 25 \\ \hline & 5 \end{array}$$

$$\therefore \sqrt{64} = 2 \times 2 \times 2 \text{ \& } \sqrt{25} = 5$$

$$\therefore \sqrt{\frac{64}{25}} = \frac{2 \times 2 \times 2}{5}$$

$$= \frac{8}{5} \text{ therefore } \sqrt{\frac{64}{25}} = \frac{8}{5}$$

2	64
2	32
2	16
2	8
2	4
	2

Try:

Find square root of the following rational numbers:

(i) $\frac{121}{625}$

(ii) $\frac{225}{49}$

From above examples following conclusions are found:

- (i) If rational number is expressed in mixed fraction then firstly write it as odd fraction.
- (ii) Evaluate square root of Numerator and denominator of so found odd fraction separately.
- (iii) Expressing roots of numerator and denominator respectively as numerator and denominator is the square root of the given rational numbers.

Example 16. A commander instructed his 3600 soldiers to stand in different rows in such a way that number of soldiers in each row is same as number of rows. Find out the number of rows soldiers are standing.

Solution: Number of soldiers in each row $= \sqrt{3600}$

First we evaluate prime factorization of 3600

2	3600
2	1800
2	900
2	450
5	225
5	45
3	9
	3

$$3600 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3} \times \overline{5 \times 5}$$

Therefore, $\sqrt{3600} = 2 \times 2 \times 3 \times 5 = 60$

Special:

Number of soldiers cannot be negative therefore, -60 is not accepted as answer.
 Number of soldiers in each row = 60.

Verify the answer.

Since, $60 \times 60 = 3600$

Hence, answer is correct.

Example 17. Find the smallest number which when multiplied to 6075 product obtained is a perfect square.

Solution: First we find prime factorization of 6075.

5	6075
5	1215
3	243
3	81
3	27
3	9
	3

$$6075 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3$$

forming pairs of prime factors

$$6075 = \overline{5 \times 5} \times \overline{3 \times 3} \times \overline{3 \times 3} \times 3$$

Here, after forming pairs of prime factors we see that prime factor 3 remained unpaired.

Therefore, if 6075 is multiplied by 3, then product will be a perfect square number.

Example 18. Number given in example 17 should be divided by what possible smallest number so that the quotient is a perfect square number.

Solution: $6075 = \overline{5 \times 5} \times \overline{3 \times 3} \times \overline{3 \times 3} \times 3$

Here we see prime factor 3 is unpaired and by dividing by 3 then quotient will be a perfect square. Therefore, 3 is the required number.

Exercise 2(b)

- Find square root of following by factorization method.
(i) 7744 (ii) 11664 (iii) 4900 (iv) 47089
- Find positive square root of the following by factorization method.
(i) $\frac{625}{1296}$ (ii) $\frac{529}{196}$ (iii) $4\frac{29}{49}$ (iv) $23\frac{26}{121}$ (v) $75\frac{46}{49}$
- There are 2304 mango trees in an orchard. Every row have same number of trees and is equal to number of rows. How many rows are there in the orchard.
- A company commander has divided his soldiers in groups such that every group has equal members and is equal to number of groups. If his army had 6561 soldiers. How many soldiers were in each group.
- A company commander forms a square by positioning their soldiers along its sides. Later he realize that 60 soldiers, remained free. If he had 8160 soldiers then find how many soldiers were deployed in each row.
- Find the smallest number which if multiplied by 1890 then product becomes a perfect square.
- Find the smallest number by which if 9408 is divided quotient becomes a perfect square.

2.5. Evaluation of square root by division method

We can find square of perfect square number by factorization method, as we can find their prime factorization easily. Those number which cannot be easily factorized into prime factors or those which are not perfect squares then we find square root by division method.

Example 19. Find positive square root of 1849 by division method?

Solution : Positive square root of 1849 = $\sqrt{1849}$

- Starting from unit's digit we draw horizontal bar on each pair.
- We select one of the number 1 to 9 whose square is less than or equal to the first pair 18 i.e. $4^2 = 16$
- At the place of square root we write 4 and after writing 16 below 18 we subtract.
- We take next pair 49 and write it adjacent to 2. In this way 249 is new dividend.
- In the next step to find new divisor we take double of previous divisor i.e. double of 4 is 8 or can get 4 added 4 to get 8.

6. By leaving units place in 249 we divide by 8 to 24, it gives quotient. Write 3 adjacent to pre found divisor 8 is right place.
7. Product of 83 and 3 is written below to 249 and subtracted. We get remainder zero.
8. Find out square root of number 1849 by division method.

