

UNIT-5 DIVISION AND FACTOR OF ALGEBRAIC EXPRESSION



- Division in algebraic expression by monomial and binomial expression (algebraic expression limited upto five degree)
- Verification of Dividend = Divisor $\times$ Quotient + Remainder
- Concept of factor of Polynomial Expressions (Not more than three term) [$a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$]
- Factor of expression like $ax^2 + bx + c$

5.1 Introduction

In previous classes we have studied about algebraic expression and identities and also learnt about factorization of algebraic expression. In this chapter we will study about polynomial expression in one variable and we will learn how to divide a polynomial by a monomial or binomial and in the end of the chapter we will learn factorization of polynomials.

5.2 A Polynomial in one variable

We know that in $12x^3y^3$ and $4xy^2$ there are variable terms x and y . Expression $y^2 + 5y + 6$ is a multinomial algebraic expression in one variable y , similarly the expression $a^2 + ab + b^2$ is multinomial algebraic expression in two variables a & b .

Note that algebraic expression $2x^4 - 4x^2 + 12$ is polynomial in x and among different terms the terms $2x^4$ has highest power 4, Term $-4x^2$ has power 2 and 12 has power 0 zero on x ($\therefore 12 = 12x^0$) therefore its degree is zero.

Similarly $(y^2 + 8y + 15)$ is polynomial of degree 2 in variable y .

Check that in the polynomial $5z^3 - 2z^5 + 7z^2 + 8$ the highest power of variable z is 5, $7y - 9 + y^3$ has highest power of variable y is 3 and in $x^6 - x^5 + x^7 - \sqrt{3}x^2$ the highest power of variable x is 7.

Remark (i) in the polynomial $3x^4 - 4x^3y^2 + 5x^2y^2 + y^3$ variable are x and y . In different terms of this expression sum of the power x and y are as follows.

In $3x^4$; sum of powers x and $y = 4 + 0 = 4$.

The sum of powers of x and y in; $-4x^3y^3$ -is $3 + 2 = 5$

The sum of powers of x and y in; $+5x^2y^2$ -is $2 + 2 = 4$

The sum of powers of x and y in; $+y^3$ -is $0 + 3 = 3$

So in the polynomial $3x^4 - 4x^3y^2 + 5x^2y^2 + y^3$ in two variable x and y . The maximum value of sum of the power of x and y is 5, so degree of the polynomial is 5.

(ii) $\because x^0 = 1$ so $11x^0 = 11$ so the power of variable is zero in term 11.

5.3.1. Division of one term expression by one term expression.

Now we discuss process of division in one term expression $12x^3y^3$ by one term expression $-4xy$.

First Method

$$12x^3y^3 = (-4xy) \times (-x^2y) \text{ (Write in multiplication)}$$

$$\text{So } 12x^3y^2 \div (-4xy) = \frac{(-4xy) \times (-3x^2y)}{(-4xy)}$$

$$\text{So } \frac{12x^3y^2}{-4xy} = -3x^2y$$

Second Method

Division of variable by variable and constant by constant

Note that in $12x^3y^2$ the constant is 12 and variable is x^3y^2 .

Similarly constant term is -4 and variable xy in $-4xy$.

$$\text{So } \frac{12}{-4} = -3 \text{ and } \frac{x^3y^2}{xy} = x^2y$$

$$\text{(Rule of Index } \frac{x^m}{x^n} = x^{m-n} \text{)}$$

Combining the two

$$\frac{12x^3y^2}{-4xy} = -3x^2y$$

For example

$$\frac{-15p^4}{-5p^3} = 3p \quad \left(\frac{-15}{-5} = 3, \quad \frac{p^4}{p^3} = p^{4-3} = p \right)$$

$$\frac{21z^9}{-7z^2} = -3z^7 \quad \left(\frac{21}{-7} = -3, \quad \frac{z^9}{z^2} = z^{9-2} = z^7 \right)$$

Conclusion :

1. Numerical coefficient of quotient of division by two monomial, is equal to the numerical coefficient of quotient of the expression.
2. Numerical coefficient of quotient of division by the monomial is equal to numerical value of quotient of variable coefficient of the expression.

Try it

Simplify the following :

$$(i) \quad \frac{36x^4}{-9x}$$

$$(ii) \quad -\frac{24a^3b^2}{-3a^2b}$$

$$(iii) \quad \frac{28x^7y^4}{-7x^3y^2}$$

$$(iv) \quad \frac{-32p^4q^3}{-8p^2q}$$

5.2.1 Division of a polynomial by a monomial

Now we think of the division of polynomial $12x^2 + 9x^3 - 36x$ by monomial $3x$. We do the operation of division in the following two terms.

Step 1. We arrange the term of dividend in decreasing powers.

$$\begin{aligned} \text{Dividend} &= 12x^2 + 9x^3 - 36x \\ &= 9x^3 + 12x^2 - 36x \end{aligned}$$

Step 2. Now on division of each term of the polynomial by given one term $3x$ we get

$$\begin{aligned} (9x^3 + 12x^2 - 36x) \div 3x &= \frac{9x^3}{3x} + \frac{12x^2}{3x} - \frac{36x}{3x} \\ &= 3x^2 + 4x - 12 \end{aligned}$$

Note : If divisor is monomial then without putting the dividend, decreasing power. We can find out quotient by step 2 above.

Example 1. Find the quotient

$$(18x + 12x^3 - 6x^2) \div (-3x)$$

$$\text{Dividend} = 18x + 12x^3 - 6x^2$$

$$= 12x^3 - 6x^2 + 18x$$

$$(12x^3 - 6x^2 + 18x) \div (-3x)$$

$$= \frac{12x^3}{(-3x)} - \frac{6x^2}{(-3x)} + \frac{18x}{(-3x)}$$

$$= -4x^2 + 2x - 6$$

Exercise 5(a)

1. Divide

(a) $8x^2y^2$ by $2xy$

(b) $15x^2y^3$ by $3x^2y^2$

(c) $a^2 - b^2$ by $(a + b)$

2. Simplify

(a) $\frac{32m^3y^2}{4m^2y}$

(b) $\frac{x^3 + 4x^2 + 4x}{x}$

(c) $\frac{16x^2 - 8x^3}{4x}$

(d) $\frac{9a^2 - 24a + 18a^3}{3a}$

5.3.2 Division of a polynomial by two term expression.

First Condition – Zero remainder

Since $(x + 5)(x + 2) = x^2 + 7x + 10$

so by rules of division in numbers

We say that

$$\frac{x^2 + 7x + 10}{x + 5} = x + 2 \quad \dots\dots\dots (1)$$

$$\frac{x^2 + 7x + 10}{x + 2} = x + 5 \quad \dots\dots\dots (2)$$

In equation (1), dividend is $x^2 + 7x + 10$, divisor is $(x + 5)$ and quotient $(x + 2)$ or in equation (2) dividend is $x^2 + 7x + 10$,

Divisor $(x + 2)$ and quotient $(x + 5)$.

