

UNIT-3

CUBE ROOT



- Concept of cube and cube roots
- To find cube root of a perfect cube number (factorization method)
- To find cube root of perfect cube of negative sign and to find cube root of product of two positive perfect cube number.
- To find cube root of a rational number whose numerator and denominator are perfect cube.
- To find cube root of a decimal number, which is a perfect cube by factorization method.
- Surds, number under surds and how to find the number under surds
- Application of cube roots

3.1 Introduction

In previous classes we have studied power and exponents. If a number is multiplied by itself for three times then it is called power three of that number.

You know that

$$2 \times 2 \times 2 = 2^3 = 8$$

$$3 \times 3 \times 3 = 3^3 = 27$$

$$5 \times 5 \times 5 = 5^3 = 125$$

Above operations can be expressed in statements as cube of 2 is 8. Cube of 3 is 27; Cube of 5 is 125, which means cube of a number means “power three” of that number.

In our daily life we use cube in calculation of volume and capacity and in stabilising its units. While cube roots are used for finding length of side of cube when volume of cubical body is known.

In previous chapter there are some numbers whose square root cannot be calculated in integer value. So square root of a number is represented in surds.

Now we will study cube and cube roots alongwith surds in this chapter.

3.2 Concept of cube and cube roots

Cube

Let us understand method of finding cube with help of some example.

you know that

$$(1) \quad 3 \times 3 \times 3 = 3^3 = 27$$

$$(2) \quad (-5) \times (-5) \times (-5) = (-5)^3 = -125$$

$$(3) \quad \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) \times \left(\frac{2}{7}\right) = \left(\frac{2}{7}\right)^3 = \frac{8}{343}$$

$$(4) \quad \left(-\frac{9}{8}\right) \times \left(-\frac{9}{8}\right) \times \left(-\frac{9}{8}\right) = \left(-\frac{9}{8}\right)^3 = -\frac{729}{512}$$

In this way a rational number n is said to be perfect cube, when for a rational number m

$$n = m \times m \times m = m^3$$

3.3 Perfect cube natural numbers :

In the table given below fill in the gaps in your exercise book.

Natural Number x	1	2	3	4	5	6	7	8	9	10
Cube of Natural Number (x^3)	1	8	27	...	...	...	...	...	...	1000

Please note there are only ten perfect cube numbers between 1 to 1000. (See by verification).

Solve these :

Make a table of the cubes of numbers 11 to 20. By observing the table list cubes of even numbers and cubes of odd numbers and find.

Conclusions.

After observing the table you can tell that cubes of odd numbers are odd and cubes of even numbers are even.

Numbers (x)	11	12	13	14	15	16	17	18	19	20
Value of cubes of Numbers (x^3)	1331	1728	2197	2744	3375	4096	4913	5832	6859	8000
	Odd	Even	Odd	Even	Odd	Even	Odd	Even	Odd	Even

Now we can guess taking cube of a natural number obtained will also be a natural number. Numbers so found are called perfect cube natural number.

Conclusion

1. Perfect cube natural numbers upto 1000 are 1, 8, 27, 64, 125, 216, 343, 512, 729 and 1000 respectively. Other natural numbers 2, 3, 5, 6, 7, 9, 10, ... 999 are not perfect cube natural numbers.
2. Smallest perfect cube natural number is 1.
3. None of natural numbers can be said the largest perfect cube natural number, because we can write a number larger to it and is perfect cube. So perfect cube natural number are infinite.
4. By taking cube of every natural number we get a perfect cube natural number.
5. Every natural number is not a perfect cube natural number.

Try :

- 1 is a perfect cube natural number. Why ?
- 216 is cube of which natural number ?
- Write cubes of 21, 31, 27 and 46.
- Write the value of 23^3 , 29^3 , 36^3
- Find cube root of 4096, 5832 and 6859
- Whether 0 is a perfect cube number.
- Find cube of $\frac{9}{13}$, $\frac{7}{22}$ and $\frac{13}{29}$

3.3.1 Cube and their Prime factorization

Observe the table given below-

Number	2	3	$\frac{4}{5}$	$\frac{-5}{6}$	0.1	0.5
Value of cube of a number	8	27	$\frac{64}{125}$	$\frac{-125}{216}$	0.001	0.125

- If a number less than 10; will its cube more than or less than 1000?
- If a number is more than 10 its cube value will be less or more than 1000?
- Write down perfect cube natural number from 1 to 100.
- Write perfect cube numbers from 1 to 1000.
- What is the smallest perfect cube natural number?

3.3.2 Geometrical representation of Cube

We have studied that cube is a solid structure with all of its sides equal. Given below three figures of cubes with sides of length 1 cm, 2 cm and 3 cm.

