



- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- Geometrical verification of the identities
- Application of identities

4.1 Introduction

In previous classes we have learned that an algebraic expression which is true for every value of the variable is called an identity. We have studied the binomial and quadratic identities and their application e.g. $(a + b)^2 = a^2 + 2ab + b^2$, $(a - b)^2 = a^2 - 2ab + b^2$, $a^2 - b^2 = (a + b)(a - b)$, in previous classes in this chapter. We will study some binomial which are cubic and some trinomial which are quadratic identities and their application.

4.2 Perception of identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

We know by expand method

$$\begin{aligned}
 (a + b)^3 &= (a + b)(a + b)^2 \\
 &= (a + b)(a^2 + 2ab + b^2) \\
 &= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \\
 &= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3 \\
 &= a^3 + (2a^2b + a^2b) + (ab^2 + 2ab^2) + b^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 &= a^3 + b^3 + 3a^2b + 3ab^2 \\
 &= a^3 + b^3 + 3ab(a + b) \\
 \text{so } (a + b)^3 &= a^3 + b^3 + 3ab(a + b)
 \end{aligned}$$

Numerical Verification

Let $a = 2$, $b = 3$ so

$$\text{L.H.S.} = (a + b)^3 = (2 + 3)^3 = 125$$

$$\text{R.H.S.} = a^3 + b^3 + 3ab(a + b) = 2^3 + 3^3 + 3 \times 2 \times 3(2 + 3)$$

$$= 8 + 27 + 3 \times 2 \times 3(2 + 3)$$

$$= 8 + 27 + 90$$

$$= 125$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Find the identity by multiplication :

$\begin{array}{r} (a + b) \\ \times (a + b) \\ \hline a^2 + ab + ba + b^2 \\ = a^2 + 2ab + b^2 \end{array}$ Again $\begin{array}{r} (a^2 + 2ab + b^2) \\ \times (a + b) \\ \hline a^3 + 2a^2b + ab^2 \\ a^2b + 2ab^2 + b^3 \\ \hline a^3 + 3a^2b + 3ab^2 + b^3 \end{array}$	$\begin{array}{l} (a + b)(a + b)(a + b) \\ = (a + b)(a + b)^2 \\ = (a + b)(a^2 + ab + b^2) \\ = a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \\ = a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3 \\ = a^3 + 3a^2b + 3ab^2 + b^3 \end{array}$ OR $= a^3 + 3ab(a + b) + b^3$
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Geometrical verification of Identity $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 + b^3$

Let a cube be of side a (it will be termed as cube of $a \times a \times a$). Let another cube be of side b (it will be termed as cube of $b \times b \times b$).

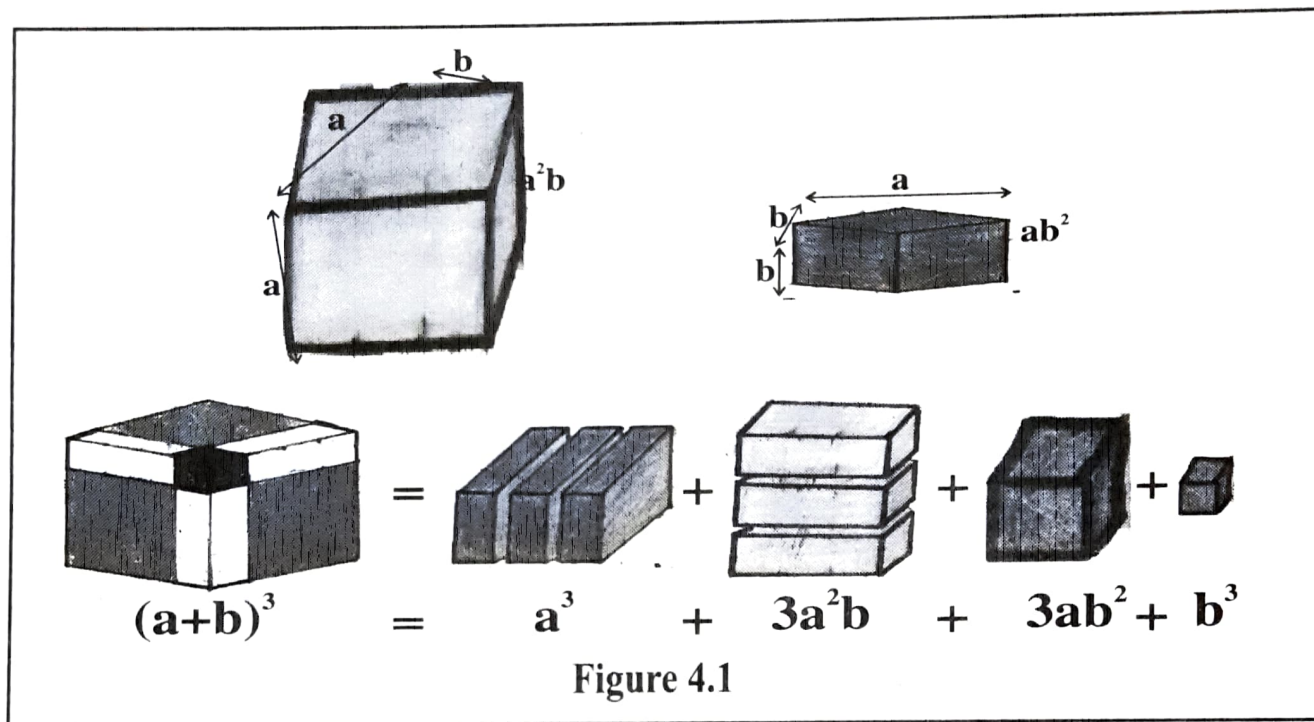
Now consider 3 cuboids with length a width a and height b (termed as cuboid of $a \times a \times b$). Consider another 3 cuboids with length b width b and height a (termed as cuboid of $a \times b \times b$).

