



- Experimental verification for following properties of parallel lines.
- Two lines parallel to a line, are parallel to each other.
- Two lines perpendicular on a line are mutually parallel.
- Construction
 - Divide a given line segment into many equal parts.
 - Divide the given line segment in the given ratio.

9.1 Parallel lines and their properties

Introduction

We remember that on a plane if two lines don't intersect each other, they are called as parallel lines. Hence two lines are parallel.

- (i) When they are on same plane and
- (ii) They do not intersect each other.

The given figure has two lines l and m . These do not meet at any point if being extended from their ends. Look at figure (2), when two lines l and m are extended backwards they intersect each other at a point P .

If two lines (on extending both ends) situated at same plane, do not meet each other, then they are called parallel lines.

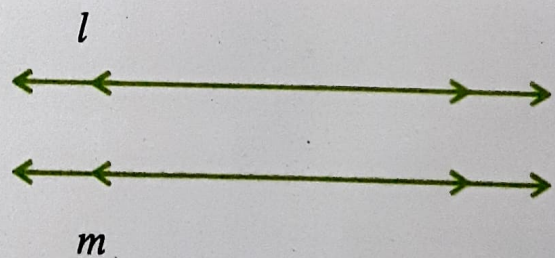


Fig. 1

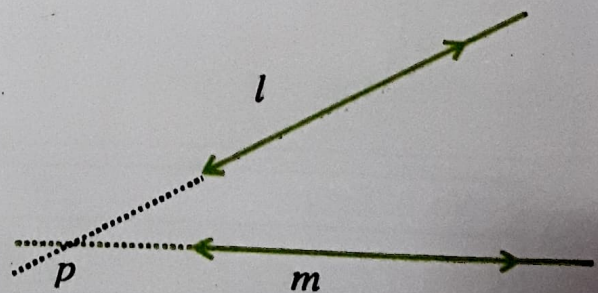


Fig. 1

Activity

Look at given figure (3) l and m are two parallel lines. Take a point P on line l . Draw perpendicular PM from point P to line m . Similarly draw a perpendicular line QN from point Q on line l to line m . Now measure PM and QN . We see that $PM = QN$.

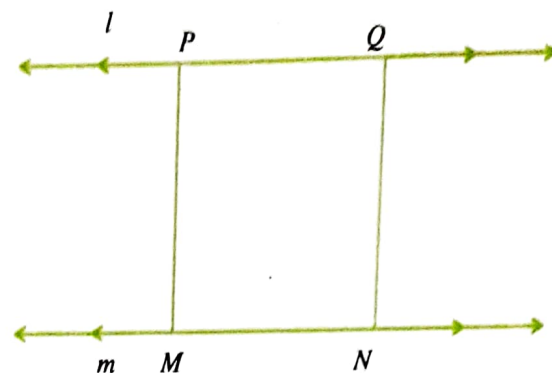


Fig. 3

9.1.1 Two lines parallel to a line are to each other

Think, Discuss and Write

If two lines are drawn parallel to a line, are they equidistant to each other?

If a line is drawn parallel to two lines, then are they equidistant from each other?

Try it

Draw two lines m and n parallel to line l with the help of set square. Examine whether these lines m and n are parallel or not.

Draw a slant line p , such that it intersects line m and n at point P and Q respectively.

Similarly draw two more figures, for convenience mark these figures as 1, 2 and 3.

Now measure $\angle P$ and $\angle Q$ and complete the following table.

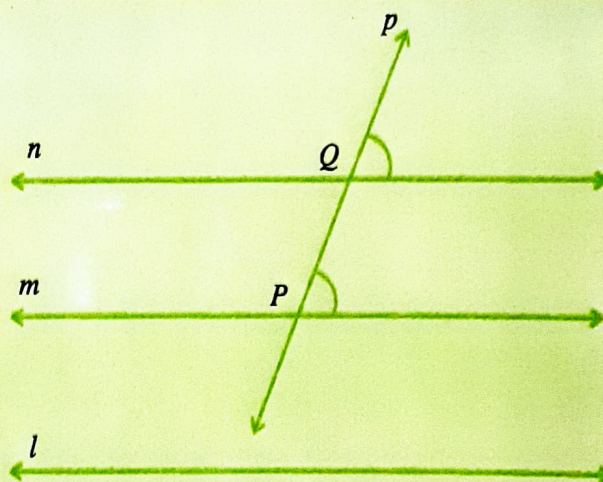


Fig. 4

No. of figure	$\angle P$	$\angle Q$	$\angle P - \angle Q$
1			
2			
3			

We observe that in every case $\angle P - \angle Q$ is zero or so negligible that its value can be also. Hence $\angle P = \angle Q$, but $\angle P$ and $\angle Q$ are corresponding angles. Therefore lines m and n are mutually parallel.