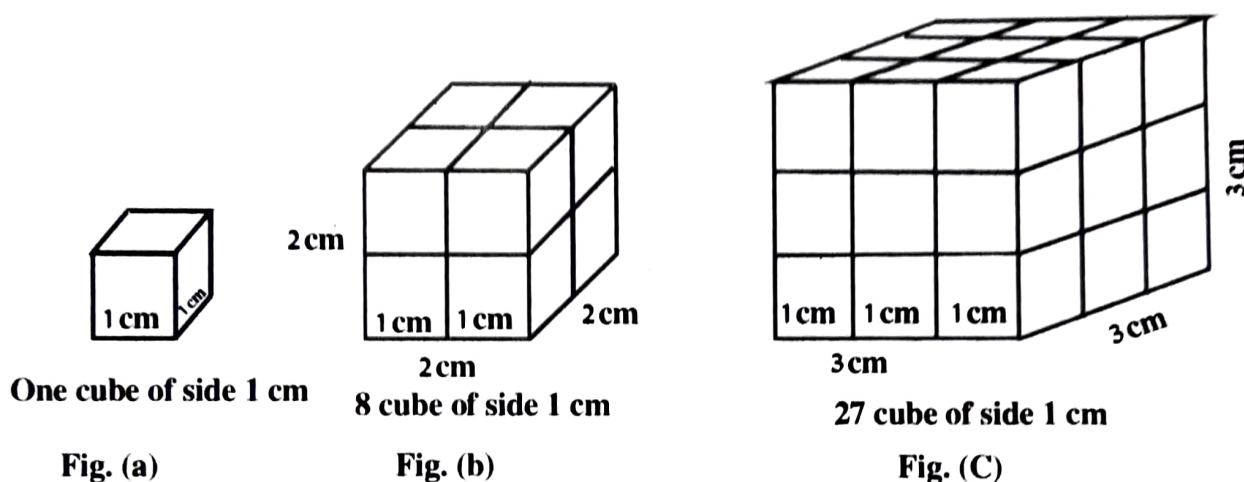


Figure (a) is a cube of side 1 cm. Fig (b) is a cube of side 2 cm. and this cube has volume 8 cm^3 . This cube has 8 cubes of side 1 cm. Fig (c) is a cube of side 3 cm whose volume is 27 cm^3 and it has 27 cubes of side 1 cm.

3.4 Cube root

We know that

$$8 = 2 \times 2 \times 2$$

$$= 2^3$$

$$-64 = (-4) \times (-4) \times (-4)$$

$$= (-4)^3$$

$$\frac{1}{1000} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$$

$$= \left(\frac{1}{10}\right)^3$$

or $0.001 = 0.1 \times 0.1 \times 0.1 = (0.1)^3$

$$-\frac{1}{8} = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3$$

From above example it is clear that cube of 2 is 8; cube of -4 is -64, cube of $\frac{1}{10}$ is $\frac{1}{1000}$ or cube of 0.1 is 0.001 and cube of $\left(-\frac{1}{2}\right)$ is $-\frac{1}{8}$. Conversely we can say cube root of 8 is 2; cube root of -64 is -4; cube root of $\frac{1}{1000}$ is $\frac{1}{10}$. Cube root of 0.001 is 0.1 and cube root of $-\frac{1}{8}$ is $-\frac{1}{2}$ these can be expressed as below :

$$\sqrt[3]{8} = 2$$

$$\sqrt[3]{-64} = -4$$

$$\sqrt[3]{\frac{1}{1000}} = \frac{1}{10} \text{ or } \sqrt[3]{0.001} = 0.1$$

and $\sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

In this way we see if we write 3 on the upper left of the surd ($\sqrt{\quad}$) symbol then it is representing cube root. From these examples it is clear that if a rational number p is cube root of another rational number q then $q = p^3$

Try

1. 27 is cube of which number ?
2. How the number 125 can be written in exponent form with base number 5?
3. $\frac{-125}{216}$ is cube of which number ?
4. Which number if multiplied for three times to itself give product 0.001.

- Cube root of any number is that number whose cube results the given number.
- Rational number r is cube root of rational number's if $r^3 = s$.

Try

Find cube root of the following numbers ?

- (i) 8 (ii) $\frac{-8}{27}$ (iii) $\frac{-64}{1331}$

Example 1. Whether 243 is a perfect cube number ?

Solution : $243 = \overline{3\ 3\ 3} \times 3 \times 3$

Here taking triplet of 3 we remain 3×3 in its prime factorization so 243 is not a perfect cube.

Example 2. Whether 729 is a perfect cube number ?

Solution : $729 = \overline{3\ 3\ 3} \times \overline{3\ 3\ 3}$

Grouping in three no factor remained

Hence 729 is a perfect cube number.

