

UNIT-7

SIMULTANEOUS EQUATIONS



- Linear equations having two variables. (Simultaneous equations and their solution)
- Solution of word problems having two variables by simultaneous equations.

7.1 Introduction

We have learned to make linear equation of one variable, to solve and its verification in the previous class. In this class, we will study the solving methods of pair of linear equations having two variables as $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Here, in the equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, x and y are two variables and $a_1, a_2, b_1, b_2, c_1, c_2$ are constant quantities. Such type of equations are called simultaneous equations. The meaning of simultaneous is to be together. Here we will see the values of x and y out of the values of y corresponding to the values of x in the first equation which satisfy the second equation, they will be the solution of simultaneous equations.

7.2 Solution of Simultaneous equations :

First we concentrate on the following three examples—

1. Think over the situation of linear equations of two variables $2x + y = 5$ and $3x - y = 10$.

To find the solution of these tabulate the different values of y corresponding to different values of x for both equations.

(i) Table of values of y corresponding to x in the equation $2x + y = 5$

x	1	2	3	4	0	-1	-2	-3	-4
y	3	1	-1	-3	5	7	9	11	13

(ii) Table of values of y corresponding to x in the equation $3x - y = 10$.

x	1	2	3	4	0	-1	-2	-3	-4
y	-7	-4	-1	2	-10	-13	-16	-19	-22

It is clear to see the both tables that only $x = 3$ and $y = -1$ are common in both. Therefore $x = 3$ and $y = -1$ satisfy the equations $2x + y = 5$ and $3x - y = 10$. So $x = 3$ and $y = -1$ become the solution of these equations.

2. Now, we think over the solution of the pair of equations $x + 3y = 5$ and $2x + 6y = 10$. Here we see that both equations are not different from each other. Second equation is obtained by multiplying 2 in the both sides of the first equation. Therefore, these both equations are not independent equations. These equations show only one equation, so infinite values of x and y will satisfy. Here mind it to find the unique solution of equations of two variables need two independent equations.

3. In the last, we think over a pair of equations $x + 3y = 5$ and $2x + 6y = 7$. Here we see that any value of x and y which satisfies the equation $x + 3y = 5$, which will never satisfies the equation $2x + 6y = 7$. Because if for any value of x and y be $x + 3y = 5$, then $2x + 6y = 2(x + 3y) = 10$ will be which never can be 7. So, there will not be any value of x and y to satisfy the both equations. In these examples we see that the unique solution of equations obtains only in first situation. So from a pair of linear equations only in the first situation, simultaneous equations are formed. Further if we pay attention, then we find that in first condition :

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{-1} = -1$$

That is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ while in remaining both position

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

Thus,

Linear equations of two unknown variables $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are called simultaneous equations if :

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

7.3 Methods of solving the simultaneous equations

We saw at the time of solving simultaneous equations to make table that the method of solving the simultaneous equations to make table is not appropriate method. Therefore, often the following two methods are used to solve the simultaneous equations :

(i) Substitution method (ii) Elimination method

7.3.1 Substitution Method

According to this method, we find the value of any one unknown variable from any equation to substitute in other equation thus this value of first unknown variable form one equation and then find the value of first unknown variable to substitute the value of variable in other equation. Therefore, this method is called substitution method.

Example 1. Solve $x + 2y = 7$... (1)
 $2x + y = 5$... (2)

Solution : From equation (1)

$$x = 7 - 2y \quad \dots (3)$$

$\therefore$ Equations (1) and (2) are simultaneous equations,

$\therefore$ The value of x obtained from equation (1) should be satisfy equation (2).

Therefore, on substituting the value of obtained from equation (3) in equation (2),

$$2(7 - 2y) + y = 5$$

$$\text{or, } 14 - 4y + y = 15$$

$$\text{or } 14 - 3y = 5$$

$$\text{or } -3y = 5 - 14 \quad (\text{on transposing})$$

$$\text{or, } -3y = -9$$

$$\text{or, } 3y = 9$$

$$\text{or, } y = \frac{9}{3}$$

$$\therefore y = 3$$

Now substituting this value of y in equation (3),

$$x = 7 - 2 \times 3$$

$$= 7 - 6$$

$$\therefore x = 1$$

Hence, $x = 1$ and $y = 3$ is the solution of simultaneous equations.