Among these place the cube of $a \times a \times a$. Two cuboids of $a \times a \times b$, one cuboid of $a \times b \times b$ as shown in figure(4.1). The base should be as per figure and height should be a . Clearly we found a square of side $(a + b)$ is formed by the base of these 4 shapes, that we made. It is like a cuboid with sides of $(a + b)$, $(a + b)$ and a .

Now, take the left 4 block (one cuboid of $a \times a \times b$ two cuboid of $a \times b \times b$ and (one cube of $b \times b \times b$) and are cube of $b \times b \times b$) and arrange them as per figure. The base should be as per figure and height should be b . Arrange these shapes on bottom part and there is no

gap in between. It will form upper part of solid body that we make. It is cuboid of sides $a + b$, $a + b$ and b . It is clear that the base of this cuboid cover completely the upper surface of bottom part. You know that the height of bottom portion cuboid is a and height of upper part of cuboid is b , so the total height of solid is $(a + b)$ as shown in figure. So we have a cube of cuboid is b , so the total height of solid is $(a + b)$ as shown in figure. So we have a cube with side $(a + b)$ it consist 8 shape that we use. The volume of solid cube with side $(a + b)$ will be equal to the sum of volume of that 8 shape used by us.

$$\text{So, } (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$



Cube of a^3 cube of b^3 , three a^2b cuboid, three ab^2 cuboid.

Put them such that we get cube of $(a + b)$

Try it

Similary for $a = 1, b = 1$

Verify $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

We see that above algebraic relation is true for every value of a and b .

So it is an identity.

Example 1. Using the identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Expand $(y + 4)^3$

Answer Comparing $(y + 4)^3$ with $(a + b)^3$ we get

$a = y$ and $b = 4$

By replacing a and b in the identity

$$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$$

$$(y + 4)^3 = y^3 + 4^3 + 3 \times y \times 4 (y + 4)$$

$$= y^3 + 64 + 12y (y + 4)$$

$$= y^3 + 64 + 12y^2 + 48y$$

$$= y^3 + 12y^2 + 48y + 64$$

Example 2. Expand $(x + 5y)^3$

Answer : $(x + 5y)^3$ with comparing $(a + b)^3$. We get that

$a = x$ and $b = 5y$ so using of identity

$$(a + b)^3 = a^3 + b^3 + 3ab (a + b) \text{ we get}$$

$$(x + 5y)^3 = x^3 + (5y)^3 + 3x (5y) (x + 5y)$$

$$= x^3 + 125y^3 + 15xy (x + 5y)$$

$$= x^3 + 125y^3 + 15x^2y + 75xy^2$$

$$= x^3 + 15x^2y + 75xy^2 + 125y^3$$

Example 3. If $x + \frac{1}{x} = \frac{5}{2}$, find the value of $x^3 + \frac{1}{x^3}$

Answer : $= (a + b)^3 = a^3 + b^3 + 3ab (a + b)$

$$\therefore \left(x + \frac{1}{x}\right)^3 = x^3 + \left(\frac{1}{x}\right)^3 + 3x \times \frac{1}{x} \left(x + \frac{1}{x}\right)$$

$$= x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$\text{Replacing } x + \frac{1}{x} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times \frac{5}{2}$$

$$\text{or } \frac{125}{8} = x^3 + \frac{1}{x^3} + \frac{15}{2}$$

$$\frac{125}{8} - \frac{15}{2} = x^3 + \frac{1}{x^3}$$

$$x^3 + \frac{1}{x^3} = \frac{65}{8}$$

Example 4. $a + b = 5$ and $ab = 6$ then find the value of $a^3 + b^3$

Answer : Identify $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Replacing $a + b = 5, ab = 6$

$$5^3 = a^3 + b^3 + 3 \times 6 \times 5$$

$$125 = a^3 + b^3 + 90$$

or $125 - 90 = a^3 + b^3$ (Transposing 90)

so $a^3 + b^3 = 35$

Try it

Expand the following :