Discuss in group for following test

1. Which number has cube 64 ?
2. Find Cube of $\frac{1}{5}$.
3. Find cube of 0.1.
4. State whether following statement's are true or false.

(i) $\sqrt[3]{\frac{16}{25}} = \frac{2}{5}$

(ii) $\sqrt[3]{10,00,000} = 100$

(iii) 1 is not a perfect cube.

ARYABHATT FIRST

Aryabhatt was a great mathematician and Astronomist. He was born in 476 A.D. In the year 499 A.D. he wrote a mathematical epic "ARYABHATIYA". In field of Arithmetic Aryabhatt described methods of finding square root and cube root. In Aryabhatiya epic among 5 Chapters, one chapter is of Mathematics and rest of Astrology. In mathematical part he gave 33 formulae on arithmetic, alzebra geometry and trigonometry.

Aryabhatt had proved very early that Earth revolves round the Sun. While Copernicus knew this fact after 1100 years in 16th century.

Exercise 3(a)

1. Find cubes of the following numbers ?

7; 12; 17; 19; 21; 100

2. Cube of $\frac{-5}{9}$ is—

(i) $\frac{25}{81}$

(ii) $\frac{-25}{81}$

(iii) $\frac{125}{729}$

(iv) $\frac{-125}{729}$

3. Which of the following numbers are perfect cubes 64, 216; 243; 900; 1728; 106480, 363000

3.4.1. Cube root of perfect cube numbers :

(i) $27 = 3 \times 3 \times 3 = 3^3$ therefore $\sqrt[3]{27} = 3$

(ii) $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
 $= 2^3 \times 3^3$
 $= (2 \times 3)^3$
 $= 6^3$

therefore $\sqrt[3]{216} = 2 \times 3 = 6$

again $216 = \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3}$

so $\sqrt[3]{216} = 2 \times 3 = 6$

(iii) $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$
 $= 2^6 \times 3^3$

therefore $\sqrt[3]{1728} = 2 \times 2 \times 3$
 $= 12$

2	1728
2	864
2	432
2	216
2	10
2	54
3	27
3	9
	3

In this way we see that for finding cube root of a perfect cube number we find a number whose cube is perfect cube number.

Try

- From above method find cube root of the number 2744; 4096; 8000 and 91125.

In this way we come to know while finding cube root of a perfect cube number we should follow the steps given below :

1. First of all evaluate prime factorization of given number
2. Form triplets of equal factors.
3. Select one factor from each triplet.
4. Multiply all the factors selected from triplets.
5. Product so obtained is the required cube root of the number.

Note : Note carefully numbers are perfect cubes only if after forming triplets in prime factorization no. of the factors remained free.

Example 3. Find Cube root of 157464.

Solution : $157464 = \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}$

$$\begin{aligned}\text{So } \sqrt[3]{157464} &= 2 \times 3 \times 3 \times 3 \\ &= 54\end{aligned}$$

2	157464
2	78732
2	39366
3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

Example 4. What smallest number should be multiplied to 43200 to make it a perfect cube ? Also find cube root of the number obtained.

Solution : $43200 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times 5 \times 5$

We see that after forming triplets (3 – 3) of equal factor 5×5 is remained free. So if one more 5's multiplied in it. It become a perfect cube. So the number required for multiplication = 5

In this way perfect cube number = $43200 \times 5 = 216000$

2	43200
2	21600
2	1080
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
	3

$$\text{Now } 21600 = \overline{2 \times 2 \times 2} \times \overline{2 \times 2 \times 2} \times \overline{3 \times 3 \times 3} \times \overline{5 \times 5 \times 5}$$

$$\text{So, } \sqrt[3]{216000} = 2 \times 2 \times 3 \times 5 = 60$$

Example 5. Find the smallest number dividing by which to 13122 quotient becomes a perfect cube ? Also find cube root of the number obtained ?

Solution : $13122 = 2 \times 3 \times 3 \times \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}$

Here after forming triplets of equal prime factors three factors 2, 3, 3 remained free.

So if given number is divided by $2 \times 3 \times 3 = 18$

Quotient obtained will be a perfect cube.

So the required divisor for quotient to be perfect cube = 18

and number obtained in quotient

$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$$

$$\text{Now } 729 = \overline{3 \times 3 \times 3} \times \overline{3 \times 3 \times 3}$$

$$\therefore \sqrt[3]{729} = 3 \times 3 = 9 \text{ is required cube root}$$

3.4.2 Perfect cubes and cube root of negative integers

$$(-1)^3 = (-1) \times (-1) \times (-1) = -1 \text{ So, } \sqrt[3]{-1} = -1$$

$$(-2)^3 = (-2) \times (-2) \times (-2) = -8 \text{ So, } \sqrt[3]{-8} = -2$$

$$(-3)^3 = (-3) \times (-3) \times (-3) = -27 \text{ So, } \sqrt[3]{-27} = -3$$

$$(-4)^3 = (-4) \times (-4) \times (-4) = -64 \text{ So, } \sqrt[3]{-64} = -4$$

Similarly if $(-x)$ is a negative integers, then

$$(-x)^3 = (-x) \times (-x) \times (-x) = -x^3$$

$$\text{So } \sqrt[3]{-x^3} = -x$$

- Show that $-1331 = (-11)^3$
- Check weather $\sqrt[3]{-1000} = -10$ is true.
- Show that cube root of -125 is -5 .