Verification : Substituting the values of x and y in equation (1)

$$\begin{aligned}\text{Left hand side} &= x + 2y = 1 + 2 \times 3 \\ &= 1 + 6 \\ &= 7 \\ &= \text{Right hand side}\end{aligned}$$

$\therefore$ Left hand side = Right hand side

On substituting the values of x and y in equation (2)

$$\text{Left hand side} = 2x + y = 2 \times 1 + 3 = 5 = \text{Right hand side}$$

Hence, the solution is correct—

Example 2. Solve

$$2x + 5y = 12 \quad \dots (1)$$

$$4x + 9y = 22 \quad \dots (2)$$

Solution : From equation (1)

$$\begin{aligned}2x &= 12 - 5y \\ x &= \frac{12 - 5y}{2} \quad \dots (3)\end{aligned}$$

On substituting this value of x in equation (2),

$$\begin{aligned}4\left(\frac{12 - 5y}{2}\right) + 9y &= 22 \\ \text{or, } 2(12 - 5y) + 9y &= 22 \\ \text{or, } 24 - 10y + 9y &= 22 \\ \text{or, } 24 - y &= 22 \\ \text{or } y &= 24 - 22 \\ y &= 2\end{aligned}$$

Now, putting this value of y in equation (3)

$$\begin{aligned}x &= \frac{12 - 5 \times 2}{2} \\ &= \frac{12 - 10}{2} \\ &= \frac{2}{2} \\ &= 1\end{aligned}$$

Therefore, the solution of above simultaneous equations is $x = 1$ and $y = 2$.

Verification : On substitution the value of x and y in equation (1).

$$\begin{aligned}\text{Left hand side} &= 2x + 5y \\ &= 2 \times 1 + 5 \times 2 \\ &= 2 + 10 \\ &= 12 \\ &= \text{Right hand side}\end{aligned}$$

On substituting the values of x and y in equation (2)

$$\begin{aligned}\text{Left hand side} &= 4x + 9y \\ &= 4 \times 1 + 9 \times 2 \\ &= 4 + 18 \\ &= 22 \\ &= \text{Right hand side.}\end{aligned}$$

Therefore, answer is correct.

7.3.2 Elimination method

Sometime elimination of one variable becomes only on adding the given simultaneous equations or on subtracting from one to other that is only one unknown variable remains in the obtained equation, that's value is found by solving the obtained equation. The value of second variable is found on substituting the obtained value of first variable in any given equation. **From this type, the process of solving the simultaneous equations is called elimination method.**

If the coefficient of any variable do not be equal (or equal with opposite sign), then the coefficients of any variable are done equal. Remaining process is same as above.

Example 3. Solve

$$x + y = 7$$

$$2x - y = 8$$

Solution : In above simultaneous equations, the coefficients of y in both equations are equal and of opposite sign. Therefore, on adding both equations

$$x + y = 7 \quad \dots \quad (1)$$

$$2x - y = 8 \quad \dots \quad (2)$$

$$\begin{array}{r} x + y = 7 \\ 2x - y = 8 \\ \hline 3x = 15 \end{array} \quad \text{became the elimination of } y$$

$$x = \frac{15}{3} \text{ or, } x = 5$$

substituting the value of x in equation (1)

$$5 + y = 7$$

$$\therefore y = 7 - 5$$

$$= 2$$

Therefore, the solution of the equations is $x = 5$ and $y = 2$.

Verification :

On substituting $x = 5$ and $y = 2$ in equation (1),

Left hand side = $x + y = 5 + 2 = 7 =$ Right hand side.

On substituting $x = 5$ and $y = 2$ in equation (2).

Left hand side = $2x - y = 2 \times 5 - 2 = 10 - 2 = 8 =$ Right hand side

Example 4. Solve

$$x + 2y = 5 \quad \dots (1)$$

$$x + 3y = 7 \quad \dots (2)$$

Solution : In the above simultaneous equations, the coefficients of x in both equations are equal and same sign.

Therefore, on subtracting equation (2) from equation (1)

$$-y = -2$$

$$\therefore y = 2$$

On substituting the value of y in equation (1).

$$x + 2 \times 2 = 5$$

$$\text{or, } x + 4 = 5$$

$$\text{or, } x = 5 - 4$$

$$\therefore x = 1$$

Verification : On substituting $x = 1$ and $y = 2$ in equation (1),

Left hand side = $x + 2y = 1 + 2 \times 2 = 1 + 4 = 5 =$ Right hand side.

On substituting $x = 1$ and $y = 2$ in equation (2).

Left hand side = $x + 3y = 1 + 3 \times 2 = 1 + 6 = 7 =$ Right hand side.

Therefore, solution of the simultaneous equations is correct.

Example 5. Solve

$$3x + y = 5 \quad \dots (1)$$

$$5x + y = 9 \quad \dots (2)$$

Solution : Since in both equations, the coefficients of y are equal same sign, therefore subtracting equation (2) from equation (1)

$$-2x = -4$$

$$\text{or, } 2x = 4$$

$$\therefore x = 2$$

On substituting the value of x in equation (1)

$$3 \times 2 + y = 5 \text{ or } y = 5 - 6 = -1$$

Verification : On substituting $x = 2$ and $y = -1$ in equation (1)

$$\text{Left hand side} = 3x + y = 3 \times 2 + (-1)$$

$$= 6 - 1$$

$$= 5 = \text{Right hand side}$$

On substituting $x = 2$ and $y = -1$ in equation (2)

$$\text{Left hand side} = 5x + y = 5 \times 2 + (-1)$$

$$= 10 - 1$$

$$= 9 = \text{Right hand side}$$

Therefore the solution of simultaneous equations is correct.