Therefore

Two lines parallel to a line are mutually parallel.

Look also the following options

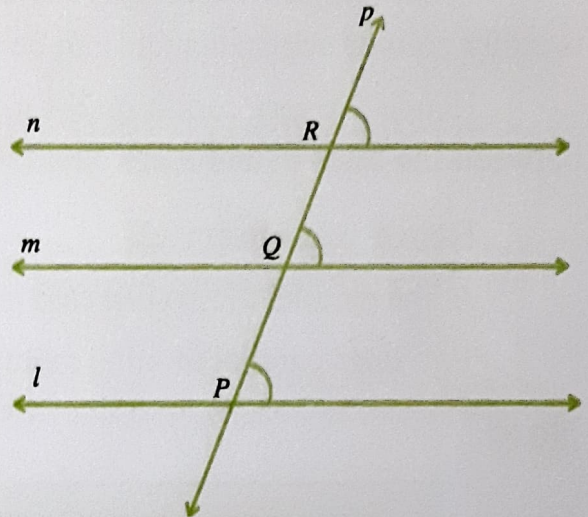
Draw two lines m and n parallel to line l . Slant line p intersects these lines at point P , Q and R respectively. Line n is drawn parallel to line l here.

$$\angle R = \angle P \text{ (corresponding angle)}$$

Similarly we see that line m is drawn parallel to line l . Therefore,

$$\angle Q = \angle P \text{ (corresponding angle)}$$

It is clear from the above result that $\angle P = \angle Q = \angle R$ that is $\angle Q = \angle R$ which are corresponding angles hence line m and line n are parallel.



Two lines parallel to a line are mutually parallel.

Activity

Draw a line l on a plane paper. With the help of ruler and set square draw two lines m and n parallel to line l . Take three points P , Q and R on line n . Draw perpendicular lines from these points PA , QB and RC on line m . Similarly draw two more figures.

For convenience mark fig.(ii) and fig. (iii). Measure PA , QB and RC

Find the $PA-QB$, $QB-RC$ and $RC-PA$ in given three figures and complete the following table.

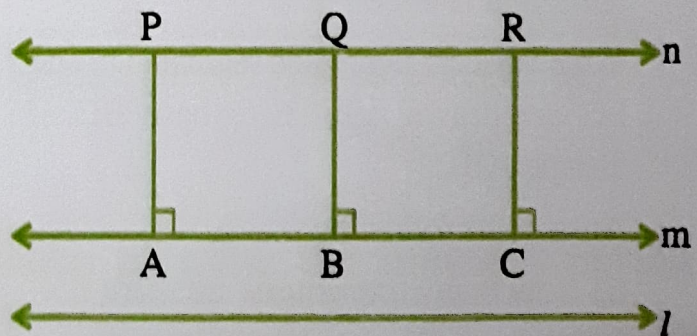


Fig. 6(l)

Figure	PA	QB	RC	PA-QB	QB-RC	RC - PA
(i)						
(ii)						
(iii)						

What did you observe ? You saw that difference PA-QB, QB-RC and RC-PA is either zero or negligible and can be ignored.

Hence in every condition $PA = QB = RC$, In other words the distance PA, QB and RC between the lines m and n are equal. Hence $m \parallel n$.

Hence you observed.

A line parallel to two lines are mutually parallel.

Two lines parallel to a line mutually parallel.

Note

Above property can be restated as follows :

Two lines parallel to a line, never intersect each other.

Exercise 9(a)

1. Draw a parallel line at distance of 5 cm. from the given line AB.
2. Draw a parallel line to a given line from a point that is not on it.
3. In the adjacent figure line segment DE and FG are drawn parallel to base BC of triangle ABC.

Answer the following :

- (i) How many trapeziums are there ?
- (ii) How many triangles are there ?

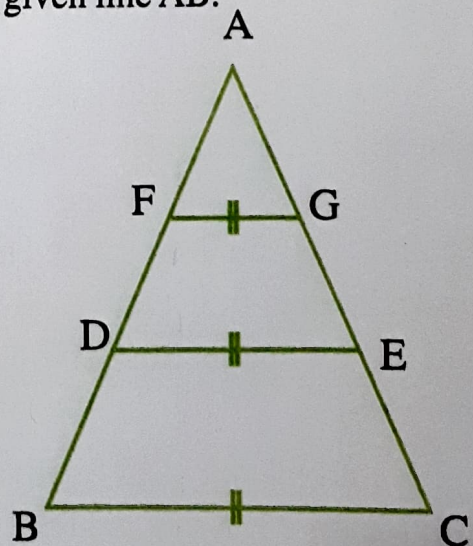


Fig. 7

4. The adjacent figure has two parallel lines l and m and a slant line t . If $\angle 1 = 30^\circ$. Find the value of remaining angles 2, 3, 4, 5, 6, 7 and 8.