In this way we conclude

Cube root of negative perfect cube integers is a negative integers. This can be obtained by taking cube root of its absolute value and their is negative sign to it, in symbolic form

$$\sqrt[3]{-x^3} = -x$$

Example 6. Find cubt root of -2197

Solution : We observe that

$$2197 = \overline{13 \times 13 \times 13}$$

$$\therefore \sqrt[3]{2197} = 13$$

$$\therefore \sqrt[3]{-2197} = -\sqrt[3]{2197} = -13$$

3.4.3 Cube root of product of two perfect cubes.

Example 7. Find value of each of the following and observe the answer carefully.

(a) $\sqrt[3]{27 \times 64}$

(b) $\sqrt[3]{27} \times \sqrt[3]{64}$

Solution : (a) $\sqrt[3]{27 \times 64}$

$$\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3}$$

$$\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4}$$

$$\sqrt[3]{27 \times 64} = \sqrt[3]{3 \times 3 \times 3 \times 4 \times 4 \times 4}$$

$$\begin{aligned}\sqrt[3]{27} \times \sqrt[3]{64} &= 3 \times 4 \\ &= 12\end{aligned}$$

(b) $\sqrt[3]{27} = 3$ and $\sqrt[3]{64} = 4$

$$\sqrt[3]{27 \times 64} = 3 \times 4 = 12$$

In both the cases (a) and (b) answers are same.

Similary, $8 = \overline{2 \times 2 \times 2} = 2^3$, So, $\sqrt[3]{8} = 2$

$$27 = \overline{3 \times 3 \times 3} = 3^3, \text{ so } \sqrt[3]{27} = 3$$

$$\text{Now } 8 \times 27 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$\begin{aligned}\text{Therefore } \sqrt[3]{8 \times 27} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} \\ &= 2 \times 3 = 6\end{aligned}$$

$$\text{Therefore } \sqrt[3]{8 \times 27} = 6 = 2 \times 3 = \sqrt[3]{8} \times \sqrt[3]{27}$$

$$\text{Means } \sqrt[3]{8 \times 27} = \sqrt[3]{8} \times \sqrt[3]{27}$$

See again :

Example 8. Find values of $\sqrt[3]{-64 \times 729}$ and $\sqrt[3]{-64} \times \sqrt[3]{729}$ and compare the results.

$$\text{Solution : } -64 = -(2 \times 2 \times 2 \times 2 \times 2 \times 2) \text{ so } \sqrt[3]{-64} = -(2 \times 2) = -4$$

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \text{ so } \sqrt[3]{729} = 3 \times 3 = 9$$

$$\begin{aligned}\text{Now, } \sqrt[3]{(-64) \times 729} &= -\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3} \\ &= -2 \times 2 \times 3 \times 3 \\ &= -36 \\ &= -4 \times 9\end{aligned}$$

$$\begin{aligned}\text{Now, } \sqrt[3]{-64} \times \sqrt[3]{729} &= -4 \times 9 \\ &= -36\end{aligned}$$

$$\text{Hence } \sqrt[3]{-64 \times 729} = \sqrt[3]{-64} \times \sqrt[3]{729}$$

Try

$$(i) \sqrt[3]{125 \times 216} = \sqrt[3]{125} \times \sqrt[3]{216}$$

$$(ii) \sqrt[3]{343 \times 512} = \sqrt[3]{343} \times \sqrt[3]{512}$$

In this way we conclude

If a and b are two positive perfect cubes then,

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

Example 9. Find cube root of $(-125) \times (-2744)$

Solution : $\sqrt[3]{-125} = -\sqrt[3]{125} = -\sqrt[3]{5 \times 5 \times 5} = -5$

$$\begin{aligned}\text{And } \sqrt[3]{-2744} &= -\sqrt[3]{2744} = -\sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} \\ &= -(2 \times 7) = -14\end{aligned}$$

$$\begin{aligned}\text{So } \sqrt[3]{(-125) \times (-2744)} &= -\sqrt[3]{-125} \times \sqrt[3]{-2744} \\ &= -5 \times -14 \\ &= 70\end{aligned}$$