Example 6. Solve

$$3x + y = 4 \quad \dots (1)$$

$$x + 2y = 3 \quad \dots (2)$$

Solution : In above simultaneous equations. Coefficient of any of x and y are not equal. Therefore, to equalize the coefficients x , multiplying by 3 in both side of the equation (2)

$$3x + 6y = 9 \quad \dots (3)$$

Now subtracting equation (3) from equation (1)

$$-5y = -5$$

$$\text{or, } 5y = 5$$

$$\text{or, } y = \frac{5}{5}$$

$$y = 1$$

Substituting the value of y in equation (1)

$$3x + 1 = 4$$

$$\text{or, } 3x = 4 - 1$$

$$\text{or, } 3x = 3$$

$$\text{or, } x = \frac{3}{3}$$

$$x = 1$$

Therefore the solution of equation is $x = 1$ and $y = 1$

Verification : On substituting $x = 1$, and $y = 1$ in equation (1).

Left hand side = $3x + y = 3 \times 1 + 1 = 3 + 1 = 4 =$ Right hand side.

On substituting $x = 1$ and $y = 1$ in equation (2)

Left hand side = $x + 2y = 1 + 2 \times 1 = 3 =$ Right hand side

Therefore, the solution is correct.

Remark : Pay attention, in which equation substituting the value of one variable, the value of second variable is found, for verification, there is no need of substituting the values in that equation. It will be sufficient to substitute the values of unknown variables in the second equation for the verification of the solution.

Example 7. Solve

$$7x - 6y = 20 \quad \dots (1)$$

$$3x + 4y = 2 \quad \dots (2)$$

Solution : In the above simultaneous equations, the coefficients of any of x and y are not equal. We know that we can eliminate any unknown by adding or subtracting the given two simultaneous equations, only if the coefficient of that unknown are same. Therefore, in above equations to eliminate y , equalise the coefficients of y . For this, we find the least common multiple 12 of coefficients of y i.e., 6 and 4. Now, to make the coefficients 12 of y multiply by 2 in both sides of equation (1) and by 3 in both sides of the equation (2).

$$\text{Therefore } 14x - 12y = 40 \quad \dots (3)$$

$$9x + 12y = 6 \quad \dots (4)$$

On adding equation (3) and (4),

$$23x = 46$$

$$\text{or, } x = \frac{46}{23}$$

$$x = 2$$

Putting the value of x in equation (1)

$$7 \times 2 - 6y = 20$$

$$\text{or, } 14 - 6y = 20$$

$$\text{or, } -6y = 20 - 14$$

$$\text{or, } -6y = 6$$

$$6y = -6 \quad (\text{on changing the sign})$$

$$\text{or } y = -\frac{6}{6}$$

$$y = -1$$

Hence, solution of equations is $x = 2$ and $y = 1$.

Verification : Substituting $x = 2$ and $y = -1$ in equation 2)

$$\text{Let hand side} = 3x + 4y = 3 \times 2 + 4 \times (-1) = 6 - 4 = 2 = \text{Right hand side}$$

Therefore, the solution is correct.

Exercise 7 (a)