Exactly in this way = $\frac{x^2 + (a+b)x + ab}{x+a} = x + b$

$$(\therefore (x + a) (x + b) = x^2 + (a + b) x + ab)$$

Algebraic expression whose numerator and denominator the power of variables are of positive integers are called rational expressions.

Example.

$$\frac{x^2 + (a+b)x + ab}{x+a}, \quad \frac{x^2 + 7x + 10}{x+5} \text{ are rational expression etc.}$$

Now we divide polynomial $(-12 - 6x^2 + 17x)$ by two term expression $(2x - 3)$.

- (i) Write the terms of dividend and divisor in power of decreasing order. So dividend $(-12 - 6x^2 + 17x)$ to $(-6x^2 + 17x - 12)$ and write divisor $(2x - 3)$ as it is so

$$2x - 3 \overline{) -6x^2 + 17x - 12}$$

- (ii) We divide first term of dividend $-6x^2 + 17x - 12$ by first term $(2x)$ of divisor $(2x - 3)$ so $-6x^2 \div 2x = -3x$ first term of quotient

$$2x - 3 \overline{) -6x^2 + 17x - 12} \quad \begin{array}{r} -3x \end{array}$$

- (iii) Now multiply divisor $(2x - 3)$ by first term of quotient $-3x$ and subtracting the product $(2x - 3) \times (-3x) = -6x^2 + 9x$ by dividend.

$$\begin{aligned} \text{In this way } (-6x^2 + 17x - 12) - (-6x^2 + 9x) &= -6x^2 + 17x - 12 + 6x^2 - 9x \\ &= 8x - 12 \end{aligned}$$

It means,

$$\begin{array}{r} -3x \\ 2x-3 \overline{) -6x^2 + 17x - 12} \\ \underline{+6x^2 + 9x} \\ 8x - 12 \end{array}$$

- (iv) Take the remainder $(8x - 12)$ as a new part and divide it from the divisor $(2x - 3)$. The first term of the new dividend $8x$ is four times the first term of $2x$ of divisor $(2x - 3)$. In this way $+4$ is second term of quotient. So quotient of second term is $+4$

$$\begin{array}{r} +4 \\ 2x-3 \overline{) 8x-12} \end{array}$$

- (v) Multiply the divisor $(2x - 3)$ by second term 4 of the quotient subtracting product $(2x - 3) \times 4 = (8x - 12)$ from new dividend $(8x - 12)$ then the remainder is zero. so

$$\begin{array}{r} +4 \\ 2x-3 \overline{) 8x-12} \\ \underline{8x-12} \\ 0 \end{array}$$

Overall, above steps can be represented like this.

$$\begin{array}{r} -3x+4 \\ 2x-3 \overline{) -6x^2 + 17x - 12} \\ \underline{+6x^2 + 9x} \\ 8x - 12 \\ \underline{8x - 12} \\ 0 \end{array}$$

sign of first expression is changed (change $+$ to $-$ and $-$ to $+$)

Note : Above question remainder is zero. So $(2x - 3)$ is a factor of $(-12 - 6x^2 + 17x)$ and obtained quotient is also a factor of $(-12 - 6x^2 + 17x)$

**If we divide a polynomial (dividend) by a polynomial (divisor) and remainder be zero then factor of dividend are divisor and quotient.
that is $\text{Dividend} = \text{Divisor} \times \text{Quotient}$**

Note : The dividend, divisor, quotient is a polynomial, binomial or monomial expression in the same variable x .

Example 2. Divide $(2y^4 + 7y^2 + 8y^3 + 4y + 3)$ by $(3 + y)$ if $(3 + y)$ a factor of $(2y^4 + 7y^2 + 8y^3 + 4y + 3)$?

Solution : Arranging in decreasing power of dividend $= (2y^4 + 8y^3 + 7y^2 + 4y + 3)$ and divisor $(y + 3)$

$$\begin{array}{r}
 2y^3 + 2y^2 + y + 1 \\
 y+3 \overline{) 2y^4 + 8y^3 + 7y^2 + 4y + 3} \\
 \underline{2y^4 + 6y^3} \\
 2y^3 + 7y^2 + 4y + 3 \\
 \underline{2y^3 + 6y^2} \\
 y^2 + 4y + 3 \\
 \underline{y^2 + 3y} \\
 y + 3 \\
 \underline{y + 3} \\
 0
 \end{array}$$

Remainder = 0

So divisor $(y + 3)$ is factor of dividend $(2y^4 + 8y^3 + 7y^2 + 4y + 3)$ and quotient $= (2y^3 + 2y^2 + y + 1)$. Thus the quotient is the second factor of the dividend.

So $(2y^4 + 8y^3 + 7y^2 + 4y + 3) = (y + 3)(2y^3 + 2y^2 + y + 1)$

Clarification :

$$\begin{aligned}\text{Divisor} \times \text{Quotient} &= (y + 3) (2y^3 + 2y^2 + y + 1) \\ &= 2y^4 + 2y^3 + y^2 + y + 6y^3 + 6y^2 + 3y + 3) \\ &= 2y^4 + 8y^3 + 7y^2 + 4y + 3) \\ &= \text{Dividend}\end{aligned}$$

Or

$$\text{Divisor} \times \text{Quotient} = \text{Dividend}$$

5.4 Second Condition = Non zero remainder

Above example obtained remainder is zero in all operation of the division. So in this case we say that dividend is fully divisible by divisor.