Exercise 3(b)

1. Among cube roots of 8 and -8 which one is greater ?
2. What smallest natural number should be multiplied to 32 so that product is a perfect cube number ?
3. Which of the following number are perfect cubes ?
(i) 432 (ii) 729 (iii) 13824 (iv) 42875 (v) 4608
(vi) 1125 (vii) 10976 (viii) 5832
4. Find cube roots of the following numbers
(i) 2744 (ii) 74088 (iii) 7408800 (iv) 134217728
(v) 2197 (vi) -0648 (vii) -64000 (viii) -17576
5. By what smallest number 2096 is to be divided so that quotient is a perfect cube ?
6. What is the smallest number that if divides 281216 quotient is a perfect cube
7. What smallest number if multiplied to 9000 results a perfect cube number ?
8. What smallest number multiplied to 83349 so that the product is a perfect cube ? Find cube root of the new number.
9. Find cube root of $(-15625) \times 512$.
10. Find cube root of $1331 \times (-1728)$
11. Value of $\sqrt[3]{125 \times 343}$ is—
(i) 35 (ii) 45 (iii) 75 (iv) 105

3.4.4 Cube root of a rational number whose numerator and denominator are perfect cubes

Look,

$$\frac{343}{512} = \frac{7 \times 7 \times 7}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$

$$= \frac{7^3}{2^3 \times 2^3 \times 2^3}$$

$$= \frac{7^3}{8^3} = \left(\frac{7}{8}\right)^3$$

So, $\sqrt[3]{\frac{343}{512}} = \frac{7}{8}$

See again

$$\frac{-125}{1728} = \frac{(-5) \times (-5) \times (-5)}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$$

$$= \frac{(-5)^3}{2^3 \times 2^3 \times 3^3}$$

$$= \frac{(-5)^3}{(2 \times 2 \times 3)^3}$$

$$= \frac{(-5)^3}{(12)^3} = \left(\frac{-5}{12}\right)^3$$

So, $\sqrt[3]{\frac{-125}{1728}} = \frac{-5}{12}$

Look at the following and discuss

By factorizing numerator and denominators into prime factors., find whether following numbers are perfect cube or not; and then find their cube root.

(i) $\frac{-125}{1331}$ (ii) $\frac{64}{125}$ (iii) $\frac{216}{343}$

See,

$$\frac{27}{125} = \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \left(\frac{3}{5}\right)^3$$

So $\sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

Again $27 = 3 \times 3 \times 3 = 3^3$

So $\sqrt[3]{27} = 3$

In the same way $125 = 5 \times 5 \times 5 = 5^3$

So $\sqrt[3]{125} = 5$

So $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$

Try

Verify that

(i) $\sqrt[3]{\frac{512}{1000}} = \frac{\sqrt[3]{512}}{\sqrt[3]{1000}}$

(ii) $\sqrt[3]{\frac{-729}{1331}} = \frac{\sqrt[3]{-729}}{\sqrt[3]{1331}}$

Conclusion :

If $\frac{a}{b}$ is a rational number, where “a” and “b” are coprime and perfect cubes, and $b \neq 0$ then $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

3.4.5 Cube root of a decimal number which is a perfect cube (by factorization method)

See,

● $(0.6)^3 = \left(\frac{6}{10}\right)^3 = \frac{6}{10} \times \frac{6}{10} \times \frac{6}{10} = \frac{6 \times 6 \times 6}{10 \times 10 \times 10} = \frac{216}{1000} = 0.216$

Means : $(0.6)^3 = 0.216$

So $\sqrt[3]{0.216} = 0.6$

● Similarly,

$(1.2)^3 = \left(\frac{12}{10}\right)^3 = \frac{12}{10} \times \frac{12}{10} \times \frac{12}{10} = \frac{1728}{1000}$

So $\sqrt[3]{1.728} = 1.2$

We know that decimal numbers can be converted into simple fractions, so if we have to find cube root of the number 0.216, then we can have it by following method.

$$\begin{aligned}\sqrt[3]{0.216} &= \sqrt[3]{\frac{216}{1000}} = \frac{\sqrt[3]{216}}{\sqrt[3]{1000}} \\ &= \frac{\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}}{\sqrt[3]{10 \times 10 \times 10}} = \frac{2 \times 3}{10}\end{aligned}$$

So $\frac{6}{10} = 0.6$

Similarly,

$$\begin{aligned}\sqrt[3]{1.728} &= \sqrt[3]{\frac{1728}{1000}} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000}} \\ &= \frac{\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}}{\sqrt[3]{10 \times 10 \times 10}} \\ &= \frac{2 \times 2 \times 3}{10} = \frac{12}{10} = 1.2\end{aligned}$$

So $\sqrt[3]{1.728} = 1.2$

Try

- Show that $\sqrt[3]{0.027} = 0.3$
- Find cube root of 0.008

Example 10. Find cube root of $29\frac{719}{1000}$

Solution : $29\frac{719}{1000} = \frac{29719}{1000} = \frac{31 \times 31 \times 31}{10 \times 10 \times 10}$

$$\begin{aligned}\text{So, } \sqrt[3]{29\frac{719}{1000}} &= \frac{\sqrt[3]{31 \times 31 \times 31}}{\sqrt[3]{10 \times 10 \times 10}} \\ &= \frac{31}{10} \text{ or } 3.1\end{aligned}$$

We find cube root of mixed fraction by converting it into simple fraction.

