

UNIT-8 QUADRATIC EQUATIONS



- Quadratic equation
- Solution of equations of the form of $x^2 = k$ (where k is a whole number).
- Solution of the equations of the form $ax^2 + bx + c = 0$
- Word problems based on the equation $ax^2 + bx + c = 0$

Introduction

You have studied in previous classes the equations of type $ax + b = cx + d$, $\frac{ax + b}{cx + d} = k$, $cx + d \neq 0$. You know that in all equations of such type, the highest power of the variable is one. All these are linear equations of one variable.

In this unit, we will study the equations in which the highest power of variable is two. These equations are called quadratic equations. See, in $x^2 = k$, $ax^2 + bx + c = 0$, the highest power of variable x is two, thus all these equations are quadratic equations.

The detail discussion of quadratic equation is done in the second part of Aryabhatiya the book of ancient Indian mathematician Arya Bhatt first. The formula to solve the quadratic equation was established by Mathematician Shri Dharacharya in ancient time, which was called Sridharacharya-formula. It is used also today. Mathematician of Arab, Alkhwazimi and Umar Khaiyam are also mathematicians, who solved the quadratic equations by their own methods.

8.2 Quadratic Equations

See the following equations :

- (i) $x^2 = 9$
- (ii) $9x^2 = 16$
- (iii) $x^2 + 5x + 6 = 0$
- (iv) $3x^2 + 10x + 8 = 0$
- (v) $2x^2 + 5x - 7 = 0$

These are called quadratic equations because the highest power in these equations is 2.

It is clear from the above equations that in quadratic equations (i) and (ii) there is no term of x having power one, only term of x with power two and a numerical term are present. Left hand side of quadratic equations (iii), (iv) and (v) are in the form of trinomial expression.

Now think over quadratic equations (i) and (ii)

(i) 9 is a constant number in $x^2 = 9$

(ii) $9x^2 = 16$

dividing by 9 in both side we get ,

$$x^2 = \frac{16}{9}$$

$\frac{16}{9}$ is a constant number

Therefore quadratic equations (i) and (ii) can be written in the form of $x^2 = k$, where in quadratic equation (i) $k = 9$ and in quadratic equation (ii) $k = \frac{16}{9}$

$x^2 = k$ is standard form of quadratic equation

Now think over quadratic equations (iii), (iv) and (v). The form of all these three equations is $ax^2 + bx + c = 0$; where $a = 1$, $b = 5$, $c = 6$ in quadratic equation (iii) $a = 3$, $b = 10$, $c = 8$ in quadratic equation (iv) and in quadratic equation (v) $a = 2$, $b = 5$, $c = -7$

The equation $3x^2 - 4x + 1 = 2x^2 - 2x + 4$ is also can be written in the form of $ax^2 + bx + c = 0$ as following method :

$$3x^2 - 4x + 1 = 2x^2 - 2x + 4$$

or $3x^2 - 4x + 1 - 2x^2 + 2x - 4 = 0$

or $x^2 - 2x - 3 = 0$

or $1 \times x^2 + (-2)x + (-3) = 0$

or $ax^2 + bx + c = 0$ where $a = 1$, $b = -2$ and $c = -3$

$ax^2 + bx + c = 0$ is also a standard form of quadratic equation

So there are following two standard form of the quadratic equation :

(i) $x^2 = k$ where k is a positive number.

and (ii) $ax^2 + bx + c = 0$, where x is variable and a , b , c are constant numbers.

8.2.1. The solution of equations of the form of $x^2 = k$

In $x^2 = k$, x is an unknown variable and k is a positive constant. It is not necessary that x always is used for denoting as unknown variable. In real any letter of alphabet as y, z, n, p excetra also can be used.

To solve the equations of type $x^2 = k$, think over the following equations :

(i) $x^2 = 16$

(ii) $x^2 = 18$

(iii) $x^2 - 25 = 0$

(iv) $x^2 - 24 = 0$

(v) $4x^2 = 9$

(vi) $16x^2 - 25 = 0$

(vii) $\frac{x}{2} - \frac{2}{x} = 0$

Solution :

(i) $x^2 = 16$

Which number on squaring obtains 16 ?