Note that, in case of such number we continue process of division till the remainder is less than divisor. Similarly in case of polynomial we continue process of division till the remainder become the polynomial of less degree than divisor.

Example 3. Divide $15z^3 - 20z^2 + 13z - 12$ by $3z - 6$

Solution :

$$\begin{array}{r} 5z^2 + \frac{10}{3}z + 11 \\ 3z - 6 \overline{) 15z^3 - 20z^2 + 13z - 12} \\ \underline{15z^3 - 30z^2} \\ 10z^2 + 13z - 12 \\ \underline{10z^2 - 20z} \\ 33z - 12 \\ \underline{33z - 66} \\ 54 \end{array}$$

$$\text{Quotient} = 5z^2 + \frac{10}{3}z + 11$$

$$\text{Remainder} = 54$$

Example 4. Divide $34x - 22x^3 - 12x^4 - 10x^2 - 75$ by $3x + 7$

Solution : $34x - 22x^3 - 12x^4 - 10x^2 - 75$ by $3x + 7$ writing in decreasing power of x .

$$\text{Divisor} = -12x^4 - 22x^3 - 10x^2 + 34x - 75$$

$$\begin{array}{r}
 -4x^3 + 2x^2 - 8x + 30 \\
 \hline
 3x + 7 -12x^4 - 22x^3 - 10x^2 + 34x - 75 \\
 -12x^4 - 28x^3 \\
 + \\
 \hline
 6x^3 - 10x^2 + 34x - 75 \\
 6x^3 + 14x^2 \\
 - \\
 \hline
 -24x^2 + 34x - 75 \\
 -24x^2 - 56x \\
 + \\
 \hline
 90x - 75 \\
 90x + 210 \\
 - \\
 \hline
 -285 \\
 \hline
 \hline
 \end{array}$$

Note: For doing division process easily, the algebraic expression came in divisor and dividend are kept in decreasing order of their degrees.

$$\text{Quotient} = 4x^3 + 2x^2 - 8x + 30$$

$$\text{Remainder} = -285$$

5.4.1 Verification of (Dividend = Divisor $\times$ Quotient + Remainder)

In earlier classes we have learnt division of number consider $35 \div 4$ so quotient 8 and remainder 3 its divisible 35 and divisor.

$$35 = (4 \times 8) + 3$$

So

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Now we see if this is also true in the division of Algebraic expression.

Example 5. Divide $x^2 + 7x + 14$ by $(x + 3)$

Solution :

$$\begin{array}{r} x + 4 \\ x + 3 \overline{) x^2 + 7x + 14} \\ \underline{x^2 + 3x} \\ 4x + 14 \\ \underline{4x + 12} \\ + 2 \end{array}$$

Above part of question dividend is $x^2 + 7x + 14$, Divisor $(x + 3)$.

Quotient $(x + 4)$ and Remainder $= 2$

$$\begin{aligned} \text{Divisor} \times \text{quotient} &= (x + 3)(x + 4) \\ &= x(x + 4) + 3(x + 4) \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + x(4 + 3) + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

$$\begin{aligned} \text{Divisor} \times \text{quotient} + \text{remainder} &= (x^2 + 7x + 12) + 2 \\ &= x^2 + 7x + 14 = \text{dividend} \end{aligned}$$

So **Dividend = Divisor $\times$ Quotient + Remainder**

Try it.

1. Verify above mentioned example 3 and 4.
2. Solve the following question and on the basis of obtained result verify the above factors.
 - (a) Divide $8y^2 + 6y + 7$ by $(2y + 1)$
 - (b) Divide $3y^3 - 3y^2 - 4y - 17$ by $(y - 2)$

From the above, It is concluded that in each part of operation.

Dividend = Divisor $\times$ Quotient + Remainder

Exercise 5(b)

1. Find the exponent of the polynomials :
 - (a) $2x^3 + 5x^2 - 7$
 - (b) $4x^2 - 5x + 2 + 3x^4$
 - (c) 7
 - (d) $3x + 4x^2 - 7$

(e) $2y^7 - 12y^6 + 48y^5 - 9$

(f) $7x^5$

(g) $20x^3 + 12x^2y^2 - 10y^2 + 20$

2. Divide :

(a) $(9z^5 + 12z^4 - 6z^2)$ by $3z^2$

(b) $(x^2 + 9x + 20)$ by $(x + 4)$

(c) $(8y^2 + 6y + 1)$ by $(2y + 1)$

(d) $(4z^3 + 6z^2 - z)$ by $-\frac{1}{2}z$

(e) $(3x^3 + 4x^2 + 5x + 18)$ by $(x + 2)$

(f) $(x^5 + y^5)$ by $(x + y)$

3. Divide the following question and write dividend, divisor, Quotient and Remainder in a table.

(a) $(14x^2 + 13x - 15)$ by $(7x - 4)$

(b) $(6y^5 - 28y^3 + 3y^2 + 30y - 9)$ by $(2y^2 - 6)$

(c) $(34x - 22x^3 - 12x^4 - 10x^2 - 75)$ by $(3x + 7)$

(d) $(15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6)$ by $(3y - 2)$

Verify from the table that :

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

4. Find by operation of division that

(a) $(x + 6)$ is a factor of $(x^2 - x - 42)$

(b) $(4x - 3)$ is a factor of $(4x^2 - 13x - 12)$

(c) $(2x - 5)$ is a factor of $(4x^4 - 10x^3 - 10x^2 + 30x - 15)$

(d) $3z^2 + 5$ is a factor of $(6z^5 + 15z^4 + 16z^3 + 4z^2 + 10z - 35)$

(e) $x^2 + 3$ is a factor of $(x^5 - 9x)$

(f) $x^2 - 1$ is a factor of $(x^6 - 1)$

5. What should be subtracted from the polynomial $(4z^2 + 7z^2 + 15)$ so that the obtained expression is completely divisible by $4z^2 - 5$?

5.5 Concept of factor in multinomials expression (Not more than three terms)

We have learned in previous classes the factor of following expression

(i) $(ax + ay)$

(ii) $ax^2 + ay^2 + bx^2 + by^2$

Now we will learn to factorize trinomial expression.

