

UNIT-6 PLAYING WITH NUMBERS



- To express two and three digits numbers in general form
- To find missing numbers in four basic mathematical operations (+, −, ×, ÷)
- Games and riddles of numbers.
- Divisibility test by numbers 2, 3, 5, 7, 9, 11 and 13.

6.1 Introduction

Numbers have always been the source of surprises and they have been surprising as the universe. Their history is as old as human civilization. It will not be hyperbolic if we say “the loving house of human civilization rests upon the development of numbers.” That is why human being has friendly relation with numbers and played many games with numbers from early ages. Today we live in the world of numbers. From morning till night men-women; boys-girls, adult, doctors, engineers, farmer, labour etc. all of us move in the world of numbers. Every activity in our daily life is associated with numbers like waking up at 5 O’clock in the morning, drinking a cup of tea, morning walk of 2-3 km; light breakfast at 9 a.m., to move for office or for work at 10 a.m. etc.

So many puzzles have been formed with the help of number and still today we form puzzles with numbers for example Japanese Sudoku, filling unknown numbers in these is the matter of pleasure for people of all age groups. Great mathematician Ramanujan had also made the world aware of many magical aspects of numbers. Decimal system itself, is not less than any magic where only ten signs (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are used to write any largest or smallest number. In this chapter, certain interesting forms of numbers, games and puzzles are discussed.

6.2 Writing and understanding of two and three digit number in their general form :

We know that

$$18 = 10 \times 1 + 8$$

$$27 = 10 \times 2 + 7$$

$$69 = 10 \times 6 + 9$$

$$80 = 10 \times 8 + 0$$

In above equalities, right hand side is expressing the numbers on the basis of Decimal system. In a two digit number right hand digit is unit's digit and left digit is ten's digit, that is why in 18, place value of 1 is $10 \times 1 = 10$. In 27 place value of digit 2 is $10 \times 2 = 20$ etc.

So this we conclude, if "a" be algebraic digit at tens place and "b" be the algebraic digit at units place, then general form of number will be —

$$10 \times a + b = 10a + b$$

A two digit number in general form is $10a + b$ where a and b are algebraic digits. Clearly 'a' is a non-zero digit and 'b' can be 0 to 9.

Try

(i) Fill in the blanks, with the help of method given above

$$15 = \boxed{} \times 1 + \boxed{}$$

$$42 = 10 \times \boxed{} + 2$$

$$60 = 10 \times \boxed{} + \boxed{}$$

$$99 = \boxed{} \times \boxed{} + \boxed{}$$

(ii) Take some two digit numbers and by above method make equalities of those numbers

See again

$$345 = 100 \times 3 + 10 \times 4 + 5$$

$$416 = 100 \times 4 + 10 \times 1 + 6$$

$$500 = 100 \times 5 + 10 \times 0 + 0$$

$$999 = 100 \times 9 + 10 \times 9 + 9$$

In a three digit number if algebraic digits are used then what will be its general form ? According to decimal system if c, b, a are digits of once, tens and hundreds place respectively then number will be $100 \times a + 10 \times b + c$.

From this we conclude :

General form of three digit number is $100a + 10b + c$ where a, b, c are algebraic digits clearly a is a non-zero digit while b and c are digits from 0 to 9.

Remark : Note if a is zero then two digit number $10a + b$ will become only one digit number and in the same way if a is zero then three digit number $100a + 10b + c$ will remain a two digit number only.

Try

With the help of above methods write 247; 484; 875; 907

- Say what is the space value of 3 in 365 ?
- What is the space value of zero in 405 ?
- What is the space value of 7 in 817 ?

6.3 To find missing numbers in four basic operation (+, -, $\times$, $\div$)

(i) Observe the following addition operation :

$$\begin{array}{r} 2 \quad \boxed{x} \\ + 4 \quad 8 \\ + 9 \quad 5 \\ \hline 1 \quad 6 \quad 6 \end{array}$$

Note that in the above addition operation “ $\boxed{x}$ ” is missing digit. Whose value is to be calculated,

$$\begin{aligned} \text{Now sum of digits at units place} &= \boxed{x} + 8 + 5 \\ &= \boxed{x} + 13 \\ &= 10 + (\boxed{x} + 3) \end{aligned}$$

$$\text{So units place in the sum} = \boxed{x} + 3$$

$$\begin{aligned} \boxed{x} + 3 &= 6 \\ \boxed{x} &= 6 - 3 \\ &= 3 \end{aligned}$$

Means 3 is the missing digit.