1. Write the value of x in terms of y :

$$(i) x - y = 4$$

$$(ii) 2x + 4y = 6$$

$$(iii) \frac{1}{3}x + y = 2$$

2. Write the value of y in terms of x :

$$(i) 5x - y = 9$$

$$(ii) 6x - 2y = 10$$

$$(iii) 2x + \frac{1}{2}y = 4$$

3. Solve (by Tabulation method) :

$$(i) x + y = 4$$

$$(ii) 5x = y$$

$$5x + 12y = 13$$

$$2x + y = 7$$

4. Solve (by substitution method) :

$$(i) x - y = 4$$

$$3x + 2y = 27$$

$$(iii) 3x + 2y = 0$$

$$2x + y = -1$$

$$(ii) x - y = -6$$

$$x + y = -18$$

$$(iv) 2x - 5y - 16 = 0$$

$$3x + 4y - 1 = 0$$

5. Solve :

(i) $x - y = 3$

$$\frac{1}{2}x + \frac{1}{2}y = 6$$

(ii) $x = -y + 1$

$$2y = x - 4$$

(iii) $\frac{1}{2}x + y = 1$

$$\frac{1}{3}x - 7y = 4$$

(iv) $1.5x + 2.5y = 21$

$$4x + y = 22$$

6. Solve :

(i) $x + y = 3$

$$x - y = 1$$

(ii) $2x + y = 3$

$$2x - y = 1$$

(iii) $x + 2y = 2$

$$x - y = -1$$

(iv) $3x - y = 4$

$$2x - y = 2$$

7. Solve and verify the answer :

(i) $2x - 3y = 13$

$$7x - 2y = 20$$

(ii) $3x - y = -2$

$$3x + 4y = -17$$

(iii) $3x + y = 4$

$$x + 2y = 3$$

8. Solve the following equations :

(i) $3(x + 2y) = 10y + 5$

$$2(x + 2y) = 3x + 2$$

(ii) $\frac{x}{7} + \frac{y}{15} = 3$

$$\frac{x}{2} + \frac{y}{5} = 4$$

(iii) $.5x - 7y = 2$

$$3.4x - 4.4y = 15.4$$

7.4 Solution of word problems of two variables by simultaneous equations :

In which word problems, two conditions are given to solve them, obtain two linear equations according to two given conditions taking x and y (or other any algebraic letter) as unknown quantities. The unknown quantities are found by solving the obtained equations. Yet such word problems also can be solved by linear equations of one variable, but to solve with the help of linear equations of two variables be more simple and precise.

Example 8. The length of a rectangle is more than its width by 5 cm. If the perimeter of the rectangle be 40 cm, find its length and breadth.

Solution : **First Method :**

With the help of linear equation of one variable—

Let the breadth of rectangle = x cm.

Since length is 5 cm more than is breadth, therefore length of the rectangle
= $(x + 5)$ cm.

Hence perimeter of the rectangle = 2 (length + breadth)

$$= 2 (x + 5 + x) \text{ cm}$$

$$= (4x + 10) \text{ cm}$$

But perimeter of the rectangle 40 cm is known.

$$\therefore 4x + 10 = 40$$

$$\text{or, } 4x = 40 - 10$$

$$\text{or, } 4x = 30$$

$$\therefore x = \frac{30}{4} = \frac{15}{2} = 7.5$$

Therefore breadth of the rectangle = 7.5 cm.

and its length = $(7.5 + 5)$ cm

$$= 12.5 \text{ cm.}$$

Second method

With the help of simultaneous equations—

Let length of the rectangle is x cm and breadth is y cm.

$\therefore$ Length is 5 cm more than breadth.

$$\therefore x - y = 5 \quad \dots (1)$$

$$\begin{aligned}\text{Perimeter of the rectangle} &= 2 (\text{length} + \text{breadth}) \\ &= 2 (x + y) \text{ cm.}\end{aligned}$$

But perimeter of the rectangle is 40 cm.

$$\therefore 2 (x + y) = 40$$

$$\text{or, } x + y = 20 \quad \dots (2)$$

Adding equations (1) and (2),

$$2x = 25$$

$$\therefore x = \frac{25}{2} = 12.5$$

Substituting the value of x in equation (2),

$$12.5 + y = 20$$

$$\therefore y = 20 - 12.5 = 7.5$$

Therefore length of the rectangle = 12.5 cm.

and its breadth = 7.5 cm.

Verification : Since length = 12.5 cm and breadth = 7.5 cm

Therefore length is 5 cm more than breadth.

Perimeter of the rectangle = 2 (length + breadth)

$$= 2 (12.5 + 7.5) \text{ cm.}$$

$$= 2 \times 20 \text{ cm.}$$

$$= 40 \text{ cm.}$$

Hence, the answer is correct.

Example 9. The price of 3 buckets and 5 mugs is ₹ 100 and the price of 5 buckets and 2 mugs is ₹ 970. Find the price of one bucket and one mug separately.

Solution : Suppose that the price of one bucket is ₹ x and price of one mug is ₹ y .
Therefore, the price of 3 buckets and 5 mugs = ₹ $(3x + 5y)$

$$\text{According to first condition } ₹ (3x + 5y) = ₹ 1000$$

$$\therefore 3x + 5y = 1000$$

$$\text{and price of 5 buckets and 2 mugs} = ₹ (5x + 2y)$$

$$\text{According to second condition } ₹ (5x + 2y) = ₹ 970$$

$$\therefore 5x + 2y = 970$$

Hence, $3x + 5y = 1000$... (1)

$5x + 2y = 970$... (2)

Multiply by 5 in both sides of the equation (1) and by 3 in both sides of the equation (2),

$15x + 25y = 5000$... (3)

$15x + 6y = 2910$... (4)

Subtracting equation (4) from the equation (3).

$19y = 2090$

or, $y = \frac{2090}{19}$

$y = 110$

Substituting the value of y in equation (1)

$3x + 5 \times 110 = 1000$

or, $3x + 550 = 1000$

or, $3x = 1000 - 550$

or, $3x = 450$

or, $x = \frac{450}{3}$

$x = 150$

Hence price of one bucket = ₹ 150

and price of one mug = ₹ 110

Verification : Price of 3 buckets and 5 mugs = ₹ $(3 \times 150 + 5 \times 110)$

= ₹ $(450 + 550)$

= ₹ 1000

Price of 5 buckets and 2 mugs = ₹ $(5 \times 150 + 2 \times 110)$

= ₹ 970

Hence, the answer is correct.