We see that $4^2 = 16$ and $(-4)^2 = (-4) \times (-4) = 16$

Therefore $x^2 = 16$

$$x^2 - 16 = 0$$

$$x^2 - 4^2 = 0$$

$$(x + 4)(x - 4) =$$

if $x + 4 = 0$

if $x - 4 = 0$

$$x = -4$$

$$x = 4$$

$$x = +4, -4$$

(iii) $x^2 = 18$

$$x^2 - 18$$

$$x^2 - (3\sqrt{2})^2 = 0$$

$$(x + 3\sqrt{2})(x - 3\sqrt{2}) = 0$$

$$x = -3\sqrt{2}, 3\sqrt{2}$$

Verification : Left hand side $= x^2 = (3\sqrt{2})^2 = 3 \times 3 \times \sqrt{2} \times \sqrt{2} = 18$ Right hand side

and Left hand side $= x^2 = (-3\sqrt{2})^2 = -3 \times -3 \times \sqrt{2} \times \sqrt{2} = 18$ Right hand side

Therefore the answer is correct

(iii) $x^2 - 25 = 0$

or $x^2 = 25$ (on transposing)

$$x = \pm\sqrt{25} = \pm 5$$

Therefore $x = 5$ and $x = -5$

Verification : $5^2 - 25 = 0$ and $(-5)^2 - 25 = 25 - 25 = 0$

Therefore the answer is correct.

(iv) $x^2 - 24 = 0$

or $x^2 = 24$ (on transposing)

$$x^2 = \pm\sqrt{24}$$

$$= \pm\sqrt{4 \times 6} = \pm\sqrt{4} \times \sqrt{6}$$

Therefore $x = \pm 2\sqrt{6}$

Therefore $x = 2\sqrt{6}$ and $x = -2\sqrt{6}$

Verification : $(2\sqrt{6})^2 = (2\sqrt{6})(2\sqrt{6}) = 2 \times 2 \times \sqrt{6} \times \sqrt{6} = 4 \times 6 = 24$ and

$$(-2\sqrt{6})^2 = (-2\sqrt{6})(-2\sqrt{6}) = -2 \times -2 \times \sqrt{6} \times \sqrt{6} = 4 \times 6 = 24$$

Therefore the answer is correct

(v) $4x^2 = 9$

or $x^2 = \frac{9}{4}$ (Division both sides by 4)

$$x^2 = \pm\sqrt{\frac{9}{4}}$$

$$= \pm\frac{3}{2}$$

Therefore $x = \frac{3}{2}$ and $x = -\frac{3}{2}$

Verification : $\left(\frac{3}{2}\right)^2 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$ and $\left(-\frac{3}{2}\right) \times \left(-\frac{3}{2}\right) = \frac{9}{4}$

Therefore the answer is correct.

(vi) $16x^2 - 25 = 0$

or $16x^2 = 25$

or

$$x^2 = \frac{25}{16}$$

$$x = \pm \sqrt{\frac{25}{16}}$$

$$= \pm \frac{5}{4}$$

Therefore

$$x = \frac{5}{4} \text{ and } x = -\frac{5}{4}$$

Verification : Left hand side = $16x^2 - 25$

$$= 16 \times \left(\frac{5}{4}\right)^2 - 25$$

$$= 16 \times \frac{25}{16} - 25$$

$$= 25 - 25$$

$$= 0$$

= Right hand side

and putting $x = -\frac{5}{4}$ in left hand side

$$\text{Left hand side} = 16 \times \left(-\frac{5}{4}\right)^2 - 25$$

$$= 16 \times \frac{25}{16} - 25$$

$$= 25 - 25$$

$$= 0$$

= Right hand side

Therefore the answer is correct.

(vii) $\frac{x}{2} - \frac{2}{x} = 0$

$$\text{or } \frac{x}{2} = \frac{2}{x}$$

$$\text{or } x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$= \pm 2$$

Therefore $x = 2$ and $x = -2$

Verification : In left hand side = $\frac{x}{2} - \frac{2}{x}$ substituting $x = 2$.

$$= \frac{2}{2} - \frac{2}{2}$$

$$= 1 - 1$$

$$= 0$$

= Right Hand Side

and substituting $x = -2$ in left hand side

$$\text{Left hand side} = \frac{-2}{2} - \left(\frac{2}{-2} \right)$$

$$= -1 - (-1)$$

$$= -1 + 1$$

$$= 0$$

= Right hand side

Therefore the answer is correct.

It is clear from the solution of above quadratic equations that there are two values of unknown number x in each quadratic equation, whose absolute values are equal, but one is positive and second is negative. **These values of x are called the “roots of the equation.”**

Therefore to solve the quadratic equations of form $x^2 = k$ we find the square roots of k . We know that there are two square roots of a number whose absolute values are equal.