	43
4	<u>18 49</u>
+4	16
	<u>249</u>
83	249
	<u>0</u>

therefore, $\sqrt{1849} = 43$

Now check the answer by multiplying 43 to 43.

$$43 \times 43 = 1849$$

Therefore, answer is correct.

Example 20. Find positive square root of 11449 by division method?

Solution: Positive square root of 11449 = $\sqrt{11449}$

	107
1	<u>114 49</u>
+1	1
	<u>0 1449</u>
207	1449
	<u>0</u>

1. Starting with unit place digit make pairs. Ten thousand place digit 1 is unpaired. Square root of 1 is 1. So write 1 at hundred place of square root. Write 1 below 1 and subtract. Now next pair on right is 14. Double of 1 is 2 or $1 + 1 = 2$.
2. Here in ten's place of 14 is 1 which divide by 2 leaves quotient 0. So we write 0 next to divisor 2 becomes 20 also write 0 next to square root 1.
3. Now we write next pair 49 adjacent to 14. In new dividend 1449 leaving 9 we get 144 which divided by 20 gives quotient 7.

4. Remaining steps are as earlier and we find required square root.

Example 21. Find positive square root of $21\frac{2797}{3364}$ by division method.

Solution: $21\frac{2797}{3364} = \frac{21 \times 3364 + 2797}{3364} = \frac{70644 + 2797}{3364} = \frac{73441}{3364}$

	271
2	7 34 41
+2	4
47	334
+7	329
541	541
	541
	0

	58
5	33 64
+5	25
108	864
	864
	0

(Note: 33 divided by 4 gives quotient 8 while 48 multiplied by 8 product is more than 334)

In this way, $\sqrt{21\frac{2797}{3364}} = \sqrt{\frac{73441}{3364}} = \frac{271}{58} = 4\frac{39}{58}$

Example 22. Find the smallest natural number which if subtracted from 194491 then remainder is a perfect square.

By finding square root of the given number by division method we get remainder 10. If 10 is subtracted from the number remainder will be a perfect square number. So the smallest number required is 10.

Solution :

	441
4	19 44 91
+4	16
84	344
+4	336
881	891
	881
	10

Example 23. Find the smallest natural number which if added to 306452, sum will be a perfect square.

Solution:

	553
5	30 64 52
+5	25
105	564
+5	525
1103	3952
	3309
	643

	554
5	30 64 52
+5	25
105	564
+5	525
1104	3952
	4416
	-464

From above it is clear that given number is larger than $(553)^2$ but less than $(554)^2$. If $(4416 - 3952)$ or 464 is added to the given number, sum will be a perfect square.

So number required is 464. Check your answer.

$$306452 + 464 = 306916$$

Find square root of 30691 by division method. Observe it will be a perfect square number.

Example 24. Find smallest six digit perfect square number.

Solution: Smallest six digit number = 100000

$$\text{Square root } 100000 = \sqrt{100000}$$

	316
3	10 00 00
+3	9
61	100
+1	61
626	3900
	3756
	144

	317
3	10 00 00
+3	9
61	100
+1	61
627	3900
	4389
	-489

While finding square root of 100000 we conclude 100000 is 489 less than smallest six digit perfect square number.

$$\text{Therefore, smallest six digit perfect square number} = 100000 + 489 = 100489$$

Example 25. Find largest possible perfect square number of six digits.

Solution : Largest six digit number is 999999

$$\text{Square root of } 999999 = \sqrt{999999}$$

	999
9	99 99 99
+9	81
189	1899
+9	1701
1989	19899
	17901
	1998

while finding square root of 999999 obtained square root number's square $(999)^2$ is 1998 less than 999999.

Therefore six digits largest perfect square number = $999999 - 1998 = 998001$

Exercise 2(c)

- Find square root of the following by division method.
 - 4489
 - 27225
 - 49284
 - 1234321
 - 4937284
- Find square root of the following by division method.
 - $\frac{361}{625}$
 - $34\frac{15}{49}$
 - $21\frac{51}{169}$
 - $10\frac{151}{225}$
 - $23\frac{394}{729}$
- Find smallest natural number which if added to 4931 sum is a perfect square number.
- Find smallest natural number which if subtracted to 18265 remainder is a perfect square number.
- Find largest five digit perfect square number.
- Find square root of 62500 by prime factorization and division method. Compare the answer in two methods.