Example 11. Find cube root of 15.625

Solution : $15.625 = \frac{15625}{1000}$

$$\begin{aligned}\text{So } \sqrt[3]{15.625} &= \sqrt[3]{\frac{15625}{1000}} = \frac{\sqrt[3]{5 \times 5 \times 5 \times 5 \times 5 \times 5}}{\sqrt[3]{10 \times 10 \times 10}} \\ &= \frac{5 \times 5}{10} = \frac{25}{10} = 2.5\end{aligned}$$

3.5 Relation between side and volume of a cube :

We know that **cube is that specific cuboid whose length, breadth and height are equal to each other**. Means edges of cube or sides are equal in size.

We also know that if side of a cube is x and volume is V , then

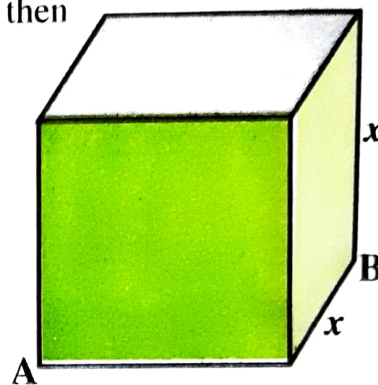
$$V = x \times x \times x = x^3$$

Means volume of a cube is equal to the cube of its side. In other words, side of the cube is equal to cube root of its volume.

Similarly $x = \sqrt[3]{V}$

Or $x = \sqrt[3]{V}$

Meaning



$$\text{Volume of the cube} = (\text{Side of the cube})^3$$

$$\text{Side of the cube} = \sqrt[3]{\text{Volume of the cube}}$$

Discuss in the group :

- (1) Find cube root of $\frac{1}{27}$.
- (2) What will be cube root of $\frac{64}{125}$?
- (3) What will be cube root of 0.001?
- (4) Find cube root of 0.027.

Exercise 3(c)

1. Find cube root of the following :

(i) $\frac{729}{1331}$ (ii) $\frac{512}{3375}$ (iii) $\frac{1728}{15625}$ (iv) $5\frac{23}{64}$

(v) $1\frac{91}{125}$ (vi) $22\frac{161}{216}$

2. Find cube root of the following decimal numbers.

(i) 1.331 (ii) 42.875 (iii) 54.872 (iv) 74.088
(v) 5.832 (vi) 0.000729

3. Write correct option for answer of the question (i) to (iii).

(i) Cube root of $\frac{27}{64}$ is—

(i) $\frac{2}{3}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{8}$ (iv) $\frac{9}{16}$

(ii) Cube root of $3\frac{3}{8}$ is—

(i) 3 (ii) $\frac{3}{8}$ (iii) $1\frac{1}{2}$ (iv) $\frac{3}{4}$

(iii) Cube root of 0.000008 is—

(i) 0.2 (ii) 0.02 (iii) 0.002 (iv) 0.004

4. Volume of a cuboidal compost pit is 13.824 m^3 . Find depth of the pit.

5. Volume of a cubical box is 8000 cm^3 . Find length of its one sides.

3.6 Application based question of cube roots

Construction of buildings, packing of goods, construction of water reservoir, decoration etc. are different daily life problems where cube root calculation can be a very useful tool in find their solutions.

These things are classified by the examples given below.

Example 12. A metallic cuboidal solid with dimensions $10 \text{ cm} \times 4 \text{ cm} \times 1.6 \text{ cm}$. A cubical block is formed by melting the cuboidal block. Find the length of side of this new block formed ?

Solution : Volume of cuboidal block = Length $\times$ Width $\times$ Height
 $= 10 \text{ cm} \times 4 \text{ cm} \times 1.6 \text{ cm}$
 $= 64 \text{ cm}^3$

Let side of new cubical block = $x \text{ cm}$. then

Its volume = $x \times x \times x \text{ cm}^3$

So according to question

$$x^3 = 64$$

$$\therefore x = \sqrt[3]{64} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 4$$

So length of side of new cubical block = 4 cm .