5.5.1 Factor of expression of the type $a^2 + 2ab + b^2$

First method – By identity

We have learned that $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}a^2 + 2ab + b^2 &= (a + b)^2 \\ &= (a + b)(a + b)\end{aligned}$$

Second method – by grouping

$$\begin{aligned}a^2 + 2ab + b^2 &= a^2 + ab + ab + b^2 \\ &= a(a + b) + b(a + b) \\ &= (a + b)(a + b) \\ &= (a + b)^2\end{aligned}$$

In this way we see that

$$a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$$

so factor of $a^2 + 2ab + b^2$ are $(a + b)(a + b)$ so algebraic expression

$$a^2 + 2ab + b^2 = (a + b)^2$$

Example 6. Factorize $x^2 + 8x + 16$

Solution :

$$\begin{aligned}x^2 + 8x + 16 &= x^2 + 2 \times x \times 4 + 4^2 \\ &= (x + 4)^2 \dots (\text{by } a^2 + 2ab + b^2 = (a + b)^2) \\ &= (x + 4)(x + 4)\end{aligned}$$

$$\text{Thus } x^2 + 8x + 16 = (x + 4)(x + 4)$$

$$\text{or } x^2 + 8x + 16 = (x + 4)^2$$

$$\therefore \text{Factor of } x^2 + 8x + 16 \text{ are } (x + 4) \text{ \& } (x + 4)$$

$$\therefore \text{we can write } x^2 + 8x + 16 = (x + 4)^2$$

By grouping

$$x^2 + 8x + 16 = x^2 + 4x + 4x + 16$$

$$= x(x + 4) + 4(x + 4)$$

$$= (x + 4)(x + 4) = (x + 4)^2$$

$$\text{Thus } x^2 + 8x + 16 = (x + 4)(x + 4)$$

Example 7. Factorize $36x^2 + 12x + 1$

Solution :

$$36x^2 + 12x + 1$$

$$= (6x)^2 + 2 \cdot 6x \cdot 1 + 1^2$$

$$= (6x + 1)^2$$

$$\text{Thus } 36x^2 + 12x + 1 = (6x + 1)(6x + 1)$$

$$\text{or } 36x^2 + 12x + 1 = (6x + 1)^2$$

By grouping

$$36x^2 + 12x + 1$$

$$= 36x^2 + 6x + 6x + 1$$

$$= 6x(6x + 1) + 1(6x + 1)$$

$$= (6x + 1)(6x + 1)$$

$$= (6x + 1)^2$$

Example 8. Factorize $p^2 + 3p + \frac{9}{4}$

Solution : $p^2 + 3p + \frac{9}{4}$

$$= p^2 + 2p \left(\frac{3}{2} \right) + \left(\frac{3}{2} \right)^2$$

$$= \left(p + \frac{3}{2} \right)^2$$



By grouping

$$\begin{aligned} & p^2 + 3p + \frac{9}{4} \\ &= p^2 + 2\left(\frac{3}{2}\right)p + \left(\frac{3}{2}\right)^2 \\ &= p^2 + \frac{3}{2}p + \frac{3}{2}p + \left(\frac{3}{2}\right)^2 \\ &= p\left(p + \frac{3}{2}\right) + \frac{3}{2}\left(p + \frac{3}{2}\right) \\ &= \left(p + \frac{3}{2}\right)\left(p + \frac{3}{2}\right) \\ &= \left(p + \frac{3}{2}\right)^2 \end{aligned}$$

So from above example it is clear that

Factors of the expression $a^2 + 2ab + b^2$ are $(a + b)$; $(a + b)$ and $(a + b)^2$ (in the form of product of factors)

Exercise 5(c)

1. Choose the type of expression ($a^2 + 2ab + b^2$) from the following expression

(i) $a^2 + 10a + 16$

(ii) $x^2 - 5x + 4$

(iii) $x^2 + 10x + 25$

(iv) $49m^2 + 140mn + 100n^2$

2. Factorize the following :

(i) $a^2 + 2a + 1$

(ii) $36 + 12x + x^2$

(iii) $4c^2 + 4c + 1$

(iv) $36x^2 + 60x + 25$

(v) $9x^2 + 6x + 1$

3. (i) Factorize $x^2 + x + \frac{1}{4}$

(ii) Factorize $p^2 + 5p + \frac{25}{4}$

5.5.2 Factor of expression of the type $(a^2 - 2ab + b^2)$.

First Method – Using Identity

We know that

$$(a - b)^2 = a^2 - 2ab + b^2$$

or

$$a^2 + b^2 - 2ab = (a - b)^2$$

Second Method – Grouping

$$\begin{aligned} & a^2 - 2ab + b^2 \\ = & a^2 - ab - ab + b^2 \\ = & a(a - b) - b(a - b) \\ = & (a - b)(a - b) \\ = & (a - b)^2 \end{aligned}$$

So $a^2 - 2ab + b^2 = (a - b)^2$

∴ Factor of $a^2 - 2ab + b^2$ is $(a - b)$ and $(a - b)$ or $a^2 - 2ab + b^2 = (a - b)^2$

Example 9. Factorize $9x^2 - 30xy + 25y^2$

Solution :

$$\begin{aligned} & 9x^2 - 30xy + 25y^2 \\ &= (3x)^2 - 2(3x)(5y) + (5y)^2 \\ &\text{by formula } a^2 - 2ab + b^2 = (a - b)^2 \\ & 9x^2 - 30xy + 25y^2 \\ &= (3x)^2 - 2(3x)(5y) + (5y)^2 \\ &= (3x - 5y)^2 \end{aligned}$$

Example 10. Factorize $36 - 12k + k^2$

Solution :

$$\begin{aligned} & 36 - 12k + k^2 \\ &= (6)^2 - 2(6)k + k^2 \\ &\text{By identity } a^2 - 2ab + b^2 = (a - b)^2 \\ & 36 - 12k + k^2 \end{aligned}$$

$$= (6)^2 - 2(6)k + k^2$$

$$= (6 - k)^2$$

Grouping method

$$36 - 12k + k^2$$

$$= 36 - 6k - 6k + k^2$$

$$= 6(6 - k) - k(6 - k)$$

$$= (6 - k)(6 - k)$$

$$= (6 - k)^2$$

from above example it is clear that

factor of the expression of the figure

$$a^2 - 2ab + b^2 \text{ is } (a - b)(a - b) \text{ or } (a - b)^2$$

Exercise 5(d)