Example 10. If increasing the length of a rectangle by 3 metre and decreasing breadth by 4 metre, its area is decreased by 72 square metre; and decreasing length by 1 metre and increasing breadth by 4 metre., its area is increased by 88 square metre, find the length and breadth of the rectangle.

Solution : Suppose that the length of rectangle = x metre
and breadth = y metre
Therefore, area of the rectangle = xy square metre

According to first condition

$$xy - (x + 3)(y - 4) = 72$$

$$\text{or, } xy - xy - 3y + 4x + 12 = 72$$

$$\text{or, } 4x - 3y = 60 \quad \dots (1)$$

According to second condition

$$(x - 1)(y + 4) - xy = 88$$

$$\text{or, } xy - y + 4x - 4 - xy = 88$$

$$\text{or, } 4x - y = 99 \quad \dots (2)$$

Subtracting equation (2) from equation (1)

$$-2y = -32$$

$$\text{or, } 2y = 32$$

$$\text{or, } y = \frac{32}{2}$$

$$y = 16$$

Substituting the value of y in equation (1)

$$4x - 3 \times 16 = 60$$

$$\text{or, } 4x - 48 = 60$$

$$\text{or, } 4x = 60 + 48$$

$$\text{or, } 4x = 108$$

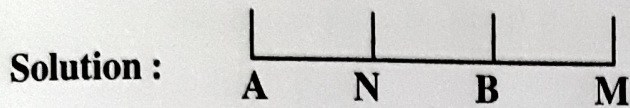
$$\text{or, } x = \frac{108}{4}$$

$$x = 27$$

Therefore, the length of the rectangle is 27 metre and breadth is 16 metre..

Verification : Verify the answer yourself.

Example 11. Two places A and B are 90 km. apart from each other. Two cars start to go from A and B simultaneously. If both cars go in the same direction, they meet each other after 9 hours and if go in opposite direction, they meet after $\frac{9}{7}$ hours. Find their speed.



Suppose that the speed of the car starting from A = x km/hour.

and speed of the car starting from B = y km/hour.

According to first condition

Both cars will meet at the place M :

Therefore, $AM - BM = AB$

(Distance covered in 9 hours by the car started from A – distance covered in 9 hours by the car started from B) = 90 km.

Therefore, $9x - 9y = 90$

or, $x - y = 10$... (1)

According to second condition

Both cars will meet a place N.

Hence, $AN + BN = 90$

(distance covered in $\frac{9}{7}$ hours by the car started from A + distance covered in $\frac{9}{7}$ hours by the car started from B)

Therefore $\frac{9}{7}x + \frac{9}{7}y = 90$

or, $x + y = 70$... (2)

Adding equation (1) and (2)

$$2x = 80$$

or, $x = \frac{80}{2}$

$$x = 40$$

Substituting the value of x in equation (1)

$$40 - y = 10$$

or, $y = 40 - 10$

$\therefore y = 30$

Therefore the speed of the car started from A = 40 km/hour.

and speed of the car started from B = 30 km/hour

(students verify the answer)

Example 12. The ratio of incomes of Sita and Geeta is 4 : 3 and ratio between their expenditures is 3 : 2. If monthly savings of each be ₹ 5000, find their income separately.

Solution : Suppose that monthly income of Sita = ₹ x
and monthly income of Geeta = ₹ y

According to first condition

$$\frac{x}{y} = \frac{4}{3}$$

or, $3x = 4y$

or, $3x - 4y = 0$... (1)

The amount expended by Sita = ₹ $(x - 5000)$

and amount expended by Geeta = ₹ $(y - 5000)$

According to the second condition

$$\frac{x - 5000}{y - 5000} = \frac{3}{2}$$

or, $2(x - 5000) = 3(y - 5000)$

or, $2x - 10,000 = 3y - 15000$

or, $2x - 3y = 10000 - 15000$

or, $2x - 3y = -5000$... (2)

Multiplying equation (1) by 2 and multiplying equation (2) by 3

$$6x - 8y = 0$$
 ... (3)

$$6x - 9y = -15000$$
 ... (4)

Subtracting equation (4) from equation (3)

$$y = 15000$$

Substituting the value of y in equation (1)

$$3x - 4 \times 15000 = 0$$

$$\therefore 3x = 60000$$

$$\therefore x = 20,000$$

Therefore monthly income of Sita = ₹ 20,000

and monthly income of Geeta = ₹ 15000

(Students verify the answer)

Example 13. Divide 700 in two parts such that 40% of one part exceeds by 80 to the 60% of the second part.