2.6. Relation of number of digits in a square number and its square root

In following example we will discuss about number of digits in a number and its square root.

a	a^2	$\sqrt{a^2}$	$\pm a$	a	a^2	$\sqrt{a^2}$	$\pm a$
1	1	$\sqrt{1}$	± 1	6	36	$\sqrt{36}$	± 6
2	4	$\sqrt{4}$	± 2	7	49	$\sqrt{49}$	± 7
3	9	$\sqrt{9}$	± 3	8	64	$\sqrt{64}$	± 8
4	16	$\sqrt{16}$	± 4	9	81	$\sqrt{81}$	± 9
5	25	$\sqrt{25}$	± 5				

See the table following of square and square roots of numbers 1 to 9 where “a” is an integer.

a	a^2	$\sqrt{a^2}$	$\pm a$	a	a^2	$\sqrt{a^2}$	$\pm a$
10				50			
25	625	$\sqrt{625}$	± 25	65			
31				85			
32	1024	$\sqrt{1024}$	± 32	99	9801	$\sqrt{9801}$	± 99

From above table we get to know square of the numbers 10 to 99 have 3 or 4 digits and square root of each have 2 digits. We observe that the numbers between 100 to 999 (i.e. 3 digits numbers) their squares will have 5 or 6 digits. It means square root of 5 or 6 digit numbers will have three digits.

Try:

In the table given below fill the gaps in your exercise book

Number of digits in a Perfect square number	Number of digits in the square roots of said numbers
1 or 2 digit	1
3 or 4 digit	
5 or 6 digit	
7 or 8 digit	4
9 or 10 digit	

2.6.1. To find number of digits in the square root of a given number

For finding number of digits in the square root of a number starting from right hand or unit's place we draw horizontal line on each pair. On the left if only one digit remained, then we draw a horizontal line on it.

Number of horizontal line will be the number of digits in the square root of the given number.

Clearly,

1. In any perfect square number with even number of digits then square root of the number will have number of digits equal to half of that in the number.
2. If number of digits in a perfect square number is odd; then number of digits in its square root will be half the successor of number of digit in the number.

Example 26. Find the number of digits in the square root of the following numbers.

256, 1225, 14641, 783225.

Solution:

- (i) $\overline{2} \overline{56}$ has two horizontal lines.
 $\therefore$ Number of digits in the square root of 256 is 2.
- (ii) $\overline{12} \overline{25}$ has two horizontal lines so its square root has two digits.
- (iii) $\overline{1} \overline{46} \overline{41}$ has three horizontal lines so its square root has three digits.
- (iv) $\overline{78} \overline{32} \overline{25}$ has three horizontal lines so its square root has three digits.

GROUP DISCUSSION

1. What is the minimum number of digits in the square of two digit numbers?
2. What is the maximum number of digits in the square of three digit numbers?
3. What number of digits in the square root of 289?
4. What number of digits are there in the square root of 15625?
5. Find number of digits in the square root of six digit number?

Exercise 2(d)

Answer the following question.

1. Find number of digits in the square root of the number 1809025.
(i) 2 (ii) 5 (iii) 4 (iv) 3
2. Number of digits in the square root of square number from 100 to 999.
(i) 2 (ii) 4 (iii) 5 (iv) 3
3. Number of digits in the square root of a seven digit square number.
(i) 2 (ii) 3 (iii) 4 (iv) 5
4. Square root of a number have only 2 digits then the number is-
(i) one digit or two digit (ii) two digit or three digit
(iii) three digit or four digit (iv) four digit or five digit
5. How many digits are there in the square root of following numbers.
(i) 2304 (ii) 75625 (iii) 166464 (iv) 32901696 (v) 64432729

2.7. Square root of a decimal number

Like perfect square numbers we can also find square root of rational numbers by division method. In this method we first represent the rational number in decimal form, then we find square root of the number by division method.

Try:

Observe following table carefully and fill in the blanks in your exercise book.
Here x is a decimal number.

x	x^2	$\sqrt{x^2}$	$\pm x$
0.3	0.09	$\sqrt{.09}$	± 0.3
0.5	0.25	$\sqrt{\dots}$	$\dots$
0.41	0.1681	$\sqrt{\dots}$	± 0.41
4.1	$\dots$	$\sqrt{16.81}$	$\dots$
7.5	$\dots$	$\sqrt{56.25}$	$\dots$
1.25	1.5625	$\sqrt{\dots}$	$\dots$

From above it is clear that

1. In square rational number, number of decimal digits is always even and number of decimal digits in their square root is half, that in the number.
2. Integer part of decimal number; we calculate number of digits in its square root by earlier method.

Example 28. Find positive square root of the number 4.6225

	2.15
2	$\overline{4.62\ 25}$
+2	4
41	62
+1	41
425	2125
	2125
	0

1. Starting from decimal; we draw a horizontal line on each pair of digits on the right hand of decimal.
2. In integer part starting from unit place digit we draw a horizontal line on each pair of digits. If only one digit remained on the left most place, we draw a horizontal line on it.
3. If number of digits on decimal part is not even, then by adding a zero on the right most place we make it even then draw the horizontal lines.