See again

$$\begin{array}{r}
 37 \\
 + \boxed{x}6 \\
 + 88 \\
 \hline
 171 \\
 \hline
 \end{array}$$

Here sum of digits on unit's place = $7 + 6 + 8 = 21$

Where 2 is at ten's place digit, so in the operation of addition it will be transferred to ten's place digit.

$$\begin{aligned}
 \text{So, sum of digits on ten's place} &= 2 + 3 + \boxed{x} + 8 \\
 &= \boxed{x} + 13
 \end{aligned}$$

which is equivalent to 17 in above sum,

$$\begin{aligned}
 \text{so } \boxed{x} + 13 &= 17 \\
 \boxed{x} &= 17 - 13 \\
 &= 4
 \end{aligned}$$

Try

● As per above, find missing digit in the following addition – operation.

$$\begin{array}{r}
 \text{(i)} \quad 3\boxed{x} \\
 76 \\
 23 \\
 \hline
 137 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(ii)} \quad 56 \\
 72 \\
 \boxed{x}7 \\
 \hline
 215 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(iii)} \quad 82 \\
 55 \\
 99 \\
 \hline
 \boxed{x}36 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(iv)} \quad 453 \\
 \boxed{x}56 \\
 128 \\
 \hline
 937 \\
 \hline
 \end{array}$$

(ii) Now observe following subtraction operation

$$\begin{array}{r}
 98 \\
 - \boxed{x}5 \\
 \hline
 53 \\
 \hline
 \end{array}$$

Here at tens place, in subtraction operation

$$9 - \boxed{x} = 5 \text{ where } \boxed{x} \text{ symbol for missing number}$$

$$\begin{aligned}
 \text{so } \boxed{x} &= 9 - 5 \\
 &= 4
 \end{aligned}$$

Again observe following subtraction operation given below :

$$\begin{array}{r} 8 \ 3 \\ - 2 \ \boxed{x} \\ \hline 5 \ 5 \\ \hline \end{array}$$

Here, at units place by subtraction of $\boxed{x}$ (missing digit) from 3 we get 5. Which means $\boxed{x}$ is more than 3. So to perform subtraction operation, one tens from 8 tens digit at tens place is to be transferred to ones place.

So, $(10 + 3) - \boxed{x} = 5$

$$\begin{aligned} \therefore \quad \boxed{x} &= 10 + 3 - 5 \\ &= 8 \end{aligned}$$

Here it is clear while transferring one tens from tens place to ones place number of remaining tens $= 8 - 1$
 $= 7$

So by subtracting 2 from 7 we get 5, which is given.

Try

As per above examples find missing digit in the following subtraction operations

(i)
$$\begin{array}{r} 8 \ 9 \\ - 6 \ \boxed{x} \\ \hline 2 \ 5 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 7 \ 6 \\ - 4 \ \boxed{x} \\ \hline 2 \ 8 \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} 8 \ 4 \\ - \boxed{x} \ 8 \\ \hline 1 \ 6 \\ \hline \end{array}$$

(iv)
$$\begin{array}{r} 8 \ 0 \ 3 \\ - 2 \ \boxed{x} \ 6 \\ \hline 5 \ 6 \ 7 \\ \hline \end{array}$$

(iii) **Observe following multiplication operation**

$$\begin{array}{r} 5 \ 6 \\ \times 2 \ \boxed{x} \\ \hline 1 \ 3 \ 4 \ 4 \\ \hline \end{array}$$

Note : Here multiplier has algebraic digit x at unit place value of x is to be calculated

$$56 \times 2x = 1344$$

$$56 \times (20 + x) = 1344$$

$$56 \times 20 + 56x = 1344$$

$$1120 + 56x = 1344$$

$$56x = 1344 - 1120$$

$$56x = 224$$

$$\therefore x = \frac{224}{56}$$

$$= 4$$

Again observe following multiplication operation

$$\begin{array}{r} \text{B A} \\ \times \text{B 6} \\ \hline 72 \text{ A} \end{array}$$

Here we have to find value of A and B. Multiplication of units place digit $A \times 6 = A$ so A can be 6 or 8.