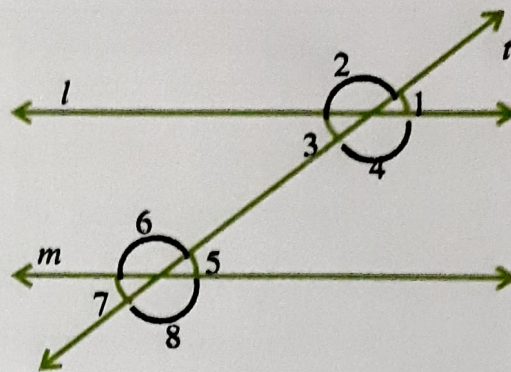


Fig. 8

5. In the adjacent figure, there is triangle ABC and line BD is parallel to side AC. $\angle ACB = 30^\circ$ and $\angle ABD = 28^\circ$. Find the value of $\angle ABC$, $\angle DBK$ and $\angle BAC$.

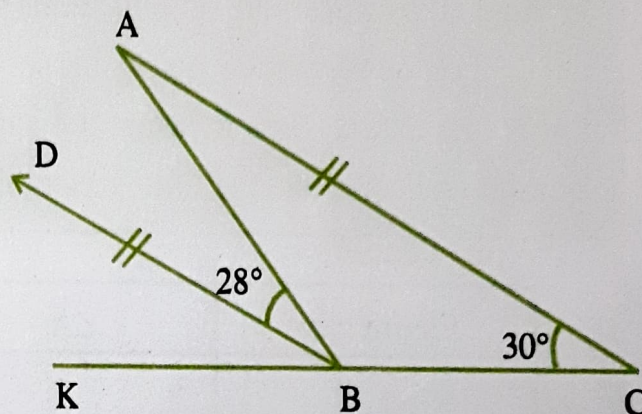


Fig. 9

6. In the adjacent figure $r \perp p$ and $r \perp q$.
- is $p \parallel q$? If so why ?
 - if $p \parallel q$ and $\angle 1 = 63^\circ$ then find out value of $\angle 2$?

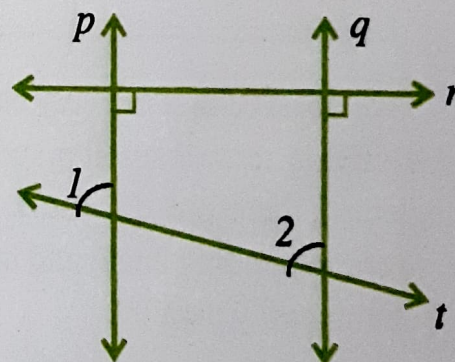


Fig. 10

9.12 Two lines which are perpendicular on a line are mutually parallel

Think, discuss and write

Can two lines perpendicular on a line be parallel to each other?

Try

Draw a line l . Take two points A and B on it. Draw perpendicular lines m and n through point A and B.

Similarly draw two more figures. Mark also these lines with l and taking two points on these lines draw perpendicular lines m and n . For convenience mark these two points as A and B. Now measure the angle A and B and complete the following table :

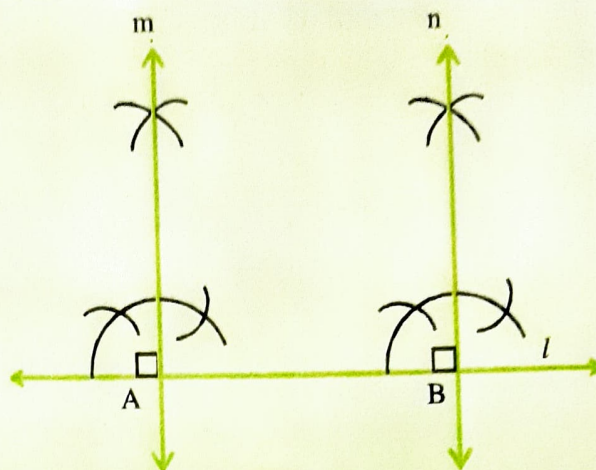


Fig. 11

Figure No.	$\angle A$	$\angle B$	$\angle A - \angle B$
1.			
2.			
3.			

We observe that in every case the value of $\angle A - \angle B$ is zero or smaller such that it can be neglected, therefore $\angle A = \angle B$. But $\angle A$ and $\angle B$ are corresponding angle and hence lines m and n are mutually parallel.