Solution : Let the first part = x

and second part = y

According to first condition

$$x + y = 700 \quad \dots (1)$$

$$40\% \text{ of first part} = x \times \frac{40}{100}$$

$$= \frac{2x}{5}$$

$$60\% \text{ of second part} = y \times \frac{60}{100}$$

$$= \frac{3y}{5}$$

According to second condition

$$\frac{2x}{5} - \frac{3y}{5} = 80$$

$$\text{or, } 2x - 3y = 400 \quad \dots (2)$$

Multiplying equation (1) by 2

$$2x + 2y = 1400 \quad \dots (3)$$

Subtracting equation (2) from the equation (3)

$$-5y = -1000$$

$$5y = 1000 \text{ (changing the sign)}$$

$$\text{or, } y = \frac{1000}{5}$$

$$\therefore y = 200$$

Substituting the value of y in equation (1)

$$x + 200 = 700$$

$$\text{or, } x = 700 - 200$$

$$\therefore x = 500$$

Therefore two parts of 700 are 500 and 200.

(Students themselves verify the answer)

It is clear from the above examples that the problems like following related to daily life can be solved with the help of simultaneous equations—

- (i) Questions related to numbers.
- (ii) Questions related to fractions
- (iii) Questions related to age
- (iv) Questions related to geometry
- (v) Questions related to ratio
- (vi) Questions related to percent

Here are given the questions for the purpose of exercise giving one example of each type. Read them carefully and solve the given questions.

Question related to Number

Example 14. Tens digit of a two digit number is 3 less than the unit digit. If the number be four times of the sum of digits, find the number.

Solution : Let the tens digit of number $= x$

and unit digit $= y$

Therefore the number $= 10x + y$

According to first condition,

$$x = y - 3$$

$$\text{or, } x - y = -3 \quad \dots (1)$$

According to second condition,

$$10x + y = 4(x + y)$$

$$\text{or} \quad 10x + y = 4x + 4y$$

$$\text{o,} \quad 10x - 4x + y - 4y = 0$$

$$\text{or,} \quad 6x - 3y = 0 \quad \dots (2)$$

Multiply equation (1) by 6,

$$6x - 6y = -18 \quad \dots (3)$$

Subtracting equation (3) from equation (2),

$$3y = 18$$

$$\text{or,} \quad y = \frac{18}{3}$$

$$\therefore y = 6$$

Substituting the value of y in equation (1)

$$x - 6 = -3$$

$$\text{or,} \quad x = 6 - 3$$

$$\therefore x = 3$$

Therefore the number = 36

Students themselves verify the answer.

Exercise 7(b)

1. The sum of two numbers is 24. Out of them one number is twice of the other. Find the numbers.
2. The sum of two numbers is 3 more than three times of small number. If difference of both is 5, find the numbers.
3. The unit digit of a two digit number is 1 more than tens digit. If the number is 3 more than the 5 times the sum of digits, find the number.
4. The tens digit of a two digit number is 5 tens than unit digit. If the places of digits are exchanged, then new number will be 7 more than two times the first number. Find that number.
5. The tens digit of a two digit number is two times of the unit digit. If the places of digits are reversed, then new number will be 36 less than the original number. Find the number.
6. 7 times of the two digit number is equal the 4 times the number made by reversing the places of the digits. If difference of unit and tens digits is 3, find the number.
7. A number of two digits is 21 times the difference of its digits. If 36 subtracts from the number, the digits of the number are reversed, then find that number.

Question related to fraction

Example 15. A fraction becomes $\frac{1}{5}$, if 1 is subtracted from both the numerator and the denominator. If 1 is added to both the numerator and the denominator the new fraction becomes $\frac{1}{3}$. Find the fraction.

Solution : Let the fraction is $\frac{x}{y}$

According to first condition

$$\frac{x-1}{y-1} = \frac{1}{5}$$

$$5(x-1) = y-1$$

$$5x-5 = y-1$$

$$5x-y = 5-1$$

$$5x-y = 4 \quad \dots (1)$$

According to second condition

$$\frac{x+1}{y+1} = \frac{1}{3}$$

$$3(x+1) = y+1$$

$$3x+3 = y+1$$

$$3x-y = 1-3$$

$$3x-y = -2 \quad \dots (2)$$

Subtraction equation (2) from equation (1),

$$2x = 6$$

or, $x = \frac{6}{2}$

$$x = 3$$

Substituting the value of x in equation (1)

$$5 \times 3 - y = 4$$

$$\begin{aligned}\text{or,} \quad & 15 - y = 4 \\ \text{or,} \quad & -y = 4 - 15 \\ & -y = -11 \\ & y = 11 \text{ (changing the sign)}\end{aligned}$$

Therefore the fraction is $\frac{3}{11}$

Students themselves verify the answer.

Exercise 7(c)

1. If 1 is added to both the numerator and the denominator of a fraction, it becomes $\frac{2}{3}$ and if 2 is subtracted from both the numerator and the denominator then it becomes $\frac{1}{3}$. Find the fraction.
2. If 1 is added to the denominator of a fraction, it becomes $\frac{1}{2}$ and if 1 is added to the numerator, it becomes 1. Find the fraction.
3. Fraction becomes $\frac{1}{2}$, if 1 is added to its numerator. Its value becomes $\frac{1}{5}$, if its denominator is increased by 1 more than its double. Find the fraction.
4. Find the fraction if 1 is subtracted from its numerator, its value becomes $\frac{2}{3}$ and if 4 is added to its denominator, then its value becomes $\frac{1}{2}$.