Now look at B, if $B = 1$ then the product of $BA \times B6$ will be $16 \times 16 = 256$ or $18 \times 18 = 288$

But here product is 72 A so B can not be 1. If $B = 3$ then the product of $BA \times B6 = 36 \times 36 = 1296$ or $38 \times 36 = 1368$ but product is 72A. So B is not equal to 3.

So B has value less than 3 say 2. So the given product is 26×26 or 28×26 .

First possibility $= 26 \times 26 = 676$ is not true.

Second possibility $28 \times 26 = 728$ is correct

So $A = 8$ and $B = 2$

Hence,

$$\begin{array}{r} 28 \\ \times 26 \\ \hline 728 \end{array}$$

again observe the following multiplication operation

$$\begin{array}{r} \boxed{x} 6 \\ \times 73 \\ \hline 6278 \end{array}$$

Here $\boxed{x}$ means at tens place missing number x is written whose value is to be calculated.



Solve

Now $\boxed{x} 6 \times 73 = 6278$

$$(10x + 6) \times 73 = 6278$$

$$730x + 438 = 6278$$

then $730x = 6278 - 438$

$$730x = 5840$$

$$\therefore x = \frac{5840}{730}$$

$$\therefore x = 8$$

Try

- As per the above example find the missing algebraic digits in the following multiplication operation

$$\begin{array}{r} \text{(i)} \quad 36 \\ \times \boxed{x} 5 \\ \hline 900 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(ii)} \quad 4 \boxed{x} \\ \times 38 \\ \hline 1596 \\ \hline \end{array}$$

(iv) Now observe following division operation :

$$\boxed{x} 2) 84 (7$$

$$\begin{array}{r} \dots \\ \hline 0 \\ \hline \end{array}$$

Here variable letter digit x is written at divisor's tens place. Whose value is to be calculated. By division operation we know that

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

means $84 = \boxed{x} 2 \times 7 + 0$

$$84 = (10x + 2) \times 7$$

$$84 = 70x + 14$$

$$84 - 14 = 70x$$

$$\therefore x = \frac{70}{70}$$

$$x = 1$$

Therefore 12 is the divisor.

Observe following division operation

$$2 \overline{) 96} (4$$

$$\begin{array}{r} \dots \\ \underline{4} \\ \hline \end{array}$$

We have to find the value of variable letter digit “x” written at the units place of divisor.

Here according to division operation

$$2 \overline{) 96} \times 4 = 96 - 4$$

$$\text{or, } (20 + x) \times 4 = 92$$

$$\text{or, } 80 + 4x = 92$$

$$4x = 92 - 80$$

$$4x = 12$$

$$x = \frac{12}{4}$$

$$x = 3$$

Try

- As per the above example find the missing algebraic digits in the following division operation

$$(i) \quad \overline{) 82} (5$$

...

$$\underline{12}$$

$$(ii) \quad \overline{) 105} (5$$

...

$$\underline{0}$$

$$(iii) \quad 2 \overline{) 110} (4$$

...

$$\underline{2}$$

$$(vi) \quad 27 \overline{) 224} (x$$

...

$$\underline{x}$$

5.4 Number puzzle and Games :

World of numbers is amazing, there are many entertaining and attractive creation. History of numbers is associated along with development of human civilization and many mathematician played interesting games with numbers formulated puzzles and solved them. Some of these are represented here.

Squaring a number whose unit place digit is 5 :

Let $(\overline{) 5})$ be a number with unit place digit 5 and tens place digit is an algebraic digit number x

$$\begin{aligned}
 \text{Now } (\overline{x} 5)^2 &= (10x + 5)^2 \\
 &= 100x^2 + 100x + 25 \\
 &= 100x(x + 1) + 25 \\
 &= \overline{x(x+1)25}
 \end{aligned}$$

Example : Suppose we have to square the number 75 then first of all write 25 the square of 5 and write again (means on the left)

$7 \times (7 + 1)$ value = 56. This is square of 75.

means $(75)^2 = 5625$

Similary $(95)^2 = \overline{9(9+1)25}$
 $= 9025$

& $(125)^2 = \overline{12(12+1)25}$
 $= 15625$

Try

As per above given examples find square of 45, 85, 135 and 165.

Digit puzzles

- (1) Take any three digit number. Write it at a proper place on a plane paper. Now write the same number again either on left or right of the number in alignment. In this way we get a six digit new number.

Now divide the number obtained by 7. Divide the quotient obtained by 11. Again divide this new quotient by 13. Observe the new quotient obtained. Are you surprised you again obtained the same number you have written in the beginning.