1. Choose the expression of the type $(a^2 - 2ab + b^2)$ from following :

(i) $81a^2 - 72ab + 16b^2$

(ii) $16b^2 - 20by + 25y^2$

(iii) $25x^2 - 10xy + 9$

(iv) $5x^2 - 30xy + 9y^2$

2. Factorize the following :

(i) $4x^2 - 12xy + 9y^2$

(ii) $9x^2 - 6x + 1$

(iii) $25x^2 - 60x + 36$

(iv) $49x^2 - 56x + 16$

(v) $x^2 - 12xy + 36y^2$

5.5.3 Factorizing of the difference of two squares or $a^2 - b^2$

$$a^2 - b^2 = a^2 - ab + ba - b^2$$

$$a^2 - b^2 = a(a - b) + b(a - b)$$

$$a^2 - b^2 = (a + b)(a - b)$$

or

$$(a + b)(a - b) = a^2 + ab - ab - b^2$$

Therefore $(a + b)$ and $(a - b)$ two factor of expression $a^2 - b^2$.

Example 11. Factorize $a^2 - 1$.

Solution : Write expression as the difference of two square.

$$a^2 - 1 = (a)^2 - (1)^2$$

$$\text{by formula} = a^2 - b^2 = (a + b)(a - b)$$

$$\begin{aligned}\text{so } a^2 - 1 &= (a)^2 - (1)^2 \\ &= (a + 1)(a - 1)\end{aligned}$$

Example 12. Factorize $a^2 - 4b^2$

$$\begin{aligned}\text{Solution : } a^2 - 4b^2 &= (a)^2 - (2b)^2 \\ &= (a + 2b)(a - 2b)\end{aligned}$$

$$\begin{aligned}\text{Verification: } (a + 2b) \cdot (a - 2b) &= a^2 + 2ab - 2ab - 4b^2 \\ &= a^2 - 4b^2\end{aligned}$$

Example 13. Factorize $4a^2 - 81$

$$\begin{aligned}\text{Solution : } 4a^2 - 81 &= (2a)^2 - (9)^2 \\ &= (2a + 9)(2a - 9)\end{aligned}$$

$$\begin{aligned}\text{Verification } (2a + 9)(2a - 9) &= 4a^2 + 18a - 18a - 81 \\ &= 4a^2 - 81\end{aligned}$$

Example 14. Factorize $64a^2 - 225b^2$

$$\begin{aligned}\text{Solution : } 64a^2 - 225b^2 &= (8a)^2 - (15b)^2 \\ &= (8a + 15b)(8a - 15b)\end{aligned}$$

Difference of two square is equal to the multiplication of addition of square root and difference of the square root or $(a^2 - b^2) = (a + b)(a - b)$

Example 15. Factorize $50a^2 - 32b^2$

Solution : $\therefore 2$ is a common factor of terms

$$\begin{aligned}\text{so } 50a^2 - 32b^2 &= 2(25a^2 - 16b^2) \\ &= 2[(5a)^2 - (4b)^2] \\ &= 2(5a + 4b)(5a - 4b)\end{aligned}$$

Example 16. Factorize $32a^4 - 162b^4$

Solution :

$$\begin{aligned}32a^4 - 162b^4 &= 2[16a^4 - 81b^4] \\ &= 2[(4a^2)^2 - (9b^2)^2] \\ &= 2(4a^2 + 9b^2)(4a^2 - 9b^2) \\ &= 2(4a^2 + 9b^2)[(2a)^2 - (3b)^2] \\ &= 2(4a^2 + 9b^2)(2a + 3b)(2a - 3b)\end{aligned}$$

Example 17. Find the value of $201^2 \times 200 - 200^3$

Solution :

$$\begin{aligned}201^2 \times 200 - 200^3 &= 200(201^2 - 200^2) \\ &= 200(201 + 200)(201 - 200) \\ &= 200(401)(1) \\ &= 80200\end{aligned}$$

Example 18. Find the factor of $a^4 - b^4$

Solution :

$$\begin{aligned}a^4 - b^4 &= (a^2)^2 - (b^2)^2 \\ &= (a^2 + b^2)(a^2 - b^2) \\ &= (a^2 + b^2)[(a)^2 - (b)^2] \\ &= (a^2 + b^2)(a + b)(a - b)\end{aligned}$$

Example 19. Factorize $\frac{9}{x^2} - \frac{y^2}{25}$

Solution :

$$\begin{aligned}\frac{9}{x^2} - \frac{y^2}{25} &= \left(\frac{3}{x}\right)^2 - \left(\frac{y}{5}\right)^2 \\ &= \left(\frac{3}{x} + \frac{y}{5}\right)\left(\frac{3}{x} - \frac{y}{5}\right)\end{aligned}$$

Exercise 5(e)

Factorize

1. $a^2 - 4$

2. $a^2 - 49b^2$

3. $x^3 - 121x$

4. $4a^2 - \frac{9}{4a^2}$

5. $\frac{18}{x^2} - \frac{2x^2}{9}$

6. $(a-b)^2 - c^2$

7. $(a-3b)^2 - 36b^2$

8. $(a+b)^2 - (a-b)^2$

9. $25(a-5b)^2 - 4(a-3b)^2$

10. $16a^4 - 81b^4$

11. $a^4 - 625$

12. Factorize $a^2b - b^3$ and using it, find the $101^2 \times 100 - 100^3$.

13. Which are the two perfect square small whole numbers. Those difference is a perfect square whole numbers ?

5.5.4 Factorization of expressions of the form $(ax^2 + bx + c)$

To factorize an expression of the form $ax^2 + bx + c$. We find two numbers expression in such that $b = l + m$ and $ac = l \times m$ so $c = \frac{l \times m}{a}$

So expression

$$\begin{aligned} ax^2 + bx + c &= ax^2 + (l + m)x + \frac{lm}{a} \\ &= ax^2 + lx + mx + \frac{lm}{a} \\ &= x(ax + l) + \frac{m}{a}(ax + l) \\ &= (ax + l) \left(x + \frac{m}{a} \right) \end{aligned}$$

For example factorization of $2x^2 + 7x + 3$. We take to numbers l and m that $l + m = 7$ and $lm = 2 \times 3 = 6$. Think two prime factor, such that their multiplication be 6 and sum be 7.