(i) $(1 + x)^3$

(ii) $(y + 2)^3$

(iii) When $x + \frac{1}{x} = 2$ find the value of $x^3 + \frac{1}{x^3}$

4.2.1. Application of Identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Example 5. Find value of $(401)^3$

Answer : $400 + 1 = 401$

$$\begin{aligned}\therefore (401)^3 &= (400 + 1)^3 \\ &= \{(4 \times 10^2) + 1\}^3 \\ &= (4 \times 10^2)^3 + (1)^3 + 3 \times (4 \times 10^2) \times 1 \{(4 \times 10^2) + 1\} \\ &= 64 \times 10^6 + 1 + 1200(400 + 1) \\ &= 64 \times 10^6 + 1 + 480000 + 1200 \\ &= 64000000 + 1 + 480000 + 1200 \\ &= 64481201\end{aligned}$$

Try it

Find the value of following with help of identity as given above :

(i) $(101)^3$

(ii) $(201)^3$

(iii) $(302)^3$

4.2.2 To derive other forms of the identity

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Subtracting $3ab(a + b)$ from both side

$$(a + b)^3 - 3ab(a + b) = a^3 + b^3 + 3ab(a + b) - 3ab(a + b)$$

$$\text{or } (a + b)^3 - 3ab(a + b) = a^3 + b^3$$

$$\text{or } a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

$$= (a + b) \{(a + b)^2 - 3ab\} \text{ (by taking common)}$$

$$= (a + b) \{a^2 + b^2 + 2ab - 3ab\}$$

$$= (a + b) (a^2 + b^2 - ab)$$

$$= (a + b) (a^2 - ab + b^2)$$

Thus

$$\begin{aligned} a^3 + b^3 &= (a + b)^3 - 3ab(a + b) \\ &= (a + b)(a^2 - ab + b^2) \end{aligned}$$

Example 6. Prove that $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

Answer : By Identity $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$$(x + 2)^3 = x^3 + 2^3 + 3 \times x \times 2(x + 2)$$

$$\text{or } (x + 2)^3 = x^3 + 8 + 6x(x + 2)$$

$$\text{or } (x + 2)^3 - 6x(x + 2) = x^3 + 8$$

$$\text{or } x^3 + 8 = (x + 2)^3 - 6x(x + 2)$$

$$= (x + 2) \{(x + 2)^2 - 6x\}$$

$$= (x + 2) (x^2 + 4x + 4 - 6x)$$

$$= (x + 2) (x^2 - 2x + 4)$$

Try it

Prove the following :

$$(a) \quad 27 + y^3 = (3 + y)(9 - 3y + y^2)$$

$$(b) \quad x^3 + 64 = (x + 4)(x^2 - 4x + 16)$$

Exercise 4(a)

1. Expand the following :

(a) $(b + 1)^3$

(b) $(c + 3)^3$

(c) $(2x + 3)^3$

(d) $(x^2 + y)^2$

(e) $(5 + 3y)^3$

(f) $(xy + 2a)^3$

2. Expand the following :

(a) $\left(\frac{a}{2} + \frac{b}{5}\right)^3$

(b) $\left(3y + \frac{1}{4y}\right)^3$

3. Find the value of the following by using identity-

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(a) $(31)^3$

(b) $(102)^3$

(c) $(201)^3$

4. Find the value of $x^3 + \frac{1}{x^3}$ if the value of $x + \frac{1}{x}$ is $\frac{10}{3}$

5. If $2x + \frac{1}{2x} = \frac{5}{2}$ then find value of $8x^3 + \frac{1}{8x^3}$

6. If $a + b = 3$ and $ab = 2$ then find value of $a^3 + b^3$.

7. If $3x + 2y = 20$ and $xy = \frac{14}{9}$ then find the value of $27x^3 + 8y^3$.

8. Show that :

(i) $125 + x^3 = (5 + x)^3 - 15x(5 + x)$

(ii) $8a^3 + 27b^3 = (2a + 3b)^3 - 18ab(2a + 3b)$

4.3 Concept of the identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Look it $(a - b)^3 = (a - b)(a - b)^2$

$$= (a - b)(a^2 + b^2 - 2ab)$$

$$= a(a^2 + b^2 - 2ab) - b(a^2 + b^2 - 2ab)$$

$$= a^3 + ab^2 - 2a^2b - ba^2 - b^3 + 2ab^2$$

$$= a^3 - b^3 + (ab^2 + 2ab^2) - (2a^2b + a^2b)$$

$$= a^3 - b^3 + 3ab^2 - 3a^2b$$

$$= a^3 - b^3 - 3ab(a - b)$$

We can also get this identity by following method :