Example : Let your number be 453. Now you again write 453 either in left or right in a line. The number obtained = 453453. Applying above said steps as under

7	453453
11	64779
13	5889
	453

In this way you can play with any three digit number.

Verification : In decimal system if we write three digit number as abc then numeral value will be $100a + 10b + c$. Now general form of a six digit number will be-

$$\begin{aligned}
 &= 100000a + 10000b + 1000c + 100a + 10b + c \\
 &= (100000a + 100a) + (10000b + 10b) + (1000c + c) \\
 &= (100100a + 10010b + 1001c) \\
 &= 1001 \times 100a + 1001 \times 10b + 1001 \times c \\
 &= 1001 (100a + 10b + c) \\
 &= 7 \times 11 \times 13 (100a + 10b + c)
 \end{aligned}$$

(2) **On subtraction of sum of digits of a number from the number difference obtained is fully divisible by 9.**

Example :

$$\begin{aligned}
 867 - (8 + 6 + 7) &= (800 + 60 + 7) - (8 + 6 + 7) \\
 &= (800 - 8) + (60 - 6) + (7 - 7) \\
 &= 792 + 54 + 0 \\
 &= 9 \times 88 + 9 \times 6 \\
 &= 9 \times (88 + 6) \\
 &= 9 \times 94
 \end{aligned}$$

$$\begin{aligned}
 726 - (7 + 2 + 6) &= (700 + 20 + 6) - (7 + 2 + 6) \\
 &= (700 - 7) + (20 - 2) + (6 - 6) \\
 &= 693 + 18 + 0 \\
 &= 9 \times 77 + 9 \times 2 \\
 &= 9 \times (77 + 2) \\
 &= 9 \times 79
 \end{aligned}$$

In this way one can enjoy the operation by selecting any number.

Now with the help of this magic we play games of missing number. You can assume any three digit number at your own. Get, the sum of its digit, subtracted from the number and have the result in your mind. or write it in a paper. Now tell all digits of new number except one digit. We can tell what digit you have skipped.

Example :

If number taken by you is 762 then sum of digits of the number ($7 + 6 + 2$) means 15 is subtracted from the number gives 747. Suppose your skipped digit is 4. Now both the remaining digits are 7. Now to find skipped number, we take sum of remaining digits ($7 + 7 = 14$) and get it subtracted from the nearest multiple of 9 to it. Which is 18 clearly $18 - 14 = 4$ is the number skipped by you. If you skip the digit 7, then sum of remaining digits is ($7 + 4 = 11$) and nearest multiple of 9 near to it is 18, which is $18 - 11 = 7$ less to it. So number skipped is 7.

Remark :

Here one point to be noted carefully is that, while skipping the digit from the number obtained by subtracting sum of digits from the number, we should not skip 0 or 9, if there in the difference. As in this case sum of remaining numbers will be exactly divisible by 9. Then uniqueness of skipped number is affected. We cannot say it will be either 0 or 9.

- (3) **Select any three digit number in which difference of units and hundred digit is more than 1. Now get a new number by reversing the order of digits. Find the difference of the original number and new number. Now in the difference obtained add the number obtained by reversing the order of digits. You will find sum is always 1089 ?**

Example : Suppose we select number 853 (Here difference of units digit and hundreds digit is more than 1) which suits to the certain said above

Now, number obtained by reversing order of digits = 358

Difference of original number and new number

$$= 853 - 358 = 495$$

Again number obtained by reversing order of digits = 594

Now we see

Sum of 495 and 594

$$= 495 + 594$$

$$= 1089$$

Now you can play by another three digit number where difference of digits at ones and hundreds should be more than 1.

Verification : Let the number of three digit is

$$10a + 10b + c \quad \dots (1)$$

where a, b, c are digits of hundreds, tens and unit place
a and c have difference more than 1.

Now reversing the order, new number

$$100 c + 10 b + a \quad \dots (2)$$

Subtracting (2) from (1)

$$100 (a - c) + 0 + (c - a)$$

$$\text{or } 100 (a - c) - 100 + 100 + (c - a)$$

$$\text{or } 100 (a - c - 1) + 90 + (10 + c - a) \quad \dots (3)$$

by reversing the order of digits in above number we get

$$100 \times (10 + c - a) + 90 + 1. (a - c - 1) \quad \dots (4)$$

adding (3) and (4)

$$\begin{aligned} & 100 (a - c - 1 + 10 + c - a) + 180 + (10 + c - a + a - c - 1) \\ & = 900 + 180 + 9 = 1089 \end{aligned}$$

Mathematical games and puzzles by participation of students.