Therefore

$$x^2 = k$$

$$\text{or } x = \pm\sqrt{k}$$

$$\text{Therefore, } x = \sqrt{k}, x = -\sqrt{k}$$

We see that to solve the quadratic equations like $x^2 = k$, we take the square roots of both sides. As in $x^2 = 64$.

Taking the square roots of both sides

$$x = \pm 8$$

Pay attention, as the square root of 64 be ± 8 , in the same way, the square root of x^2 will be $\pm x$

Therefore, $\pm x = \pm 8$ should be written

Writing such the following values are obtained

(i) $+x = +8$

(ii) $+x = -8$

(iii) $-x = +8$

(iv) $-x = -8$

But in the above, from (i) and (iv) $+x = +8$ and $-x = -8$

Since these both express only one value of x . Therefore out of these one value $x = +8$ is taken.

In the same way, with the help of (ii) and (iii) $+x = -8$ and $-x = +8$. Since also these both express the same value. Therefore $x = -8$ is taken.

Therefore it is clear that all above four values are in $x = \pm 8$. This is the reason that plus and minus values of only one side are written at time of taking square root. Practically numerical value of square root is written with signs plus and minus, but only positive value of unknown quantity is taken.

8.2.2. Second method of solving the equations like $x^2 = k$

Solution : $x^2 = k$

or $x^2 - k = 0$ (Transposing to k)

or $x^2 - (\sqrt{k})^2 = 0$ [Writing in the form of $(a^2 - b^2)$]

or $(x - \sqrt{k})(x + \sqrt{k}) = 0$ [According to identity $a^2 - b^2 = (a - b)(a + b)$]

See, here the product of two expressions is zero.

Therefore out of these either $x - \sqrt{k}$ will be zero or $(x + \sqrt{k})$ will be zero.

when $x - \sqrt{k} = 0$

$$x = \sqrt{k}$$

and when $x + \sqrt{k} = 0$

$$x = -\sqrt{k}$$

Example : Solve $x^2 = 121$

Solution : $x^2 = 121$

or $x^2 - 121 = 0$

(Transposing 121)

or $x^2 - 11^2 = 0$

(Writing in the form of $x^2 - 11^2$)

or $(x - 11)(x + 11) = 0$

$[x^2 - 11^2 = (x - 11)(x + 11)]$

when $x - 11 = 0$, then $x = 11$

when $x + 11 = 0$, then $x = -11$

Therefore, $x = 11, x = -11$

Exercise 8(a)

Solve the following quadratic equations and verify the answer :

1. $x^2 - 49 = 0$

2. $16x^2 - 9 = 0$

3. $ax^2 - b = 0$ (where a and b are positive integers)

4. $\frac{4}{9}x^2 = 1$

5. $64p^2 = 25$

6. $6y^2 = 20$

7. $7x^2 = 8$

8. $5x^2 = x^2 + 1$

9. $x = \frac{4}{x}$

10. $\frac{x}{5} - \frac{5}{x} = 0$

$$11. -ax^2 + c = 0$$

$$12. 2x^2 - 18 = 0$$

$$13. \frac{x^2}{4} = 9$$

$$14. \frac{x}{a} - \frac{a}{x} = 0$$

$$15. x^2 - 256 = 0$$

$$16. 0.04x^2 - 0.25 = 0$$

8.3 Simple word problems on the basis of equation $x^2 = k$

We can solve many problems relating to daily life by using equations. Doing so, we will have to follow the following four steps.

1. Denote the unknown quantity by any letter of alphabet as x, y, z, n, p etc.
2. Change the statement given in language of equation.
3. Solve the equation
4. Verify the answer to substitute the obtained value in original problem.

Example 1. The area of a square is 64 square cm. Find its side.

Solution : Suppose that the side of square is x cm.

Area of square = (side of the square)²

$$= x^2 \text{ square cm}$$

But area of the square is 64 cm².

$$\therefore x^2 = 64$$

$$\therefore x = \pm\sqrt{64}$$

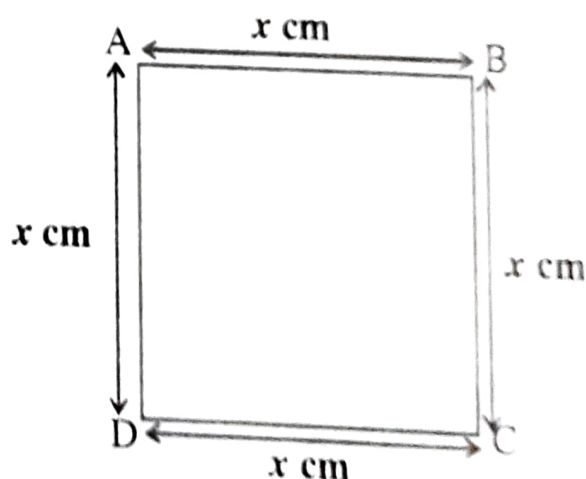
$$= \pm 8$$

$$\therefore x = 8 \text{ and } x = -8$$

But the side of the square cannot be negative,

therefore $x = -8$ is not applicable.

$$x = 8$$



Therefore side of the square = 8 cm

Verification of the answer :

Area of the square = 8×8 square cm = 64 square cm

Therefore the answer is correct.