Example 13. A fruit exporter packs the apple in cubical boxes in such a way that the number of apples in each row is same as that in the columns (vertical lines) or (rows) is also same as number of layers in the cubical box. If one cubical packet has 3375 apples then find the number of apples kept in each row ?

Solution : Since every row has as much number of apples as the number of line (columns) and also same to the number of layers.

So if there are x apples in a row,

Then number of apples in the cubical box

$$= x \times x \times x = x^3$$

According to question 3375 apples are kept in the box.

$$\text{So } x^3 = 3375$$

$$\text{therefore } x = \sqrt[3]{3375}$$

$$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$$

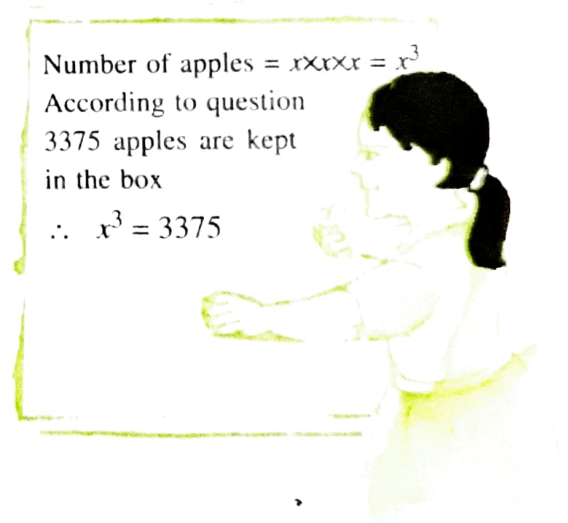
$$= 3 \times 5 = 15$$

So each line or row have 15 apples.

$$\text{Number of apples} = x \times x \times x = x^3$$

According to question
3375 apples are kept
in the box

$$\therefore x^3 = 3375$$



Special :

There are certain numbers that can be written as the sum of two perfect cubes. For example 1729 is smallest number that can be written as the sum of two perfect cube numbers in two ways only. This number is called Hardy Ramanujan number.

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

Hardy Ramanujan's number 1729

Once Prof. G.H. Hardy went to meet Ramanujan. At that time Ramanujan was admitted in a hospital for the treatment of his illness. While talking Hardy said to Ramanujan the number of the taxi by which I came here, is not a lucky number. Ramanujan replied instantly no, it is an interesting number. He told that it is the smallest number which can be expressed as the sum of two perfect cube numbers in two ways. From then 1729 is known as Hardy Ramanujan's number. In this way we can estimate the intellectual intensity of Ramanujan that he had discovered this fact at Matric level.

Exercise 3(d)

1. In a cubical box apples are kept in such a way that in every layer each row (line) contains as many apples as the number lines in the layer. If number of rows (lines) equal to the number of layers and 1728 apples are kept in the box; How many apples are kept in each row ?
2. In a box 216 mangoes are arranged in the form of a cube. Find how many layers of mangoes are there in each row ?
3. In a box; chalk boxes are put in the form a cube. Number of chalk boxes is 2744. Find how many boxes are there in one layer of chalk boxes.
4. In a show room colored cubical boxes are placed adjacently so that it forms a larger cube. If number of cubes so placed are 512 in number then find out number of layers in large cubical shape so formed.
5. Every student of a class in a school donates money equal to cube of the number of student in the class for Prime Minister Relief Fund. In this way total money contributed is ₹15625. Find how many students are there in the class.
6. A cubical tank contains 1331 litre of water. Find size of the tank if 1000 litre of water is equal to 1 cubic metre.

Efficiency practice 3

1. Find value of x in the following equation :
$$10^3 + x^3 = 1^3 + 12^3$$
2. Find cube root of the following—
 - (i) 250047
 - (ii) 110592
 - (iii) 1404928
 - (iv) 1771561

3. Find cube root of the following :

(i) $3\frac{920}{1331}$

(ii) $1\frac{13084}{29791}$

(iii) $3\frac{2042}{3375}$

(iv) $132\frac{651}{1000}$

4. Find cube root of the following—

(i) 226.981

(ii) 0.373248

(iii) 0.0008847736

(iv) 0.000001157625

5. What will be the edge of a solid metallic cube to which we can form three cubes of sizes 3 cm, 4 cm and 5 cm.
6. Edges of two solid metallic cubes are 9 cm and 10 cm, respectively. Both of these melted and a volume of 1 cm^3 is taken out from their combined volume. A new solid cube is formed by the remaining volume. Find edge of this new cube ?
7. Find the smallest integer, which is divided, 963144 quotient becomes a perfect cube ?
8. What smallest natural number should be multiplied to 26244 so that product is a perfect cube ?
9. By prime factorization find whether 2097152 is a perfect cube ?
10. Find edge of a cubical gas chamber whose volume is 6028.568 cm^3 .
11. Dimensions of a cuboid is $98\text{ cm} \times 42\text{ cm} \times 18\text{ cm}$. Find side of the cube equal to the volume of said cuboid.
12. In a box 42875 glass balls are kept in the form of a cube. How many layers of glass balls are in the box.