(1) Game between two students

Suppose A and B are two colleagues. They are said to say any digit between 1 to 6. Game starts from A. B after A then A after B then B after A, in the same order continued. Simultaneously progressive sum of the said numbers is done. Limit of the sum 50, 60, ... maximum hundred is fixed and then game is started. Condition of the game will be that player who reaches the fixed number by his uttered number progressive sum will be the winner. It is obvious that each candidate has to be careful while uttering the number and has to be careful about his progressive total.

This game can be played by taking numbers upto 6, 7, 8, 9 students himself find the method of winning.

(2) Game of guessing numbers :

In this game all student of the class can participate. All students are said to take any number from 1 to 9. They are said to multiply the taken number by 5 and add 6 to the product. Again multiply the sum obtained by 4 and add 9 to the product. They are again said to multiply by 5 in this new sum.

Now ask this last product to each student one-to-one then subtract 165 from the product told by the student and remainder found is to be divided by 100. Whatever quotient you got is the number taken by the student.

How the number guessed is to be found by participation of the students.

In the same way frame some question with help of students.

(3) One more puzzle of guessing a number :

A student is said to take any two numbers less than 10. Again the taken number multiplied by 5 and 7 added to product, then found sum is said to multiply by 2. Now second number is to be added in the product obtained finally. Student is said to tell this sum.

By subtracting 14 from the sum told by the student; digits obtained in the difference are the numbers taken by the student originally.

Reason of guessing this puzzle should be found with the help of students participation.

Example : Let us suppose student has taken two numbers 3 and 8 less than 10.

Now as per procedure discussed above.

$$\begin{aligned}\{2 (5 \times 3 + 7) + 8\} - 14 &= 44 + 8 - 14 \\ &= 38\end{aligned}$$

which is formed by the digits 3 and 8, taken by the student,

One more example

If numbers taken by the students be 7 and 3 then by above said procedure :

$$\begin{aligned}\{2 \times (5 \times 7 + 7) + 3\} - 14 &= \{84 + 3\} - 14 \\ &= 73\end{aligned}$$

which is made of taken numbers 7, 3.

6.5 Divisibility and steps to be performed for the division of two or three digit general form number by 2, 3, 5, 7, 9, 11, 13.

Rule of divisibility by 2 :

Any number with even digit on unit place (means 2, 4, 6, 8, 0 will be divisible by 2.

Any number abc can be written as $100a + 10b + c$

100 a and 10 b are obviously divisible by 2 as 100 and 10 are divisible by 2. If c is an even number then it will also be divisible by 2 so in order for abc to be divisible by 2, c must to be 0, 2, 4, 6 or 8.

Try

Check divisibility of the Numbers 54; 592; 575; 1890; 12884 by 2.

Divisibility rule of 3 and 9 :

We know the general form of a two digit number is

$$\begin{aligned} &= 10a + b &= (9 + 1)a + b \\ &= 9a + (a + b) \end{aligned}$$

in the above first part 9a is divisible by 9 and 3 both.

so if second part (a + b) is divisible by 3 and 9 only then the considered number will be divisible by 3 and 9, otherwise not.

In the same way we see the general form of a three digit number

$$\begin{aligned} &= 100a + 10b + c &= (99 + 1)a + (9 + 1)b + c \\ & &= 99a + 9b + a + b + c \\ & &= 9(11a + b) + a + b + c \end{aligned}$$

In this also first part is divisible by 3 and 9. Now if second part (a + b + c) is also divisible by 3 and 9 then the number considered above will be divisible by 3 and 9 otherwise not.

So any number is divisible by 3 and 9 only if the sum of its digits is divisible by 3 and 9.

Example

- (1) Sum of digits in the numbers 72 $= 7 + 2$
 $= 9$ is divisible by 3 and 9.

so 72 is divisible by 3 and 9.

(2) Sum of digits of the number 729 = $7 + 2 + 9$
 $= 18$ is divisible by 3 and 9.

So 729 is also divisible by 3 and 9.

From above examples we conclude :

Any two or more digit number is divisible by 3 and 9 only if its sum of digits is divisible by 3 and 9.