Example 2. The length of a rectangular garden is thrice of its breadth. If the area of the garden be 243 square metre, how much is the length of the garden ?

Solution : Suppose that the breadth of the garden is x m.

Since the length of the garden is thrice its breadth

Therefore length of the garden = $3x$ breadth.

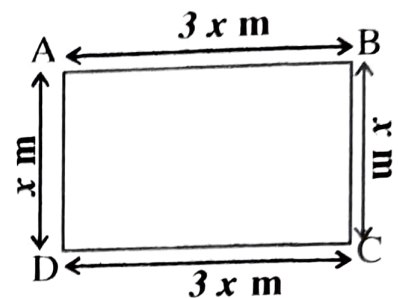
$$= 3 \times x \text{ m}$$

$$= 3xm$$

Area of the garden = length $\times$ breadth

$$= 3x \times x \text{ square m}$$

$$= 3x^2 \text{ square m}$$



But the area of the garden is given 243 square m.

$$3x^2 = 243$$

$$\text{or } x^2 = \frac{243}{3}$$

$$= 81$$

$$x = \pm\sqrt{81}$$

$$= \pm 9$$

But breadth of garden does not negative

Therefore breadth of the garden = 9 m

Length of the garden = $3xm$

$$= 3 \times 9 \text{ m} = 27 \text{ m}$$

Verification of the answer :

Area = length $\times$ breadth

$$= 27 \times 9 \text{ square m}$$

$$= 243 \text{ square m}$$

Therefore the answer is correct.

Exercise 8(b)

1. The area of a square shaped field is 441 square metre find the perimeter of the field.
2. The ratio between length and breadth of a rectangular garden is 3 : 2. If area of the garden is 600 square m, find its length and breadth.
3. The area of a square shaped field is 225 square metre. Find its perimeter.
4. 144 students of class 8 has to stand in such a way that the number of students in each row as much as are the total number of rows. Find the number of rows.
5. Thrice of the square of a number is 192. Find the number.

8.4 The solution of quadratic equation $ax^2 + bx + c = 0$ (By factorisation method)

We know that if $x \times y = 0$, then

$$x = 0$$

or $y = 0$

Similarly if $x(x - 4) = 0$, then

$$x = 0$$

or $x - 4 = 0$

or $x = 4$

and if $(x - 5)(x - 7) = 0$, then

$$(x - 5) = 0 \quad \text{or } x = 5$$

or $x - 7 = 0 \quad \text{or } x = 7$

Therefore, if product of two (or more than two) expressions be zero, then the value of one expression of them necessarily will be zero.

Now multiply the expressions of left hand side of the equation $(x - 5)(x - 7) = 0$

$$x^2 - 7x - 5x + (-5)(-7) = 0$$

or $x^2 - (7 + 5)x + 35 = 0$

or $x^2 - 12x + 35 = 0$

We see that this is a quadratic equation whose form is $ax^2 + bx + c = 0$ where $a = 1$, $b = -12$ and $c = 35$.

Thus multiplying the expressions of left hand side of $(x - 8)(x + 6) = 0$

$$x^2 + 6x - 8x + (-8) \times 6 = 0$$

or $x^2 - 2x - 48 = 0$

It is also in the form of $ax^2 + bx + c = 0$ where $a = 1$, $b = -2$ and $c = -48$

Therefore the solutions of quadratic equations of the form $ax^2 + bx + c = 0$ are known to factorise their left hand side.

See the following quadratic equations :

(i) $(x + 1)(x - 2) = 0$

or $x^2 + (1 - 2)x + 1 \times (-2) = 0$

or $x^2 + (-1)x + (-2) = 0$

or $x^2 - x - 2 = 0$

(ii) $(x - p)(x - q) = 0$

or $x^2 - (p + q)x + pq = 0$

Thus, we see that we can obtain the quadratic equation $x^2 - (p + q)x + pq = 0$ by multiplying the linear equations $x - p = 0$, $x - q = 0$ whose solution is $x = p$ and $x = q$. Conversely, we can know the solution of the quadratic equation $ax^2 + bx + c = 0$ to factorise it.