$$a - b = a + (-b)$$

$$\therefore (a - b)^3 = \{a + (-b)\}^3$$

Now compare,

$(a + b)^3$ and $\{a + (-b)\}^3$. Here $b = -b$ in this equation,

$$\begin{aligned}\therefore \{a + (-b)\}^3 &= (a)^3 + (-b)^3 + 3a(-b)\{a + (-b)\} \\ &= a^3 - b^3 - 3ab(a - b)\end{aligned}$$

$$\text{thus } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Numerical verification

Let $a = 5$, $b = 3$ then

$$\text{L.H.S. } (a - b)^3 = (5 - 3)^3 = (2)^3 = 8$$

$$\begin{aligned}\text{R.H.S. } &a^3 - b^3 - 3ab(a - b) \\ &= 125 - 27 - 3 \times 5 \times 3(5 - 3) \\ &= 98 - 90 \\ &= 8\end{aligned}$$

$$\text{L.H.S. } = \text{R.H.S.}$$

Try it

If $a = 4$, $b = 1$ then verify $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

We see that for all values of a and b the algebraic relation $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ is true—

So it is an identity.

It is observed from above :

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
so, Cube of Difference two terms = (First term)³ - (Second term)³
-3 First term $\times$ Second term (First term - Second term)

Example 7. Expand $(a - 1)^3$

Solution : $(a - 1)$ composing with $(a - b)$ then
 $a = a$ and $b = 1$

$$\begin{aligned}
 (a - b)^3 &= a^3 - b^3 - 3ab(a - b) \\
 \therefore (a - 1)^3 &= a^3 - 1^3 - 3a \cdot 1(a - 1) \\
 &= a^3 - 1 - 3a(a - 1) \\
 &= a^3 - 1 - 3a^2 + 3a \\
 &= a^3 - 3a^2 + 3a - 1
 \end{aligned}$$

Example 8. Expand $(5x - 3y)^3$

Solution : $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ in replacing $a = 5x$, $b = 3y$

$$\begin{aligned}
 (5x - 3y)^3 &= (5x)^3 - (3y)^3 - 3(5x) \times 3y(5x - 3y) \\
 &= (5x)^3 - (3y)^3 - 45xy(5x - 3y) \\
 &= 125x^3 - 27y^3 - 225x^2y + 135xy^2 \\
 &= 125x^3 - 225x^2y + 135xy^2 - 27y^3
 \end{aligned}$$

Example 9. If $x - \frac{1}{x} = \frac{3}{2}$, then find the value of $x^3 - \frac{1}{x^3}$

Solution : $\therefore (a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$\therefore \left(x - \frac{1}{x}\right)^3 = x^3 - \left(\frac{1}{x}\right)^3 - 3x \times \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$\text{Replacing } \left(x - \frac{1}{x}\right) = \frac{3}{2}$$

$$\left(\frac{3}{2}\right)^3 = x^3 - \frac{1}{x^3} - 3 \times \frac{3}{2}$$

$$\text{or } \frac{27}{8} = x^3 - \frac{1}{x^3} - \frac{9}{2}$$

$$\text{or } \frac{27}{8} + \frac{9}{2} = x^3 - \frac{1}{x^3}$$

$$\text{or } x^3 - \frac{1}{x^3} = \frac{63}{8}$$

Example 10. If $a - b = 4$ and $ab = 5$ so find the value of $a^3 - b^3$.

Solution : $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Replacing $a - b = 4$ and $ab = 5$
 $(4)^3 = a^3 - b^3 - 3 \times 5 \times 4$
 or $64 = a^3 - b^3 - 60$
 or $a^3 - b^3 = 64 + 60 = 124$

Try it

Express the following :

(i) $(1 - x)^3$ (ii) $(y - 3)^3$

(iii) If $x - \frac{1}{x} = \frac{8}{3}$ so find value of $x^3 - \frac{1}{x^3}$

4.3.1 Other forms of identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\therefore (a - b)^3 = a^3 - b^3 - 3ab(a - b)$
 Adding $3ab(a - b)$ on both sides we get
 $(a - b)^3 + 3ab(a - b) = a^3 - b^3 - 3ab(a - b) + 3ab(a - b)$
 $(a - b)^3 + 3ab(a - b) = a^3 - b^3$
 $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

Try it

Prove by above method :