Select correct option and write in the question 13 to 19.

13. A cube has volume 6859 cm^3 . Its edge will be

(i) 13 cm

(ii) 15 cm

(iii) 17 cm

(iv) 19 cm

14. $(-343 \times 512)^{1/3}$ has value—

(i) -56

(ii) -42

(iii) -84

(iv) 56

15. What is the smallest perfect cube number divisible by 12, 18 and 25.

(i) 1200

(ii) 1800

(iii) 2700

(iv) 27000

16. By what smallest number 10000 be multiplied so that product becomes a perfect cube; among the following is—

(i) 10

(ii) 25

(iii) 100

(iv) 800

17. By what smallest number 8192 be divided so that quotient is perfect cube, among the following is—
- (i) 2 (ii) 4
(iii) 16 (iv) 32
18. Value of $\left(\sqrt{27} - \frac{9}{\sqrt{3}} - \sqrt{\frac{1}{9}} + \sqrt[3]{\frac{1}{27}}\right)$ is
- (i) $\sqrt{3}$ (ii) $2\sqrt{3}$
(iii) 0 (iv) $3\sqrt{3}$
19. If $2^n - 2^{n-1} = 4$ then n^n is equal to—
- (i) 1 (ii) $\frac{3}{2}$
(iii) 2 (iv) 27
20. A cubical box has interior volume 32.768 m^3 . Find length of its internal side.
21. In a school a cubical dustbin is kept in each classroom, whose volume is 27000 cubic centimeter. Find side of the dustbin.
22. In the corner of a garden a cubical pit is dug where their leaves and garbage is dump to form compost, while digging the soil its volume increases by 10% which is taken out of the garden with the help of Tractor trolley making 5 trips fully loaded to its capacity. If measurement of trolley is 2.75 metre $\times$ 2 metre $\times$ 0.625 metre then find depth of the pit dug out ?

We have learnt :

1. Cube root of a number is that number whose cube gives the given number.
2. While finding cube root of a number, prime factorization of the number is grouped in triplets with same prime factors, again we select one factor corresponding to each triplet and multiply then this product is cube root of the given number.
3. We make use of cube roots find solutions of many problems in our daily life.
4. Cube root of numbers are both positive and negative.
5. Cube root of particular number cannot be positive and negative simultaneously.

Answers

Exercise 3(a)

1. 343; 1728; 4913; 6859, 9261; 1000000; 2. (iv) $-\frac{125}{729}$ 3. 64, 216, 1728

Exercise 3(b)

1. cube root of 8; 2. 2, 3. (ii) 729 (iii) 13824 (iv) 42875 (viii) 5832, 4. (i) 14 (ii) 42 (iii) 420 (iv) 512 (v) -13 (vi) -22 (vii) -40 (viii) -26 5. 262 6. 2, 7.3 8. 3, 63 9 - 200 10. -132 11. (i) 35

Exercise 3(c)

1. (i) $\frac{9}{11}$ (ii) $\frac{8}{15}$ (iii) $\frac{12}{25}$ (iv) $1\frac{3}{4}$ (v) $1\frac{1}{5}$ (vi) $2\frac{5}{6}$ 2. (i) 1.1 (ii) 3.5 (iii) 3.8 (iv) 4.2 (v) 1.8 (vi) 0.09 3. (ii) $\frac{3}{4}$ 4. (iii) $1\frac{1}{2}$ 5. (ii) 0.020, 6. 2.4 metre 7. 20 cm.

Exercise 3(d)

1. 144 apples 2. 6 layers 3. 196 boxes 48 layers 5. 25 student 6. side of cubical tankies 1.1 metre.

Efficiency Practice 3

1. 9 2. (i) 63 (ii) 48 (iii) 112 (iv) 121; 3. (i) $1\frac{6}{11}$ (ii) $1\frac{4}{31}$ (iii) $1\frac{8}{15}$ (iv) $5\frac{1}{10}$ 4. (i) 61. (ii) 0.72 (iii) 0.096, (iv) 0.0150 5. 6 cm. 6. 12 cm. 7. 13 8. 6m 9. yes 128 cubes 10. 18.2 cm. 11. 42 cm., 12. 35 layers, 13. (iv) 19 cm, 14. (i) -56 15. (iv) 27000, 16. (iii) 100, 17. (i) 2 18. (iii) 0 19. (iv) 27 20. 3.2 metre 21. 30 cm 22. 2.5 m.