Example 3. If 21y5 is a multiple of 9. Where y is a digit. Find the value of the digit.

Solution : 21 y 5 is a multiple of 9 means divisible by 9

So the sum of digits $2 + 1 + y + 5 = 8 + y$ should be divisible by 9 it is possible only if $8 + y$ has value 9 or 18 as y is a digit.

$$\therefore 8 + y = 9$$

$$y = 9 - 8$$

$$y = 1$$

so value of y is 1

Try

(1) Check divisibility of following numbers by 3 and 9

216; 726; 525; 1008; 3735.

(2) Whether 71; 98 and 112 divisible by 3 ? If no, why ?

Rules of divisibility by 5 :

We see that

55 is divisible by 5; 70 is divisible by 5; 125 is divisible by 5, but 62; 78; 89; 91; 94 etc. are not divisible by 5.

It is clear that in general form of two digit number $10a + b$ first part $10a$ is divisible by 5. Now if second part is also divisible by 5 then the number will be divisible by 5.

Similarly in a three digit number general form

$$\begin{aligned} &= 100a + 10b + c \\ &= 100a + 10b + c \\ &= 10(10a + b) + c \end{aligned}$$

in this first part $10(10a + b)$ is divisible by 5.

Now if second part c is divisible by 5 then the number considered will be divisible by 5, otherwise not. It is to be noted that c will be divisible by 5 only if it has values 0 or 5. So by this we can conclude.

Any two or three digit number will be divisible by 5 only if their unit place digit is 0 or 5.

Note : above conclusion also applies for the numbers with more than three digits.

Divisibility test of 7:

For divisibility test of number by 7 digits of the number on units, tens, hundreds; thousands, ten thousands are multiplied in order by 1, 3, 2, -1, -3, -2, ... respectively and sum of the product is calculated. If sum is divisible by 7 then number will be divisible by 7. If number is of more than six digits then these multipliers are repeated again. Take an example to clarify :

Example : Check divisibility of the numbers 2,39,10,551 by 7.

From above rules we see.

Units digit	$\times 1 = 1 \times 1 = 1$
tens digit	$\times 3 = 5 \times 3 = 15$
hundreds digit	$\times 2 = 5 \times 2 = 10$
thousands digit	$\times -1 = 0 \times (-1) = 0$
ten thousand digit	$\times -3 = 1 \times (-3) = -3$
lakhs digit	$\times -2 = 9 \times (-2) = -18$
ten lakhs digit	$\times 1 = 3 \times 1 = 3$
crores digit	$\times 3 = 2 \times 3 = 6$

Sum = 14 which is divisible by 7.

So the number 2,39,10,551 is divisible by 7.

Similarly take some numbers and check their divisibility by 7.

Verification of above rule :

Three digit general number

$N = 100a + 10b + c$ (where a, b, c are hundreds, tens and ones digit respectively)

Now according to above said rule c, b, and a are multiplied by 1, 3, 2 respectively and added sum $S = 2a + 3b + c$

So $6S = 12a + 18b + 6c$

$$\begin{aligned}N + 6S &= (100a + 10b + c) + (12a + 18b + 6c) \\&= 112a + 28b + 7c \\&= 7 (16a + 4b + c)\end{aligned}$$

In this way $N + 6S$ is divisible by 7. If N is divisible by 7 then $6S$ will also be divisible by 7.

It implies S will also be divisible by 7.

Conversely if S is divisible by 7 then N is also divisible by 7.

Divisibility test of 11

To test divisibility of any number by 11, digits of units, tens, hundreds, thousands, lakh, ten lakh, of the number are multiplied by 1, -1, 1, -1, 1, -1, respectively and product is added.

If the total is completely divisible by 11 then the given number will also be divisible by 11.

Example : Check divisibility of 99, 62, 425 by 11.

by above rule we see that

$$\begin{aligned} & 5 \times 1 + 2 \times (-1) + 4 \times 1 + 2 \times (-1) + 6 \times 1 + 9 \times (-1) + 9 \times 1 \\ &= 5 - 2 + 4 - 2 + 6 - 9 + 9 \\ &= 11 \text{ is divisible by 11} \end{aligned}$$

So the given number 9962425 is divisible by 11.