(a) $1 - 8y^3 = (1 - 2y)^3 + 6y(1 - 2y)$

(b) $27 - z^3 = (3 - z)^3 + 9z(3 - z)$

Again, $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
 $= (a - b) \{(a - b)^2 + 3ab\} = (a - b) \{(a^2 + b^2 - 2ab + 3ab)\}$
 $= (a - b)(a^2 + ab + b^2)$

Try it

Prove as above mentioned

(a) $64 - x^3 = (4 - x)(16 + 4x + x^2)$

(b) $1 - 27y^3 = (1 - 3y)(1 + 3y + 9y^2)$

Like this

$$\begin{aligned} a^3 - b^3 &= (a - b)^3 + 3ab(a - b) \\ &= (a - b)(a^2 + ab + b^2) \end{aligned}$$

4.3.2 Application of $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Example 11. Using identity, find the value of $(99)^3$

Solution : $99 = (100 - 1)$

$$\therefore (99)^3 = (100 - 1)^3$$

In identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ replacing $a = 100$, $b = 1$

$$\therefore (100 - 1)^3 = (100)^3 - 1^3 - 3 \times 100 \times 1 \times (100 - 1)$$

$$= 1000000 - 1 - 300 \times 99$$

$$= 999999 - 29700$$

$$99^3 = 970299$$

Try it

Find the value of the following by above method

(i) $(999)^3$

(ii) $(98)^3$

(iii) $(499)^3$

There are two methods to find product of identities :

(a) Multiplication method

$$\begin{array}{r} (a + b + c)^2 \\ a + b + c \\ \times a + b + c \\ \hline ac + bc + c^2 \\ ab + b^2 + bc \\ a^2 + ab + ac \\ \hline a^2 + b^2 + c^2 + 2ab + 2bc + 2ac \end{array}$$

(b) Expand method

$$\begin{aligned} (a + b + c)^2 \\ (a + b + c)(a + b + c) &= a(a + b + c) \\ &+ b(a + b + c) + c(a + b + c) \\ &= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2 \\ &= a^2 + b^2 + c^2 + 2bc + 2ab + 2ac \end{aligned}$$

Example 12. Simplify $\left(\frac{x}{x} + \frac{y}{5}\right)^3 - \left(\frac{x}{x} - \frac{y}{5}\right)^3$

Solution : $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 \quad \dots (i)$

and $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2 \quad \dots (ii)$

Subtract equation (ii) in equation (i)

$$(a + b)^3 - (a - b)^3 = 2b^3 + 6a^2b$$

there $a = \frac{x}{3}, b = \frac{y}{5}$

$$\begin{aligned} \text{so } \left(\frac{x}{3} + \frac{y}{5}\right)^3 - \left(\frac{x}{3} - \frac{y}{5}\right)^3 &= 2\left(\frac{y}{5}\right)^3 + 6\left(\frac{x}{3}\right)^2 \cdot \frac{y}{5} \\ &= \frac{2y^3}{125} + \frac{2x^2y}{15} \end{aligned}$$

Exercise 4(b)

1. Expand the following :

(a) $(x - 7)^3$

(b) $(2 - y)^3$

(c) $\left(3p - \frac{1}{4q}\right)^3$

(d) $\left(4 - \frac{1}{3y}\right)^3$

(e) $(7x - 3y)^3$

(f) $(8 - 2a)^3$

2. If $x - \frac{1}{x} = \frac{24}{5}$ so find value of $x^3 - \frac{1}{x^3}$

3. If $x^2 - \frac{1}{x^2} = \frac{82}{9}$ so find value of $x^3 - \frac{1}{x^3}$

4. Find the value of following by using identities :

(i) 98^3

(ii) $(9.9)^3$

(iii) $(598)^3$

5. Simplify

(i) $(x + 7)^3 - (x - 7)^3$

(ii) $(3x + 8y)^3 - (3x - 8y)^3$

(iii) $\left(\frac{y}{2} + \frac{z}{3}\right)^3 - \left(\frac{y}{2} - \frac{z}{3}\right)^3$

(iv) $(7k - 5l)^3 - (7k + 5l)^3$

6. If $x - y = 4$ and $xy = 21$ so find the value of $x^3 - y^3$.

7. If $7x - 5y = 6$ and $xy = 9$ then find the value of $343x^3 - 125y^3$

4.4 Concept of identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$a + b + c = (a + b) + c$$

$$\text{Like this } (a + b + c)^2 = \{(a + b) + c\}^2$$

$$= (a + b)^2 + 2(a + b)c + c^2$$

$$= a^2 + b^2 + 2ab + 2ac + 2bc + c^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\text{Identity (i) } (a + b)^3 + (a - b)^3 = 2a(a^2 + 3b^2)$$

$$\text{and (ii) } (a + b)^3 - (a - b)^3 = 2a(3a^2 + b^2)$$

$$\text{Verification of } (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Numerical