Example 1. Solve : Equation $x^2 + 7x + 10 = 0$

Solution : $x^2 + 7x + 10 = 0$

Here $a = 1$, $b = 7$, $c = 10$

or $x^2 + (2 + 5)x + 10 = 0$

$a \times c = 1 \times 10 = 10$

or $x^2 + 2x + 5x + 10 = 0$

$10 = 2 \times 5$

or $x(x + 2) + 5(x + 2) = 0$

and $2 + 5 = 7$

or $(x + 2) + (x + 5) = 0$

Therefore $x + 2 = 0$ or $x + 5 = 0$

$\therefore x = -2$ or $x = -5$

Example 2. Solve equation $x^2 + x - 6 = 0$

Solution : $x^2 + x - 6 = 0$

or $x^2 + (3 - 2)x - 6 = 0$

or $x^2 + 3x - 6 = 0$

$$\text{or } x(x + 3) - 2(x + 3) = 0$$

$$\text{or } (x + 3)(x - 2) = 0$$

Therefore if $x + 3 = 0$, then $x = -3$ and if $x - 2 = 0$, then $x = 2$

See the following quadratic equations

$$(i) \quad (2x - 3)(x + 1) = 0$$

$$\text{or } 2x^2 + 2x - 3x - 3 = 0$$

$$\text{or } 2x^2 - x - 3 = 0$$

$$(ii) \quad (3x + 4)(2x - 5) = 0$$

$$\text{or } 6x^2 - 15x + 8x - 20 = 0$$

$$\text{or } 6x^2 - 7x - 20 = 0$$

Thus, we see that if the coefficient of x^2 does not be 1, then the solution also can be obtained by factorising it.

Example 3. Solve the equation $3x^2 + 10x + 8 = 0$

Solution : $3x^2 + 10x + 8 = 0$

$$\text{or } 3x^2 (4 + 6)x + 8 = 0$$

$$\text{or } 3x^2 + 4x + 6x + 8 = 0$$

$$\text{or } x(3x + 4) + 2(3x + 4) = 0$$

$$\text{or } (3x + 4)(x + 2) = 0$$

$$\text{Here } \left\{ \begin{array}{l} a = 3, b = 10 \text{ and } c = 8 \\ a \times c = 3 \times 8 = 24 \\ 24 = 4 \times 6 \\ \text{and } 4 + 6 = 10 \end{array} \right\}$$

$$\text{Therefore } 3x + 4 = 0 \text{ or } x + 2 = 0$$

$$\text{From which } 3x = -4 \text{ or } x = -2$$

$$\text{or } x = -\frac{4}{3} \text{ or } x = -2$$

Example 4. Solve the equation $12x^2 - 11x - 15 = 0$

Solution : $12x^2 - 11x - 15 = 0$

$$\text{or } 12x^2 + (-20 + 9)x - 15 = 0$$

$$\text{or } 12x^2 - 20x + 9x - 15 = 0 \quad \text{Here } a = 12, b = -11 \text{ and } c = -15$$

$$\text{or } 4x(3x - 5) + 3(3x - 5) = 0$$

$$\begin{aligned} \text{here, } a \times c &= 12 \times (-15) = -180 \\ &= (-20) \times 9 \end{aligned}$$

$$\text{or } (3x - 5)(4x + 3) = 0$$

$$\text{and } -20 + 9 = -11$$

$$\text{Therefore } 3x - 5 = 0 \text{ or } 4x + 3$$

$$\text{From which } 3x = 5 \text{ or } 4x = -3$$

$$\text{or } x = \frac{5}{3} \text{ and } x = -\frac{3}{4}$$

Verification of the answer : Substituting $x = \frac{5}{3}$ in the left hand side of the equation

$$\text{Left hand side} = 12\left(\frac{5}{3}\right)^2 - 11\left(\frac{5}{3}\right) - 15$$

$$= 12 \times \frac{25}{9} - 11 \times \frac{5}{3} - 15 = \frac{100}{3} - \frac{55}{3} - \frac{45}{3} = 0$$

$$= \text{Right hand side}$$

Similarly substituting $x = -\frac{3}{4}$ in the left hand side of the equation

$$\text{Left hand side} = 12\left(-\frac{3}{4}\right)^2 - 11\left(-\frac{3}{4}\right) - 15$$

$$= 12 \times \frac{9}{16} + \frac{33}{4} - \frac{60}{4} = 0$$

$$= \text{Right hand side}$$

Hence the answer is correct.