Question related to age

Example 16. Before 5 years, the age of father was thrice the age of his son. After 10 years, the age of father will be twice the age of his son. Find the present ages of father and son.

Solution : Suppose that the present age of father = x years and present age of son = y years

The age of father before 5 years = $(x - 5)$ years

The age of son before 5 years = $(y - 5)$ years

After 10 years, father's age = $(x + 10)$ years

After 10 years son's age = $(y + 10)$ years

According to first condition,

$$x - 5 = 3(y - 5)$$

$$\text{or, } x - 5 = 3y - 15$$

$$\text{or, } x - 35 = 5 - 15$$

$$\text{or, } x - 3y = -10 \quad \dots (1)$$

According to second condition,

$$x + 10 = 2(y + 10)$$

$$\text{or, } x + 10 = 2y + 20$$

$$\text{or, } x - 2y = 20 - 10$$

$$\text{or, } x - 2y = 10 \quad \dots (2)$$

Subtracting equation (2) from equation (1),

$$-y = -20$$

$$\text{or, } y = 20$$

Substituting the value of y in equation (1)

$$x - 3 \times 20 = -10$$

$$\text{or, } x - 60 = -10$$

$$\text{or, } x = 60 - 10$$

$$x = 50$$

Therefore father's present age = 50 years

Son's present age = 20 years

The students themselves verify the answer.

Exercise 7(d)

1. Before 10 years, the age of father was three times the age of his son. After 10 years, the age of father will be two times of his son. Find the present ages of father and son.
2. The age of a man at present is four times of the age of his son. After 18 years from now his age will be twice the age of his son. Find present ages of both.
3. The present age of father of Radhey is 7 times of his age. Before one year, father's age was 9 times of the age of Radhey. Find the ages of both at present.

4. At present, the age of Meera is $\frac{4}{5}$ times of the age of Reeta. Before 4 years Meera's age was $\frac{3}{4}$ of Reeta's age. What is the age of both at present ?
5. Father's age is 3 times the age of his son. After 12 years, Father will be 2 times the age of his son. Find the present ages of both.

Question related to geometry

Example 17. The ratio between $\angle A$ and $\angle B$ of parallelogram ABCD is 1 : 2. Find the angles of the quadrilateral.

Solution : Suppose that

$$\angle A = x^\circ, \angle B = y^\circ$$

According to given condition,

$$\frac{x}{y} = \frac{1}{2}$$

$$\text{or, } 2x = y$$

$$2x - y = 0 \quad \dots (1)$$

Since ABCD is a parallelogram,

Therefore

$$\angle A + \angle B = 180^\circ$$

$$\text{or, } x^\circ + y^\circ = 180^\circ$$

$$x + y = 180 \quad \dots (2)$$

Adding equation (1) and equation (2),

$$3x = 180$$

$$\therefore x = 60$$

$$\therefore y = 2x = 2 \times 60 = 120$$

$$\angle A = 60^\circ, \angle B = 120^\circ$$

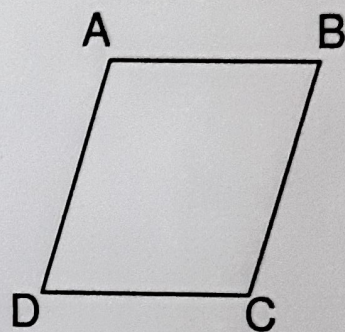
Since opposite angle of parallelogram are equal,

$$\therefore \angle C = \angle A = 60^\circ$$

$$\text{and } \angle D = \angle B = 120^\circ$$

Students themselves verify the answer

(Further in solution, there will be written x at the place of x° and y at the place of y°)



(The sum of adjacent angles of parallelogram is 180°)

Exercise 7(e)

1. The ratio between two angles of a triangle is $5 : 4$. if one angle of them is 10° more than the other angle, find its angles.
2. Two complementary angles are such that smaller angle is 10° more than $\frac{3}{5}$ times of the other angle. Find the angles.
3. Find all the angles of $\triangle ABC$, if $\angle A = x^\circ$, $\angle B = 3x^\circ$, $\angle C = y^\circ$ and $3y - 5x = 30$.

Efficiency Practice 7

Solve the following pair of equations and verify the answer

1. $3x + 2y = 8$
 $5x - 2y = 16$
2. $4x + 6y = 9$
 $4x - 2y = -11$
3. $x + y = 7$
 $3x - 2y = 11$
4. $7x - 2y = 1$
 $3x + 4y = 15$
5. The sum of a number of two digits and the obtained number by reversing the place of digits of that number is 121 and difference between digits is 3. Find the number.
6. The sum of digits of a two digit number is 9. By reversing the place of digits of given numbers, the obtained number is 27 more than original number. Find the original number.
7. If 1 is added to the numerator of a fraction and 1 is subtracted from the denominator, the fraction becomes 1. If 1 is added only to the denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.
8. A fraction is such that if 1 is added to its both the numerator and the denominator, the value of fraction becomes $\frac{4}{5}$. If 5 is subtracted from both the numerator and denominator, the value of fraction becomes $\frac{1}{2}$. Find the fraction.
9. Before five years, my age was three times the age of my son and after 10 years, my age will be two times the age of my son. Tell that how much is my age.