$$\therefore \text{ Let } a = 1, b = 2, c = 1$$

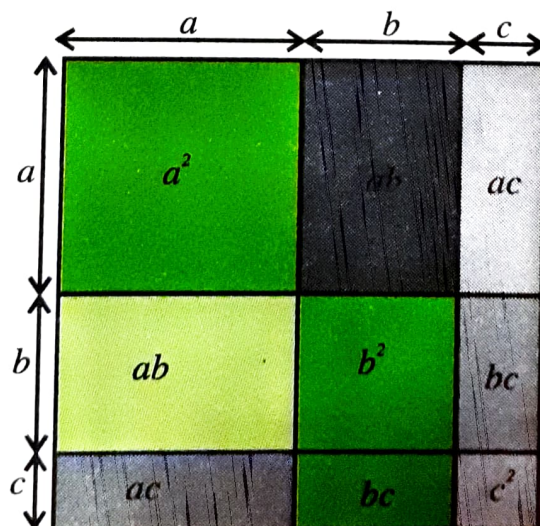
$$\begin{aligned} \text{Left Side} &= (a + b + c)^2 = (1 + 2 + 1)^2 = 4^2 \\ &= 16 \end{aligned}$$

$$\begin{aligned} \text{Right side} &= (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \\ &= 1^2 + 2^2 + 1^2 + 2 \times 1 \times 2 + 2 \times 2 \times 1 + 2 \times 1 \times 1 \\ &= 1 + 4 + 1 + 4 + 4 + 2 \\ &= 16 \end{aligned}$$

$$\therefore \text{ Left side } = \text{ Right side}$$

Geometrical representation and verification :

In the following figure there is square of side $(a + b + c)$,



It is therefore, clearly observed that

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Square of Addition of three terms = (First term)² +
(Second term)² + (Third terms)² + 2 (First term × Second term) +
2 (Second term × Third term) + 2 (Third term × First term)

4.4.1 Application of Identity

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\begin{aligned} \text{(i)} \quad (a + b + c)^2 &= [a + b + (-c)]^2 \\ &= a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2(-c)a \\ &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (a - b + c)^2 &= [a + (-b) + c]^2 \\ &= a^2 + (-b)^2 + c^2 + 2a(-b) + 2(-b)c + 2ca \\ &= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad (-a + b + c)^2 &= [(-a) + b + c]^2 \\ &= (-a)^2 + b^2 + c^2 + 2(-a)b + 2bc + 2c(-a) \\ &= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca \end{aligned}$$

Example 13. Expand the following :

$$\text{(i)} \quad (2x + 3y + 4z)^2 \quad \text{(ii)} \quad (x - 2y + 3z)^2$$

Solution :

$$\begin{aligned} \text{(i)} \quad (2x + 3y + 4z)^2 &= (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(2x)(4z) \\ &= 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16zx \\ \text{(ii)} \quad (x - 2y + 3z)^2 &= \{x + (-2y) + 3z\}^2 \\ &= x^2 + (-2y)^2 + 3(z)^2 + 2x(-2y) + 2(-2y)(3z) + 2(3z)x \\ &= x^2 + 4y^2 + 9z^2 - 4xy - 12yz + 6zx \end{aligned}$$

Example 14. If $x + y + z = 12$ and $x^2 + y^2 + z^2 = 64$,

Then find the value of $xy + yz + zx$

Solution : $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$$12^2 = 64 + 2(xy + yz + zx)$$

$$144 = 64 + 2(xy + yz + zx)$$

$$144 - 64 = 2(xy + yz + zx)$$

$$\text{or } 80 = 2(xy + yz + zx)$$

$$\text{or } 40 = xy + yz + zx$$

$$\text{or } xy + yz + zx = 40$$

Try it

(i) If $a + b + c = 9$ and $ab + bc + ca = 23$.

Find the value of $(a^2 + b^2 + c^2)$.

(ii) If $a^2 + b^2 + c^2 = 70$ and $ab + bc + ca = 37$.