Take two number with sum and product $(2 \times 3) = 6$ such number are 6 and 1.

So $l = 6$ and $m = 1 \quad \therefore l + m = 7$ and $l \cdot m = 6$

$$\begin{aligned}\therefore 2x^2 + 7x + 3 &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (x + 3)(2x + 1)\end{aligned}$$

thus $2x^2 + 7x + 3 = (x + 3)(2x + 1)$

Factorization of trinomials of the form $(x^2 + bx + c)$.

To factorize a trinomials of the form $x^2 + bx + c$

We find the use expression $(x + a)(x + b) = x^2 + (a + b)x + ab$

Example 20. Factorize $x^2 + 8x + 12$

Solution : Comparing with identity

$$a + b = 8, ab = 12$$

if $a = 6, b = 2$

Comparing $x^2 + 8x + 12$ with the identity $x^2 + (a + b)x + ab$

$$\begin{aligned}x^2 + 8x + 12 &= x^2 + (6 + 2)x + 6 \times 2 \\ &= x^2 + 6x + 2x + 12 \\ &= x(x + 6) + 2(x + 6) \\ &= (x + 2)(x + 6)\end{aligned}$$

Second Method

$$\begin{aligned}x^2 + 8x + 12 &= x^2 + (6 + 2)x + 12 \\&= x^2 + 6x + 2x + 12 \\&= x(x + 6) + 2(x + 6) \\&= (x + 6)(x + 2)\end{aligned}$$

Example 21. Factorize $x^2 - x - 6$

Solution : Expression $x^2 - x - 6$ to compare $[x^2 + (a + b)x + ab]$

$$a + b = -1 = (-3) + 2$$

$$\text{and } ab = -6 = -3 \times 2$$

$$\begin{aligned}\therefore x^2 - x - 6 &= x^2 + (-3 + 2)x + (-3) \times 2 \\&= (x - 3)(x + 2)\end{aligned}$$

$$[\because (x+a)(x+b) = x+(a+b)x+ab]$$

Second method :

$$\begin{aligned}x^2 - x - 6 &= x^2 + (-3 + 2)x - 6 \\&= x^2 - 3x + 2x - 6 \\&= x(x - 3) + 2(x - 3) \\&= (x - 3)(x + 2)\end{aligned}$$

Example 22. Factorize $x^2 - 7x + 12$

Solution : Compare the expression $x^2 - 7x + 12$ with $[x^2 + (a + b)x + ab]$

$$a + b = -7 = (-4) + (-3)$$

$$ab = 12 = (-4) \times (-3)$$

$$\begin{aligned}\text{and } x^2 - 7x + 12 &= x^2 + (-4 - 3)x + (-4)(-3) \\&= (x - 4)(x - 3)\end{aligned}$$

$$[\because (x+a)(x+b) = x+(a+b)x+ab]$$

Second method :

$$\begin{aligned}x^2 - 7x + 12 &= x^2 + (-4 - 3)x + 12 \\&= x^2 - 4x - 3x + 12\end{aligned}$$

$$= x(x-4) - 3(x-4)$$

$$= (x-4)(x-3)$$

Example 23. Factorize $x^2 + 2x - 15$

Solution : $x^2 + 2x - 15$

$$(\because a+b = 2 = (5-3); a \times b = 15 = 5 \times 3)$$

$$= x^2 + (5-3)x - 15$$

$$= x^2 + 5x - 3x - 15$$

$$= x(x+5) - 3(x+5)$$

$$= (x+5)(x-3)$$

Example 24. Factorize $2x^2 + 9x - 5$

To factorize $2x^2 + 9x - 5$ we need two numbers a & b such that

$$a + b = 9, |10 + (-1)| = 9 \text{ and } a \times b = -10 = 10 \times -1 = -10$$

$$\text{So, } 2x^2 + 9x - 5 = 2x^2 + [10 + (-1)]x - 5$$

$$= 2x^2 + 10x - x - 5$$

$$= 2x(x+5) - 1(x+5)$$

$$= (x+5)(2x-1)$$

Example 25. Factorize $12y^2 - y - 1$

We need number a & b such that

Solution : $a + b = -1$ and $ab = 12 \times (-1) = -12$

$$\because -4 + 3 = -1 \text{ and } (-4) \times 3 = -12$$

Therefore $a = -4$ & $b = 3$ are the suitable numbers

$$\text{So, } 12y^2 - y - 1 = 12y^2 + (-4 + 3)y - 1$$

$$= 12y^2 - 4y + 3y - 1$$

$$= 4y(3y-1) + 1(3y-1)$$

$$= (3y-1)(4y+1)$$

Example 26. Factorize $-5x^2 - x + 4$

Solution : $-5x^2 - x + 4$
or $- [5x^2 + x - 4]$
 $= - [5x^2 + 5x - 4x - 4]$
for $[a + b = 1 = 5 + (-4)]$ and $ab = 5 \times (-4)$, $a = 5$, $b = -4$
 $= - [5x(x + 1) - 4(x + 1)]$
 $= -(x + 1)(5x - 4)$
 $= (1 + x)(4 - 5x)$

Example 27. Factorize $12x^3 - 14x^2 - 10x$

$$\begin{aligned} 12x^3 - 14x^2 - 10x &= 2x(6x^2 - 7x - 5) \\ &= 2x[6x^2 - 10x + 3x - 5] \\ &= 2x[2x(3x - 5) + 1(3x - 5)] \\ &= 2x(3x - 5)(2x + 1) \end{aligned}$$

[For $a + b = -7 = -10 + 3$,
 $ab = 6 \times (-5)$
 $= -30 = (-10) \times 3$
 $a = -10, b = 3$]