4. Next to above steps we follow same division method of finding square root as that in square numbers. Only difference is before taking first decimal digit pair we assign a decimal to the square root value.

Example 29. Find square root of 0.00053361.

Solution : Square root of 0.00053361 = $\sqrt{0.00053361}$

	0.0231
2	0.00 05 33 61
+2	4
43	133
+3	129
461	461
	461
	0

therefore, $\sqrt{0.00053361} = 0.0231$ Similarly ± 0.0231 are square root of 0.00053361

2.8. Finding square root of non perfect square and non-integer number

In this case, given number is decimal fraction or can be expressed as decimal fractions; we add some zeroes to last decimal digit as required number decimal places in its square root.

Example 30. Find positive square root of 0.9 correct upto two decimal places.

Solution : First we add that number of zeroes next to 9, so that three (=2+1) pairs decimal digits formed. As

	0.948
9	0.90 00 00
+9	81
184	900
+4	736
188	16400
	15104
	1296

At the third decimal place of square root is 8, which is more than 5, so to write square root correct to two decimal places we add 1 to second decimal place digit.

therefore, $\sqrt{0.9} = 0.95$

Example 31. Find the value of $\sqrt{2}$ correct to three places of decimals.

Solution : First we add as much number of zeroes next to 2, so that four digit ($=3+1$) pairs can be formed.

	1.4142
1	<u>2.00 00 00 00</u>
+1	1
+24	100
4	96
281	400
+1	281
2824	11900
+4	11296
28282	60400
	56564
	3836



$\therefore$ In square root at fourth decimal place digit is “2” which is less than 5, so decimal third place digit remain unchanged

therefore, $\sqrt{2} = 1.414$

Example 32. Find positive square root of $10\frac{2}{3}$ correct to three decimal places.

Solution : $10\frac{2}{3} = \frac{32}{3} = 10.66666666$

therefore, $\sqrt{10\frac{2}{3}} = 10.66666666$

	3.2659
3	10.66 66 66 66
+3	9
62	166
+2	124
646	4266
+6	3876
6525	39066
+ 5	32625
65309	644166
	587781
	56385



In square root four decimal place digit is 9 which is more than 5, so third decimal place digit will be increased by 1.

Therefore, $\sqrt{10\frac{2}{3}} = 3.266$

Here by observation of example 30, 31, and 32 answers

$$\sqrt{.9} = 0.948.....$$

$$\sqrt{2} = 1.4142.....$$

$$\sqrt{10\frac{2}{3}} = 3.2659.....$$

So square root of all numbers is not a rational number.

All those numbers which are not rational are called Irrational.

Arrange a group discussion

1. Find square root of 0.16?
2. Number 0.3 is square root of what number?
3. Number 0.5 is square root of what number?

Exercise 2(e)

- Find positive square root of the following
(i) 84.8241 (ii) 150.0625 (iii) 477.4225
(iv) 225.6004 (v) 0.00008281
- Find positive square root of the following numbers correct to three places of decimals.
(i) 1.7 (ii) 23.1 (iii) 5
(iv) 237.615 (v) 0.016
- Convert following rational numbers into decimal fractions and find their positive square root correct upto three places of decimals:
(i) $\frac{5}{12}$ (ii) $2\frac{1}{12}$ (iii) $287\frac{5}{8}$ (iv) $367\frac{2}{7}$
- Find the decimal number, which if multiplied to itself product is 1227.8016.
- Area of a square is 0.00037636 m^2 . Find the length of the side of square approximately in centimeters.
- If $\sqrt{2} = 1.4142$ then find value of $\sqrt{8}$ correct upto three places of decimals.

Efficiency practice 2

- Find whether 5400 is a perfect square number?
- Evaluate $\sqrt{41^2 - 40^2}$
- Find square root of the following by factorization method.
(i) 15876 (ii) 148225 (iii) 69696
- Solve following by taking positive square root.
(i) $\sqrt{5^2 + 5^2}$ (ii) $\sqrt{(5^2 + 12^2)}$ (iii) $30 + \sqrt{900}$
(iv) $\sqrt{400} + \sqrt{0.04} + \sqrt{0.000004}$
- Find positive square root of the following by division method.
(i) 4225 (ii) 75625 (iii) 3915380329
- Find positive square root of the following by factorization method.
(i) $\frac{625}{121}$ (ii) $38\frac{139}{169}$ (iii) $28\frac{189}{289}$
- Find smallest positive number which if multiplied to 9792 then product is a perfect square.