Find value of $a + b + c$

Exercise 4(c)

1. Expand :

(i) $(a + 2b + 3c)^2$

(ii) $(2p - q - 3r)^2$

(iii) $(-3x + 2y + z)^2$

(iv) $(2x - 3y + 4)^2$

(v) $(m + 3n - 4p)^2$

(vi) $(7 + 4a - 3b)^2$

(vii) $(2x - 9 + 5y)^2$

(viii) $(xy + yz + x)^2$

(ix) $\left(a + \frac{b}{2} + \frac{c}{3}\right)^2$

(x) $\left(a - \frac{b}{3} + \frac{2c}{5}\right)^2$

(xi) $\left(a - \frac{b}{2} - 6\right)^2$

2. Simplify

(i) $(x + y + z)^2 + (x - y + z)^2 + (x + y - z)^2$

(ii) $(3x - 2y + 2)^2 - (3x + 2y - z)^2$

(iii) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

Group Discussion

1. Expand-

(i) $(x + 2y + 3z)^2$

(ii) $(x - 2y + 2)^2$

(iii) $(a + 2b)^3$

(i) $(2 - 3p)^3$

Efficiency Practice 4

1. Find the value of the following :

(i) 997^3

(ii) $(10.5)^3$

(iii) 501^3

(iv) $(9.5)^3$

2. Find the value :

(i) If $x - y = 4$ and $xy = 21$ then $x^3 - y^3$

(ii) if $p + q = 5$ and $pq = 6$ then $p^3 + q^3$

(iii) if $x = 3$ then $x^3 + 9x^2 + 27x + 27$

(iv) If $4x - 5z = 16$ and $xz = 12$, then $64x^3 - 125z^3$

Choose the correct option of the following questions :

3. Value of $(1 - y)^3$

(i) $(1 - y)(1 + y)^2$

(ii) $(1 - y)(1 + y^2 - y)$

(iii) $1 - y^3 - 3y(1 - y)$

(iv) $1 + y^3 + 3y(1 + y)$

4. Value of $8 + x^3$ is—

(i) $(2 + x)(4 + x^2 + 2x)$

(ii) $(2 + x)(4 + x^2 - 2x)$

(iii) $(2 + x)(4 + x^2)$

(iv) $(2 - x)(4 + x^2 + 2x)$

5. Value of $(1 - 27p^3)$ is—

(i) $(1 - 3p)(1 + 9p^2 + 3p)$

(ii) $(1 + 3p)(1 + 9p^2 + 3p)$

(iii) $(1 - 3p)(1 + 9p^2 - 3p)$

(iv) $(1 + 3p)(1 - 9p^2)$

6. Value of $(2 + x)^3$ is—

(i) $8 + x^3 + 2x(2 + x)$

(ii) $8 + x^3 + 6x(2 - x)$

(iii) $8 + x^3 + 6x(2 + x)$

(iv) $(8 + x^3 + 6x)$

7. Value of $(1 + 2x + y)^2$ is—
- $1 + 4x^2 + y^2 + 4x + 4xy + 2y$
 - $1 + 4x^2 - y^2 + 4x + 4xy + 2y$
 - $1 - 4x^2 + y^2 + 4x + 4xy + 2y$
 - $1 + 4x^2 + y^2 + 4x + 4xy - 2y$
8. Same form of identity $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ is-
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
 - $(a + b)^3 = a^3 - b^3 + 3ab(a + b)$
 - $(a + b)^3 = a^3 + b^3 - 3ab(a + b)$
 - $(a + b)^3 = a^3 + b^3 + 3ab(a - b)$
9. Value of $(a - b)^3$ is—
- $a^3 + b^3 - 3ab(a - b)$
 - $a^3 - b^3 - 3ab(a - b)$
 - $a^3 - b^3 + 3ab(a - b)$
 - $a^3 - b^3 - 3ab(b - a)$

We learned in this unit :

- Verification and Geometrical representation of Identity
 $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- Verification of the identity $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
- Verification and Geometrical representation of identity
 $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- Verification of $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$
- Verification of $(a + b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$
- Verification of $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$
- Verification of $a^3 - b^3 = (a - b)(a^2 + b^2 + ab) = (a - b)^3 + 3ab(a - b)$
- Application of above mentioned identities.

Facts you may know

Indian mathematicians also contribute in the field of identity. Umar Khayyam was born in 11th century. He was a poet, Astrologer, Mathematician and Philosophical person. He wrote a granth on algebra after that he became famous. The method to express identity $(a + b)^n$ had been used earlier in east as compared to west. Uclid knew the specific value of this identity for $n = 2$, but for the other value of n it was given by Umar K hayyam. He knew the method to find fourth, fifth, sixth roots of numbers by a principle, but the book which contain these method could not be found till now. He introduced binomial as simple equation and trinomial and tetranomial as combine equation. There are many other mathematician such as Suryadas, Ganesh Daivagya, Narayan Pandit etc. who also contributed in this field.