Example 28. Factorize $15x^4 + 3x^2 - 18$

Solution : $15x^4 + 3x^2 - 18 = 3[5x^4 + x^2 - 6]$
 $= 3[5y^2 + y - 6]$ [Here $x^2 = y$]
 $= 3[5y^2 + 6y - 5y - 6]$ [for $a + b = 1 = 6 + (-5)$
 $ab = 5 \times (-6)$
 $a = 6, b = -5$]
 $= 3[y(5y + 6) - 1(5y + 6)]$
 $= 3(5y + 6)(y - 1)$ [y = x² Replacing]
 $= 3(5x^2 + 6)(x^2 - 1)$
 $= 3(5x^2 + 6)(x + 1)(x - 1)$

Exercise 5(f)

1. Find two integers a and b such that :

- (i) $a + b = 8$ and $ab = 15$
- (ii) $a + b = 13$ and $ab = 12$
- (iii) $a + b = 1$ and $ab = -20$
- (iv) $a + b = -5$ and $ab = 4$
- (v) $a + b = -1$ and $ab = -12$
- (vi) $a + b = -11$ and $ab = 10$
- (vii) $a + b = 8$ and $ab = -20$

2. Factorize :

- (i) $x^2 + 5x + 6$
- (ii) $q^2 + 6q + 8$
- (iii) $m^2 + 11m + 24$
- (iv) $y^2 + 9y - 36$
- (v) $a^2 + 3a - 10$
- (vi) $k^2 - 11k - 102$
- (vii) $p^2 - 5p - 176$
- (viii) $48 + 2x - x^2$
- (ix) $-11p + p^2 + 24$
- (x) $y^2 - 26y + 69$
- (xi) $a^4 - 5a^2 + 36$
- (xii) $y^4 + 4y^2 - 32$
- (xiii) $2x^3 + 10x^2 - 28x$
- (xiv) $-2y^3 + 22y^2 + 24y$

(Hint : Take 2 as a common factor)

- (xv) $12x + 15 - 3x^2$
- (xvi) $40p^3 + 16p^3q - 2p^3q^2$
- (xvii) $-18k^4 + 3k^5 - 48k^3$
- (xviii) $b^2c^3 + 8bc^4 + 12c^5$
- (xix) $-18 + a^2b^2 - 3ab$

Factorize

- 3. $4x^2 + 5x + 1$
- 4. $2x^2 + 11x + 14$
- 5. $2y^2 - 5y - 12$
- 6. $13k^2 + 37k - 6$
- 7. $40y^2 + y - 6$
- 8. $6 - 9e - 27e^2$
- 9. $1 - t - 6t^2$
- 10. $2a^2 + 7ab - 15b^2$
- 11. $4y^2 + 24y + 20$
- 12. $12a^2 + 2a - 4$

13. $21k^3 + 15k^2 - 6k$

14. $6x^5 - 22x^3 - 8x$

15. $-21z^4 - 12z^2 + 6z^6$

16. $-12b^5 - b^3 + b$

17. The area of a rectangular park is $5x^2 + 17x + 6$ square metre. Of one side is $(x + 3)$. Find out second adjacent side.
18. The area of a square is $(4x^2 - 36x + 81)$ square metre. Find out side of this square. If $x = 5$ then find out the value of area of the square and side of the square.

GROUP DISCUSSION

By division find out that—

- (a) $x^3 - 8$ is a factor of $(x - 2)$
- (b) $x^3 + 27$ is a factor of $(x + 3)$
- (c) $x^3 + 64$ is a factor of $(x^2 - 4x + 16)$
- (d) $x^4 - y^4$ is a factor of $x + y$
- (e) $x^3 - 216$ is a factor of $(x - 6)$

Efficiency Practice

1. Divide :

- (i) $y^2 - 18y + 69$ by $(y + 5)$
- (ii) $x^5 - y^5$ by $(x - y)$
- (iii) $x^4 - y^4$ by $(x + y)$
- (iv) $(6a^2 + 7ab - 20b^2)$ by $(3a - 4b)$
- (v) $10x^3 - 39x^2 + 41x - 15$ by $(2x - 5)$

2. Find out quotient and remainder

- (i) $(9x^3 - 45x^2 + 71x - 40) \div (3x - 8)$
- (ii) $(27a^3 - 54a^2b + 36ab^2 - 8b^3) \div (3a - b)$
- (iii) $(6x^3y^2 - 4x^2y - 8y^2) \div (-2xy)$
- (iv) $(-9x^3y^3 - 6x^2y^2 + 12x^2y^3) \div (-3x^2y^2)$
- (v) $(4x^5y^5 - 2x^3y^3 + 6x^3y^5) \div (-2x^3y^3)$

3. Factorize the following expressions :

- (i) $1 - x^2$
- (ii) $64a^6 - 49b^2c^4$
- (iii) $x^2y^2 - 4$
- (iv) $\frac{1}{4}b^2 - 49$

$$(v) \quad c^2 - 0.36 \quad (vi) \quad \frac{x^2}{9} - \frac{y^2}{4}$$

$$(vii) \quad (a+b+c)^2 - (a-b-c)^2 \quad (viii) \quad 49 - x^2 - y^2 + 2xy$$

4. Using factorization simplify the following

$$(i) \quad \frac{a^2 - b^2}{a + b} \quad (ii) \quad \frac{9a^2 - 16b^2}{3a - 4b}$$

$$(iii) \quad \frac{a^3b - b^3a}{ab} \quad (iv) \quad \frac{50a^2 - 98b^2}{10a - 14b}$$

$$(v) \quad \frac{(x-1)(x-2)(x^2 - 9x + 14)}{(x-7)(x^2 - 3x + 2)}$$

5. Factorize :

$$(i) \quad 25 - 4y^2 + 21y \quad (ii) \quad 5x^2 + 3x - 14$$

$$(iii) \quad 2x^2 + 5x - 25 \quad (iv) \quad 3x^2 + 5x - 28$$

$$(v) \quad 24 - 3a^2 + 34a \quad (vi) \quad y^2 + 18y - 40$$

$$(vii) \quad a^2 + 10a - 24 \quad (viii) \quad x^2 + 22x - 48$$

We have learnt

1. The numerical coefficient of the quotient of two monomial expression is quotient of numerical coefficient of this expression. The variable coefficient of quotient of two monomial expression is quotient of variable coefficient of the same expression.
2. Those algebraic expression in which the power of variable in the form of numerator and denominator positive integer are called rational expression.
3. If the remainder in dividing a polynomial by any other polynomial is zero, then this way of division the divisor and obtained quotient are factor of the dividend.