Verification of above rule :

Let us consider general form of a four digit number $N = 1000a + 100b + 10c + d$ whose divisibility by 11 is to be checked. Here units, tens, hundreds and thousands digit are d, c, b and a respectively.

by above rule

$$\begin{aligned} S &= d(1) + c(-1) + b(1) + a(-1) \\ &= -a + b - c + d \\ \therefore N - S &= (1000a + 100b + 10c + d) - (-a + b - c + d) \\ &= 1001a + 99b + 11c \\ &= 11(91a + 9b + c) \text{ which is divisible by 11 completely.} \end{aligned}$$

Now if N is divisible by 11 then S is also divisible by 11.

Conversely if S is divisible by 11 then N is also divisible by 11.

Try

Choose few numbers on your own and check their divisibility by 11.

Rule of divisibility by 13 :

To check divisibility of any number by 13, we multiply its digits of units, tens, hundreds, thousand, ten thousands, lakh, ten lakh, by the numbers -1, 3, 4, 1, -3, -4, respectively and add the products obtained. If sum is divisible by 13 then the number is also divisible by 13.

Example : Check divisibility of 1066195 by 13.

According to above said rule

$$= 5 \times (-1) + 9 \times 3 + 1 \times 4 + 6 \times 1 + 6 \times (-3) + 0 \times (-4) + 1 \times (-1)$$

$$= -5 + 27 + 4 + 6 - 18 + 0 - 1$$

$$= 13 \text{ which is divisible by 13.}$$

So the number 1066195 is divisible by 13.

Verification of the above rule :

Let N be any three digit number

So; $N = 100a + 10b + c$ where c, b, a are digits as units, tens and hundreds place.

$$\text{Now} \quad S = a \times (4) + b \times (3) + c \times (-1)$$

$$\text{or} \quad S = 4a + 3b - c$$

$$\text{Therefore } N + S = 104a + 13b = 13(8a + b)$$

which is divisible by 13.

So if N is divisible by 13 then S is also divisible by 13.

Conversly if S is divisible by 13 then N is also divisible by 13.

Try

Choose few numbers at your own and check their divisibility test by 13.

Divisibilit rules of 7, 11 and 13.

Teachers can tell the divisibility test of number 452912356 by 7, 11, 13 by the following steps.

- (i) Number 452912356 is grouped in triplets starting from right.

452 912 356

- (ii) Take sum triads at odd place

$$452 + 356 = 808$$

- (iii) Sum obtained will get its difference from the sum at even place triads.

$$912 - 808 = 104$$

- (iv) Number obtained if divisible by 7, 11, 13 then number considered will be divisible by 7, 11, 13. Here 104 is divisible by 13 so our considered number 452912356 is divisible by 13 only.

Note : If any number have no triplet on the left most side then add zero and frame triplet. Example number 4256 have no left side triplet so number written as 004256 has triplet 004 and 256.

Verification :

Suppose number abcdefg has to be checked for divisibility by 7, 11 and 13.

- (i) Number abcdefg is framed with triplets starting from Right

$$00a \quad bcd \quad efg \quad \dots\dots\dots (i)$$

- (ii) Difference of totals at add place triplets is to that of even place triplets is

$$00a + efg - bcd$$

- (iii) $a \times 10^6 + b \times 10^5 + c \times 10^4 + d \times 10^3 + e \times 10^2 + f \times 10 + g$
 $- a - e \times 10^2 - f \times 10 - g + b \times 10^2 + c \times 10 + d$
 $= a (10^6 - 1) + b (10^5 + 10^2) + c (10^4 + 10) + d (10^3 + 1)$
 $= a (10^3 - 1) (10^3 + 1) + b \times 10^2 \times (10^3 + 1) + c \times 10 (10^3 + 1) + d \times 1001$
 $= a \times 1001 \times (10^3 - 1) + b \times 10^2 \times 1001 + c \times 10 \times 1001 + d \times 1001$
 $= 1001 [a \times (10^3 - 1) + b \times 10^2 + c \times 10 + d]$

So difference is divisible by 1001. and $(7 \times 11 \times 13 = 1001)$

So the number considered is divisible by 7, 11, 13.