Example 5. Solve the equation $3x^2 - 10x - 8 = 0$

Solution : $3x^2 - 10x - 8 = 0$

Find two factors of $(-8) \times 3$ such that whose sum be -10 that is $-8 \times 3 = -24$
and $-24 = -12 \times 2, -12 + 2 = -10$

$$\text{Thus } 3x^2 - 10x - 8 = 0$$

$$\text{or } 3x^2 + (-12 + 2)x - 8 = 0$$

$$\text{or } 3x^2 - 12x + 2x - 8 = 0$$

$$\text{or } 3x(x - 4) + 2((x - 4) = 0$$

$$\text{or } (x - 4)(3x + 2) = 0$$

$$\text{If } x - 4 = 0, \text{ then } x = 4$$

$$\text{and if } 3x + 2 = 0 \text{ then } 3x = -2$$

$$x = -\frac{2}{3}$$

Verification of the answer : Substituting $x = 4$

$$3x^2 - 10x - 8 = 3 \times 4^2 - 10 \times 4 - 8$$

$$= 48 - 40 - 8$$

$$= 48 - 48 = 0$$

$$= \text{Right hand side}$$

$$\text{and substituting } x = -\frac{2}{3}$$

$$3x^2 - 10x - 8 = 3 \times \left(-\frac{2}{3}\right)^2 - 10 \times \left(-\frac{2}{3}\right) - 8$$

$$= 3 \times \frac{4}{9} + \frac{20}{3} - 8$$

$$= \frac{4}{3} + \frac{20}{3} - 8$$

$$= \frac{24}{3} - 8 = 8 - 8 = 0$$

$$= \text{Right hand side}$$

Therefore the answer is correct

Exercise 8(c)

Solve and verify the answer :

1. $3x^2 + 10x + 8 = 0$

2. $(2x - 3)(x + 2) = 0$

3. $x(x - 4) = 0$

4. $x^2 + 7x = 44$

5. $3x^2 + 10x - 8 = 0$

6. $(2x + 1)(x + 3) + 3 = 0$

7. $6x^2 - x = 1$

8. $4y^2 = 11y + 3$

9. $a^2 + a = 90$

10. $x + 1 = 7 - x^2$

11. $2x^2 = 12 - 5x$

12. $2x + \frac{4}{x} = 9$

13. $9x^2 - 3x - 2 = 0$

14. $x^2 + 3x - 18 = 0$

15. $x^2 - 3x - 10 = 0$

16. $x^2 - 6x + 9 = 0$

17. $4x^2 - 20x + 25 = 0$

18. $16x^2 + 24x + 9 = 0$

19. $x^4 - 25x^2 + 144 = 0$

(Hint : supposing $x^2 = y$, the transformation of the equation is $y^2 - 25y + 144 = 0$)

20. $x^4 - 13x^2 + 36 = 0$

Remark : Left hand side of the equation in above questions 16, 17 and 18 are complete square from which only one value of x is obtained. Note that the maximum number of roots (values of unknown variable) of the quadratic equation is 2.

Example 1. Solve the equation $\frac{x^2}{x^2+2} + \frac{x^2}{x^2-1} = 2$

Solution :

$$\frac{x^2}{x^2+2} + \frac{x^2}{x^2-1} = 2$$

or $\frac{x^2+2-2}{x^2+2} + \frac{x^2-1+1}{x^2-1} = 2$

or $\frac{x^2+2}{x^2+2} - \frac{2}{x^2+2} + \frac{x^2-1}{x^2-1} + \frac{1}{x^2-1} = 2$

or $1 - \frac{2}{x^2+2} + 1 + \frac{1}{x^2-1} = 2$

or $-\frac{2}{x^2+2} + \frac{1}{x^2-1} = 0$

or $\frac{1}{x^2-1} = \frac{2}{x^2+2}$

or $2x^2 - 2 = x^2 + 2$

or $2x^2 - x^2 = 2 + 2$

or $x^2 = 4$

(By cross multiplication)

Taking square root of both sides

$$x = \pm\sqrt{4}$$

$$= \pm 2$$

Therefore $x = 2$ and $x = -2$

Verification of the answer :

Left hand side of the quadratic equation = $\frac{x^2}{x^2+2} + \frac{x^2}{x^2-1}$

$x^2 = 4$ (putting $x = 2$)

Putting the value of x^2 in the equation

$$= \frac{4}{4+2} + \frac{4}{4-1}$$

$$= \frac{4}{6} + \frac{4}{3}$$

$$= \frac{2}{3} + \frac{4}{3}$$

$$= \frac{6}{3}$$

$$= 2$$

$$= \text{Right hand side}$$

Similarly putting $x = -2$, Left hand side = Right hand side

Therefore the answer is correct.