ANSWERS

Exercise 4(a)

1. (a) $b^3 + 3b^2 + 3b + 1$ (b) $c^3 + 9c^2 + 27c + 27$ (c) $8x^3 + 36x^2 + 54x + 27$
 (d) $x^6 + 3x^4y + 3x^2y^2 + y^3$ (e) $125 + 225y + 135y^2 + 27y^3$ (f) $x^3y^3 + 6ax^2y^2 + 12a^2xy + 8a^3$

2. (a) $\frac{a^3}{8} + \frac{3ab}{10} \left(\frac{a}{2} + \frac{b}{5} \right) + \frac{b^3}{125} = \frac{a^3}{8} + \frac{3a^2b}{20} + \frac{3ab^2}{50} + \frac{b^3}{125}$ (b) $27y^3 + \frac{9}{4} \left(3y + \frac{1}{4y} \right) + \frac{1}{64y^3}$
 $= 27y^3 + \frac{27}{4}y + \frac{9}{16y} + \frac{1}{64y^3}$ 3. (a) 29791 (b) 1061208 (c) 8120601 4. $\frac{730}{27}$ 5. $\frac{65}{8}$ 6. 9 7. 7440

Exercise 4(b)

1. (a) $x^3 - 21x(x - 7) - 343 = x^3 - 21x^2 + 147x - 343$ (b) $8 - 6y(2 - y) - y^3 = 8 - 12y + 6y^2 - y^3$ (c) $27p^3 - \frac{9p}{4q} \left(3p - \frac{1}{4q} \right) - \frac{1}{64q^3} = 27p^3 - \frac{27p^2}{4q} + \frac{9p}{16q^2} - \frac{1}{64q^3}$ (d) $64 - \frac{4}{y} \left(4 - \frac{1}{3y} \right) - \frac{1}{27y^3} = 64 - \frac{16}{y} + \frac{4}{3y^2} - \frac{1}{27y^3}$ (e) $343x^3 - 63xy(7x - 3y) - 27y^3 = 343x^3 - 441x^2y + 189xy^2 - 27y^3$ (f) $512 - 48a(8 - 2a) - 8a^3 = 512 - 384a + 96a^2 - 8a^3$; 2. $\frac{15624}{125}$ 3. $\frac{728}{27}$;
 4. (i) 941192 (ii) 970.299 (iii) 213847192; 5. (i) $42x^2 + 686$ (ii) $432x^2y + 1024y^3$ (iii) $\frac{y^2z}{2} + \frac{2z^3}{27}$ (iv) $-1470k^2l - 250l^3$ 6. 316; 7. 5886

Exercise 4(c)

1. (i) $a^2 + 4b^2 + 9c^2 + 4ab + 12bc + 6ca$ (ii) $4p^2 + q^2 + 9r^2 - 4pq + 6qr - 12pr$
 (iii) $9x^2 + 4y^2 + z^2 - 12xy + 4yz - 6xz$ (iv) $4x^2 + 9y^2 + 16 - 12xy - 24y + 16x$
 (v) $m^2 + 9n^2 + 16p^2 + 6mn - 24np - 8pm$ (vi) $49 + 16a^2 + 9b^2 + 56a - 24ab - 42b$
 (vii) $4x^2 + 81 + 25y^2 - 36x - 90y + 20xy$ (viii) $x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z + 2xyz^2 + 2x^2yz$
 (ix) $a^2 + \frac{b^2}{4} + \frac{c^2}{9} + ab + \frac{2ac}{3} + \frac{bc}{3}$ (x) $a^2 + \frac{b^2}{9} + \frac{4c^2}{25} - \frac{2ab}{3} - \frac{4bc}{15} + \frac{4ac}{5}$ (xi) $a^2 + \frac{b^2}{4} +$
 $36 - ab - 12a + 6b$ 2. (i) $3x^2 + 3y^2 + 3z^2 + 2xy - 2yz + 2zx$ (ii) $-24xy + 12xz = 6x(2z - 4y)$
 $= 12x(z - 2y)$ (iii) $2x^2(2y^2 - 2z^2) = 4x^2(y^2 - z^2) = 4x^2(y + z)(y - z)$

Group Discussion

1. (i) $x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6xz$ (ii) $x^2 + 4y^2 + z^2 - 4xy - 4yz + 2xz$
 (iii) $a^3 + 6a^2b + 12ab^2 + 8b^3$ (iv) $8 - 18p(2 - 3p) - 27p^3 = 8 - 36p + 54p^2 - 27p^3$

Efficiency Practice 4

1. (i) 991026973 (ii) 1157.625 (iii) 125751501 (iv) 857.375 2. (i) 316 (ii) 35
 (iii) 216 (iv) 15616 3. (iii) $1 - y^3 - 3y(1 - y)$ 4. (ii) $(2 + x)(4 + x^2 - 2x)$
 5. (i) $(1 - 3p)(1 + 9p^2 + 3p)$ 6. (iii) $8 + x^3 + 6x(2 + x)$ 7. (i) $1 + 4x^2 + y^2 + 4x + 4xy + 2y$ 8. (i)
 $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ 9. (ii) $a^3 - b^3 - 3ab(a - b)$