GROUP DISCUSSION

1. By interchanging the digits of 83 and number obtained is subtracted from original number. What number we obtain ?

2. Find numerical value of x in the following question :

$$\begin{array}{r} \text{(i)} \quad 28 \\ + 32 \\ + \boxed{x}7 \\ \hline 127 \end{array}$$

$$\begin{array}{r} \text{(ii)} \quad 732 \\ - 2\boxed{x}7 \\ \hline 445 \end{array}$$

$$\begin{array}{r} \text{(iii)} \quad 48 \\ \times \boxed{x} \\ \hline 144 \end{array}$$

$$\begin{array}{r} \text{(iv)} \quad \boxed{x}) 366 \\ \dots \\ \hline 0 \end{array}$$

3. If three digit number 42x is divisible by 9 then find the value of x. Ans. = 3

Exercise 6 (a)

1. Find missing digit (or algebraic digit) in the following :

$$\begin{array}{r} \text{(i)} \quad 2\boxed{} \\ + \boxed{}8 \\ + 95 \\ \hline 186 \end{array}$$

$$\begin{array}{r} \text{(ii)} \quad 88x \\ + 905 \\ + x12 \\ \hline 2400 \end{array}$$

$$\begin{array}{r} \text{(iii)} \quad 72x \\ + 6x9 \\ + x18 \\ \hline 1702 \end{array}$$

$$\begin{array}{r} \text{(iv)} \quad 87 \\ - 2\boxed{} \\ \hline 58 \end{array}$$

$$\begin{array}{r} \text{(v)} \quad 4\boxed{}5 \\ - 281 \\ \hline 194 \end{array}$$

$$\begin{array}{r} \text{(vi)} \quad 672 \\ - 2\boxed{x}5 \\ \hline \boxed{x}87 \end{array}$$

$$\begin{array}{r} \text{(vii)} \quad 68 \\ \times \boxed{x} \\ \hline 612 \end{array}$$

$$\begin{array}{r} \text{(viii)} \quad 93 \\ \times \boxed{x}7 \\ \hline 2511 \end{array}$$

$$\begin{array}{r} \text{(ix)} \quad 872 \\ \times 3\boxed{x} \\ \hline 30520 \end{array}$$

$$\begin{array}{r} \text{(x)} \quad 2\boxed{x}) 2168 \\ \dots \\ \hline 0 \end{array}$$

$$\begin{array}{r} \text{(xi)} \quad 42) 886(\boxed{x}1 \\ \dots \\ \hline 4 \end{array}$$

$$\begin{array}{r} \text{(xii)} \quad \boxed{x}7) 907(24 \\ \dots \\ \hline 19 \end{array}$$

2. Find which number among 828; 2340; 38046; 77514; 893408 and 100116 is not divisible by 3 ?

3. Numbers in question 2; which among them is divisible by 9.
4. Find out the number that are divisible by 7, 11 and 13 among the following and list them :
329623, 63271, 492895, 25179, 38632, 96283, 25137.
5. Find the remainder in the following numbers if divided by 5.
8034, 97446, 85405, 83560
6. Whether 722722 is a multiple of 7, 11, 13 ?
7. A number is 576. Sum of its digits subtracted from the number. Find difference is how many times of 9 ?
8. A number $\overline{x46}$ with algebraic digit x . Difference obtained by subtraction of a reverse digit number obtained from it and the original number is 297. Find value of $\overline{x}$.
9. Find the sum of divisible numbers by 2 or 5 between 1 to 1000.
(a) 2550 (b) 3050 (c) 3550 (d) 3600

Hint : Sum of natural numbers 1 to n is $\frac{n(n+1)}{2}$

We have learnt

1. General form of two digit number is $10a + b$ where a, b are algebraic digits clearly a is non-zero and b can take values 0 to 9.
2. General form of a three digit number is $100a + 10b + c$ where a, b, c are algebraic digits, a is non-zero b, c can take values 0 to 9.
3. We discussed some number puzzle and number game.
4. Divisibility tests of 2, 3, 5, 7, 9, 11 and 13 were demonstrated. These rules were verified algebraic.



ANSWERS

Exercise 6(a)

1. (i) 3 in first row; 6 in second row; (ii) 3 in first row, 6 in third row, (iii) 5 in first row, 5 in second row, 3 in third row, (iv) 9 (v) 7 (vi) 8 in second row, 3 in third row, (vii) 9 (viii) 2 (ix) 5 (x) 7 (xi) 2 (xii) 3. 2. 893408 3. 828; 2340; 100116; 4 numbers divisible by 7 are 329623, 25179 and 25137, number divisible by 11 are 25179; 38632; 96283 numbers divisible by 13 are 63271; 492895; 5. 4, 1, .0, 0, 6. Yes, 7. 62 times, 8. 9 9. (b) 3050.