Example 2. A steamer-driver returns back to the same place to cover a distance of 10 km in the direction of flow of river and against flow of its flow in 50 minutes. If the speed of the steamer in still water be 25 km/hour, find the speed of river.

Solution : Suppose that the speed of river is x km/hour. Speed of steamer in still water = 25 km/hour

The speed of steamer in the opposite direction of flow of the river = $(25 - x)$ km/hour and the speed of the steamer in the direction flow of the river = $(25 + x)$ km/hour

Therefore time taken to cover the distance of 10 km in the opposite direction of flow = $\frac{10}{25 - x}$. and time taken to cover the distance of 10 km in the direction

of the flow = $\frac{10}{25 + x}$

$$\text{Total time} = \left(\frac{10}{25 - x} + \frac{10}{25 + x} \right) \text{ hour}$$

$$\text{But the total time} = 50 \text{ minutes} = \frac{50}{60} \text{ hours} = \frac{5}{6} \text{ hours}$$

$$\therefore \frac{10}{25 - x} + \frac{10}{25 + x} = \frac{5}{6}$$

$$\text{or} \quad \frac{2}{25 - x} + \frac{2}{25 + x} = \frac{1}{6}$$

$$\text{or } \frac{2(25+x) + 2(25-x)}{(25-x)(25+x)} = \frac{1}{6}$$

$$\text{or } \frac{(50+x) + (50-x)}{(25-x)(25+x)} = \frac{1}{6}$$

$$\text{or } \frac{100}{625-x^2} = \frac{1}{6}$$

$$\text{or } 600 = 625 - x^2$$

$$\text{or } x^2 = 25$$

Taking square root of both sides,

$$x = \pm\sqrt{25}$$

$$= \pm 5$$

$$x = 5 \text{ and } x = -5$$

But negative value is not acceptable

$$x = 5$$

Therefore the speed of the river = 5 km/hour

Students verify the answer themselves.

Example 3: Solve the equation $\frac{x+1}{x-1} + \frac{x+2}{x-2} = \frac{2x+13}{x+1}$

Solution : $\frac{x+1}{x-1} + \frac{x+2}{x-2} = \frac{2x+13}{x+1}$

$$\text{or } \frac{x-1+2}{x-1} + \frac{x-2+4}{x-2} = \frac{2x+2+11}{x+1}$$

$$\text{or } \frac{x-1}{x-1} + \frac{2}{x-1} + \frac{x-2}{x-2} + \frac{4}{x-2} = \frac{2(x+1)+11}{x+1}$$

$$\text{or } 1 + \frac{2}{x-1} + 1 + \frac{4}{x-2} = \frac{2(x+1)}{x+1} + \frac{11}{x+1}$$

$$\text{or } 2 + \frac{2}{x-1} + \frac{4}{x-2} = 2 + \frac{11}{x+1}$$

$$\text{or } \frac{2}{x-1} + \frac{4}{x-2} = \frac{11}{x+1}$$

$$\text{or } \frac{2(x-2)+4(x-1)}{(x-1)(x-2)} = \frac{11}{x+1}$$

$$\text{or } \frac{2x-4+4x-4}{x^2-3x+2} = \frac{11}{x+1}$$

$$\text{or } \frac{6x-8}{x^2-3x+2} = \frac{11}{x+1}$$

$$\text{or } 11x^2 - 33x + 22 = (6x - 8)(x + 1)$$

$$\text{or } 11x^2 - 33x + 22 = 6x^2 + 6x - 8x - 8$$

$$\text{or } 11x^2 - 33x + 22 = 6x^2 - 2x - 8$$

$$\text{or } 11x^2 - 33x + 22 - 6x^2 + 2x + 8 = 0$$

$$\text{or } 5x^2 - 31x + 30 = 0$$

$$\text{or } 5x^2 - 25x - 6x + 30 = 0$$

$$\text{or } 5x(x - 5) - 6(x - 5) = 0$$

$$\text{or } (x - 5)(5x - 6) = 0$$

$$\text{If } x - 5 = 0, \text{ then } x = 5$$

$$\text{and if } 5x - 6 = 0$$

$$\text{then } 5x = 6$$

$$x = \frac{6}{5} = 1\frac{1}{5}$$

$$\text{Therefore } x = 5 \text{ and } x = 1\frac{1}{5} = 1.2$$

Students themselves verify the answer

Efficiency Practice 8

Solve :