8. Find smallest possible number which if divides 3675 quotient will be a perfect square.
9. Find smallest four digit perfect square number.
10. Find largest four digit perfect square number.
11. Find smallest four digit perfect square number, which is perfectly divisible by 16, 18 and 45.
12. A basket contains 1250 flowers. A person attributes as much flowers in each temple equal to the number of temples in the city. If he attributes 8 basket of flowers, then find number of temples in the city.
13. On the occasion of 15th August each student of class 6 is given as much number of grams of sweets, as the number of students in the class. If 1.6 kilogram of sweets distributed to the students of class 6. Find the number of student in the class and number of decagram of sweets each student got?
14. Find smallest number subtracted to 16160 results a perfect square number?
15. Find smallest possible natural number added to 594 results a perfect square sum?
16. To maintain to square garden clean, fair and pollution free per month. Expense Rs 2.25 per m^2 . If monthly expense of the garden is Rs. 3600. Find the length of side of the garden?
17. On the occasion of Independent day students planted 289 trees. Each row has as number of trees as the number of rows. Find how many trees are planted in a row?

We have learnt

1. Square of any number x is x^2 which means $x \times x$.
2. Squares of non zero integers are positive integers.
3. Square root of $x^2 = \pm x$ where x is an integer.
4. Square of zero is zero and square root of zero is also zero.
5. Square root of square rational numbers are rational numbers.
6. If x is any rational number then $y = x^2 \geq 0$ then rational number is called perfect square number.
7. If a number is a perfect square number then while making pairs of its same prime factors no prime factor remains alone.

8. Square of even number is even and square of odd number is odd number.
9. Square root of any number can be obtained by division method or factorization method.
10. In a perfect square number if number of digits even then number of digits in its square root will be half in the number. If number of digit in perfect square numbers is odd then number of digits in its square root will be half the successor of the number of digits in the number.
11. In square root of any number, number of digits in its integer part can be obtained as described in above 10 serial.
12. Like perfect square integer square root of rational numbers is also evaluated by division method.
13. The real number that are not rational are called irrational.



ANSWERS

Exercise 2(a)

1. 9; 25; 49; 81; 121; 169, 2. (i) 3136 (ii) 4225 (iii) 6724 (iv) 5625, 3. (i) 25
 (ii) $\frac{169}{289}$ (iii) $\frac{36}{49}$ (iv) $\frac{225}{361}$ (v) 15625 (vi) $\frac{81}{529}$

Exercise 2(b)

1. (i) ± 88 (ii) ± 108 (iii) ± 70 (iv) ± 217 , 2. (i) $\frac{25}{36}$ (ii) $1\frac{9}{14}$ (iii) $2\frac{1}{7}$ (iv) $4\frac{9}{11}$
 (v) $\pm 8\frac{5}{7}$, 3. 48, 4. 81, 5. 90, 6. 210, 7. 3

Exercise 2(c)

1. (i) ± 67 (ii) ± 165 (iii) ± 222 (iv) ± 1111 (v) ± 2222 , 2. (i) $\pm \frac{19}{25}$ (ii) $\pm 5\frac{6}{7}$
 (iii) $\pm 4\frac{8}{13}$ (iv) $\pm 3\frac{4}{15}$ (v) $\pm 4\frac{23}{27}$, 3. 110, 4. 40, 5. 99856, 6. 250

Exercise 2(d)

1. (iii) 4, 2. (i) 2, 3. (iii) 4, 4. (iii) three digit or four digit, 5. (i) 2 (ii) 3 (iii) 3 (iv) 4 (v)

4.

Exercise 2(e)

1. (i) 9.21 (ii) 12.25 (iii) 21.85 (iv) 15.02 (v) 0.0091, 2. (i) 1.304 (ii) 4.806 (iii) 2.236 (iv) 15.415 (v) 0.126, 3. (i) 0.645 (ii) 1.443 (iii) 16.960 (iv) 19.165, 4. 35.04, 5. 2 cm, 6. 2.828

Efficiency practice 2

1. No, 2. 9, 3. (i) ± 126 (ii) ± 385 (iii) ± 264 , 4. (i) $5\sqrt{2}$ (ii) 13 (iii) 60, 0 (iv) 20.202, 5. (i) 65 (ii) 275 (iii) 62573, 6. (i) $2\frac{3}{11}$ (ii) $6\frac{3}{13}$ (iii) $5\frac{6}{17}$, 7. 17, 8. 3, 9. 1024, 10. 9801, 11. 3600, 12. 100, 13. 40 students; 4 decagrams, 14. 31, 15. 31, 16. 40 metre, 17. 17