10. A fraction is such that subtracting 2 from its numerator and adding 3 to the denominator, that becomes $\frac{1}{4}$ and adding 6 to the numerator and multiply by 3 the denominator it becomes $\frac{2}{3}$. Find the fraction.
11. If in the age (in years) of father, the two times the age of his son is added, the sum becomes 70 and if the two times of the age of father is added to the age of son, the sum becomes 95. Find the ages of father and son.
12. In cyclic quadrilateral ABCD, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$ and $\angle D = (4x - 5)^\circ$, find the four angles of the quadrilateral.
13. The price of 3 mugs and 2 buckets is ₹ 700 and the price of 5 mugs and 3 buckets is ₹ 1100. Find the price of one mug and one bucket separately.
14. At preset, the age of A is twice the age of B. After 8 years, the ratio between their ages will be 7 : 4. How much is age of each at this time ?
15. A person has lended a part of his total capital of ₹ 35000 at the rate of interest 12% yearly and rest at the rate of interest 14% yearly. If he got total yearly interest of ₹ 4460, then how much capital had he lended separately ?

We learned in this Unit

1. Equations of two unknown variables $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ (where $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$) are called simultaneous equations.
2. The solution of simultaneous equations can be found by tabulation method.
3. To obtain the solution of simultaneous equations by tabulation method is not so easy. For this reason, the solutions of these can be found by other two methods (1) Substitution method and (2) Elimination method.
4. Converting the word problems of two variables in the form of simultaneous equations, their method of solution is learned and it is learned to verify the accuracy of the solution on the basis of given conditions.

ANSWER

Exercise 7(a)

1. (i) $x = 4 + y$, (ii) $x = 3 - 2y$, (iii) $x = 6 - 3y$; 2. (i) $y = 5x - 9$ (ii) $y = 3x - 5$, (iii) $y = 8 - 4x$; 3. (i) $x = 5$, $y = -1$ (ii) $x = 1$, $y = 5$; 4. (i) $x = 7$, $y = 3$, (ii) $x = -12$, $y = -6$, (iii) $x = -2$, $y = 3$, (iv) $x = 3$, $y = -2$; 5. (i) $x = 7.5$, $y = 4.5$, (ii) $x = 2$, $y = -1$, (iii) $x = 2\frac{20}{23}$, $y = -\frac{10}{23}$, (iv) $x = 4$, $y = 6$; 6. (i) $x = 2$, $y = 1$, (ii) $x = 1$, $y = 1$, (iii) $x =$

- 0, $y = 1$, (iv) $x = 2$, $y = 2$; 7. (i) $x = 2$, $y = -3$; (ii) $x = -\frac{5}{3}$, $y = -3$, (iii) $x = 1$, $y = 1$; 8. (i) $x = 3\frac{1}{2}$, $y = 1\frac{3}{8}$, (ii) $x = 14$, $y = 15$, (iii) $x = \frac{110}{24}$, $y = \frac{1}{24}$.

Exercise 7(b)

1. 8, 16; 2. 7, 2; 3. 78; 4. 38; 5. 84; 6. 36; 7. 84

Exercise 7(c)

1. $\frac{3}{5}$; 2. $\frac{2}{3}$; 3. $\frac{5}{12}$; 4. $\frac{5}{6}$

Exercise 7(d)

1. 70 years, 30 years; 2. 36 years, 9 years; 3. 28 years, 4 years; 4. 16 years age of Meera, 20 years age of Reeta; 5. 36 years, 12 years

Exercise 7(e)

1. 50° , 40° ; 2. 40° , 50° ; 3. $\angle A = 30^\circ$, $\angle B = 90^\circ$, $\angle C = 60^\circ$

Efficiency Practice-7

1. $x = 3$, $y = -\frac{1}{2}$; 2. $x = -\frac{3}{2}$, $y = \frac{5}{2}$; 3. $x = 5$, $y = 2$; 4. $x = 1$, $y = 3$; 5. 74 or 47; 6. 36; 7. $\frac{3}{5}$; 8. $\frac{7}{9}$; 9. 50 years, 20 years; 10. $\frac{4}{5}$; 11. 40 years, 15 years; 12. 70° , 53° , 110° , 127° ; 13. Price of one mug = ₹100, Price of one bucket = ₹ 200; 14. 48 years, 24 years; 15. ₹ 22000 at the rate of interest 12% yearly and ₹ 13000 at the rate of 14% yearly are invested.