$$1. \quad 4 - x^2 = 0$$

$$2. \quad 2(x^2 - 121) = 0$$

$$3. \quad \frac{1}{x^2-6} = \frac{2}{x^2+2}$$

4. $\frac{x^2}{x^2+1} + \frac{x^2-5}{x^2-6} = 2$
5. $\frac{x-2}{x+2} + \frac{x+2}{x-2} = \frac{5}{2}$
6. Find a number in which sum of its and its reciprocal $\frac{50}{7}$.
7. The area of a square shaped bed is 16 square m. Find the perimeter of this bed.
8. ₹289 have to distribute among some persons such that each person gets as much rupees as is the total number of persons. Find the number of persons.
9. The length of a room is $\frac{5}{4}$ times of its breadth.
If the area of the floor of the room is 20 square m, then find its length and breadth.
10. A boat, whose speed in still water is 15 km/hour, takes total time 4 hours 30 minutes for going 30 km in the direction of flow of stream and to come back at the same place. Find the speed of stream.

Solve the following equations :

11. $12x^2 + 25x + 12 = 0$
12. $x^2 - 4x - 2\frac{1}{4} = 0$
13. $(a + 1)x^2 + 2ax + (a - 1) = 0$
14. $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{8}$
15. $\frac{x+4}{x+5} - \frac{x}{x+1} = \frac{1}{8}$
16. $5x^2 - 16x - 21 = 0$
17. $x^2 - 23x + 132 = 0$
18. $14x^2 + 19x - 3 = 0$
19. $6x^2 + 17x + 12 = 0$
20. $24x^2 + 65x + 21 = 0$

We have learnt

1. To solve the equations of the form of $x^2 = k$ (where k is a whole number)
2. To solve the equations of the form of $ax^2 + bx + c = 0$
3. To solve the word problems on the basis of equation $ax^2 + bx + c = 0$

ANSWERS

Exercise 8 (a)

1. ± 7 ; 2. $\pm \frac{3}{4}$; 3. $\pm \sqrt{\frac{b}{a}}$; 4. $\pm \frac{3}{2}$; 5. $\pm \frac{5}{8}$; 6. ± 2 ; 7. $\pm 2\sqrt{\frac{2}{7}}$; 8. $\pm \frac{1}{2}$; 9. ± 2 ; 10. ± 5 ; 11. $\pm \sqrt{\frac{c}{a}}$; 12. ± 3 ; 13. ± 6 ; 14. $\pm a$; 15. $x = \pm 16$; 16. $x = \pm 2.5$

Exercise 8 (b)

1. 84 m; 2. 30 m, 20 m; 3. 60 ml; 4. 12; 5. ± 8

Exercise 8 (c)

1. $-2, -\frac{1}{3}$; 2. $\frac{1}{2}, -2$; 3. 0, 4; 4. $-11, 4$; 5. $-4, \frac{2}{3}$; 6. $-2, -\frac{1}{2}$; 7. $\frac{1}{2}, -\frac{1}{3}$; 8. $3, -\frac{1}{4}$; 9. $-10, 9$; 10. $-3, 2$; 11. $-4, \frac{1}{2}$; 12. $4, \frac{1}{2}$; 13. $\frac{2}{3}, -\frac{1}{3}$; 14. $(-6, 3)$; 15. $5, -2$; 16. 3 ; 17. $\frac{5}{2}$; 18. $-\frac{3}{4}$; 19. $\pm 3, \pm 4$; 20. $\pm 2, \pm 3$

Efficiency Practice 8

1. ± 2 ; 2. ± 11 ; 3. $\pm \sqrt{14}$; 4. $\pm \sqrt{14}$; 5. ± 6 ; 6. $7, \frac{1}{7}$; 7. 5 m, 4 m; 8. 5 km/hr; 9. ± 1 ; 10. 16 m; 11. $-\frac{1}{3}, -\frac{3}{4}$; 12. $4\frac{1}{2}, -\frac{1}{2}$; 13. $-1, -\frac{a-1}{a+1}$; 14. $3, \frac{1}{3}$; 15. $-9, 3$; 16. $4\frac{1}{5}, -1$; 17. 11, 12; 18. $-\frac{1}{2}, \frac{1}{7}$; 19. $-\frac{1}{3}, -\frac{1}{2}$; 20. $2\frac{1}{3}, \frac{3}{8}$