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

If the remainder is other than zero then

$$\text{Dividend} = \text{Divisor} \times \text{quotient} + \text{Remainder}$$

4. $\therefore a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$ so
 $(a + b)$ and $(a + b)$ are factor of $a^2 + 2ab + b^2$
5. For expression of the type $a^2 - 2ab + b^2$ the form of factor is $(a - b)^2$.
6. Difference of two square number is equal product of sum and difference of their square roots.
 that is $a^2 - b^2 = (a + b)(a - b)$
7. The factor of algebraic expression $x^2 + (a + b)x + ab$ is of the type $(x + a)(x + b)$.
8. In factorizing the algebraic expression $ax^2 + bx + c$ take numbers l and m such that $l + m = b$ and $lm = ac$.



ANSWERS

Exercise 5(a)

1. (a) $4xz$ (b) $5y$ (c) $(a - b)$ 2. (a) $8my$ (b) $(x + 2)^2$ (c) $2x(2 - x)$ (d) $(6a^2 + 3a - 8)$

Exercise 5(b)

1. (a) 3, (b) 4, (c) Zero, (d) 2 (e) 7, (f) 5, (g) 4, 2. (a) $3z^3 + 4z^2 - 2$, (b) $(x + 5)$,
 (c) $(4y + 1)$, (d) $-8z^2 - 12z + 2$, (e) $3x^2 - 2x + 9$, (f) $x^4 - x^3y + x^2y^2 - xy^3 + y^4$,
 3. (a) Quotient $2x + 3$ Remainder $= -3$, (b) Quotient $3y^3 - 5y + \frac{3}{2}$ Remainder $= \text{Zero}$,
 (c) Quotient $-4x^3 + 2x^2 - 8x + 30$ Remainder $= -285$, (d) Quotient $5y^3 - 2y^2 + \frac{5}{3}y$
 Remainder $= 6$, 4. (a) Yes, Quotient $= (x - 7)$, Remainder $= 0$ (b) Quotient $\left(x - \frac{5}{2}\right)$
 Remainder $\left(-\frac{39}{2}\right)$, No, (c) Quotient $2x^3 - 5x + \frac{5}{2}$ Remainder $= \frac{-5}{2}$ No, (d) Quotient $2z^3$
 $+ 5z^2 + 2z - 7$ Remainder Zero, Yes, (e) Quotient $x^3 - 3x$ Remainder Zero, yes, (f) Quotient
 $x^4 + x^2 + 1$ Remainder Zero, Yes.

Exercise 5(c)

1. (iii) $x^2 + 10x + 25$, (iv) $49m^2 + 140mn + 100n^2$ 2. (i) $(a+1)^2$, (ii) $(6+x)^2$ (iii) $(2c+1)^2$ (iv) $(6x+5)^2$ (v) $(3x+1)^2$ 3. (i) $\left(x + \frac{1}{2}\right)^2$, (ii) $\left(p + \frac{5}{2}\right)^2$

Exercise 5(d)

1. (i) $81a^2 - 72ab + 16b^2$ 2. (i) $(2x-3y)^2$ (ii) $(3x-1)^2$ (iii) $(5x-6)^2$ (iv) $(7x-4)^2$ (v) $(x-6y)^2$

Exercise 5(e)

1. $(a+2)(a-2)$ 2. $(a+7b)(a-7b)$ 3. $x(x+11)(x-11)$ 4. $\left(2a + \frac{3}{2a}\right)\left(2a - \frac{3}{2a}\right)$ 5. $2\left(\frac{3}{x} + \frac{x}{3}\right)\left(\frac{3}{x} - \frac{x}{3}\right)$ 6. $(a-b+c)(a-b-c)$ 7. $(a+3b)(a-9b)$ 8. $4ab$ 9. $(7a-31b)$ $(3a-19b)$ 10. $(4a^2+9b^2)(2a+3b)(2a-3b)$ 11. $(a^2+25)(a+5)(a-5)$ 12. $b(a+b)(a-b)$, 20100, 13. 25, 16

Exercise 5(f)

1. (i) 5, 3 (ii) 12, 1 (iii) 5, -4 (iv) -4, -1 (v) -4, 3 (vi) -10, -1 (vii) 10, -2;
2. (i) $(x+3)(x+2)$ (ii) $(q+4)(q+2)$ (iii) $(m+8)(m+3)$ (iv) $(y+12)(y-3)$ (v) $(a+5)(a-2)$ (vi) $(k-17)(k+6)$ (vii) $(p-16)(p+11)$ (viii) $-(x-8)(x+6)$ (ix) $(p-8)(p-3)$ (x) $(y-23)(y-3)$ (xi) $(a-3)(a+3)(a^2+4)$ (xii) $(y^2+8)(y+2)(y-2)$ (xiii) $2x(x+7)(x-2)$ (xiv) $-2y(y-12)(y+1)$ (xv) $-3(x-5)(x+1)$ (xvi) $-2p^3(q-10)(q+2)$ (xvii) $3k^3(k-8)(k+2)$ (xviii) $c^3(b+6c)(b+2c)$ (xix) $(ab-6)(a+3)$; 3. $(x+1)(4x+1)$; 4. $(x+2)(2x+7)$; 5. $(2y+3)(y-4)$; 6. $(13k-2)(k+3)$; 7. $(5y+2)(8y-3)$; 8. $3(2+3e)(1-3e)$; 9. $(1+2t)(1-3t)$; 10. $(a+5b)(2a-3b)$; 11. $4(y+1)(y+5)$; 12. $(3a+2)(4a-2)$; 13. $3k(k+1)(7k-2)$; 14. $2x(x+2)(x-2)(3x^2+1)$; 15. $3z^2(2z^2+1)(z+2)(z-2)$; 16. $-b(3b^2+1)(2b+1)(2b-1)$; 17. $(5x+2)$; 18. $(2x-9)m, 1m, 1m^2$.