Again take two points P and Q outside of line l . From these points draw perpendicular PA and QB . Measure angle $\angle A$ and $\angle B$ are they equal ? What is the relation between A and B ?

Both angles are equal and they are corresponding angles. Hence m and n are mutually parallel.

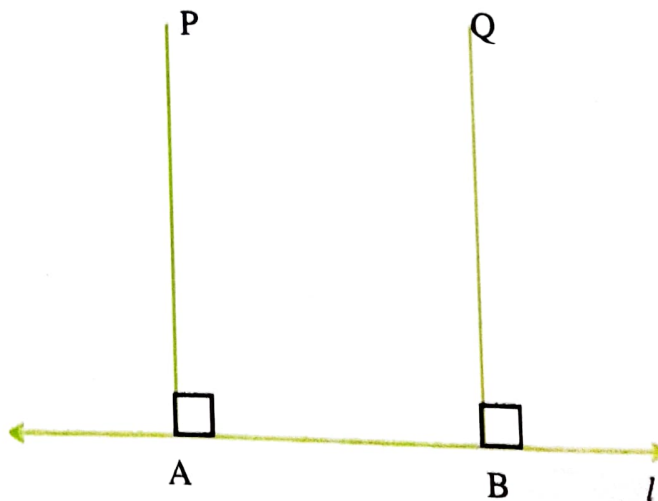


Fig. 12

Hence lines perpendicular at a line are mutually parallel.

Think, Discuss and Write

In the figure 13 $l \parallel m$ and $m \parallel n$

If $\angle 1 = 65^\circ$ then find out $\angle 2$?

In the fig. $l \parallel m$ and $m \parallel n$ then $l \parallel n$ why ?

$\angle 2 = \angle 1 = 65^\circ$ (write the reason)

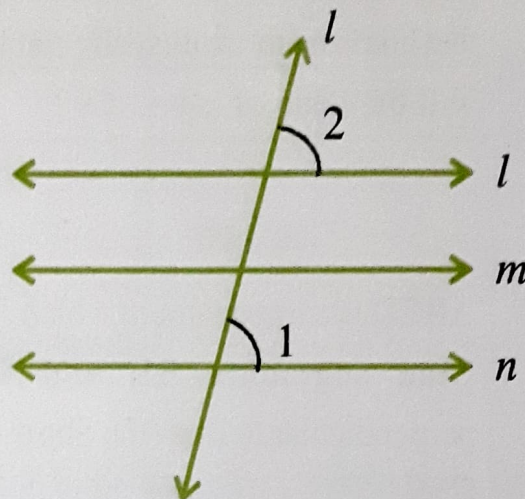


Fig. 13

Do this :

In the following figure 14 $m \perp l$, $n \perp l$ and slant line t , form $\angle 1$ and $\angle 2$ with lines n and m . If $\angle 1 = 80^\circ$ then find out $\angle 2$.

$m \perp l$ and $n \perp l$ (is given) in Fig. 14
 $m \parallel n$ (write the reason)

$\angle 2 = \angle 1 = 80^\circ$ (why)

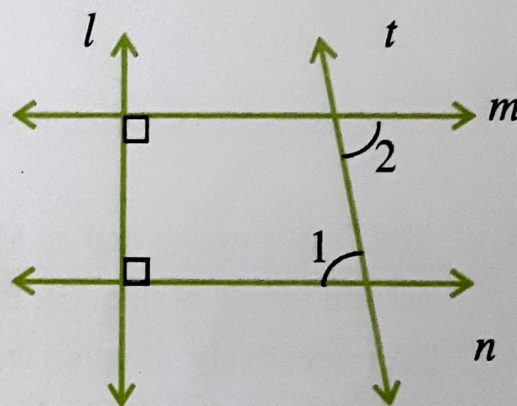


Fig. 14

Exercise 9(b)

1. In Fig. 15 every angle of quadrilateral ABCD is right angle. Prove that $AB \parallel CD$ and $AD \parallel BC$.

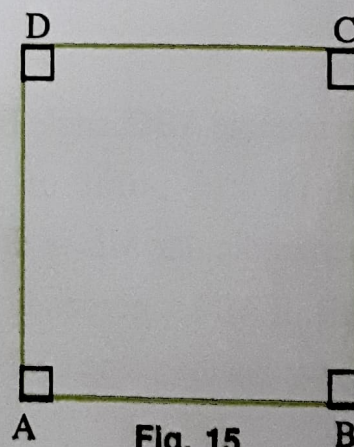


Fig. 15

2. In the adjacent figure slant line t intersects the two lines l and m at points P and Q respectively. If $\angle 2 = \angle 3 = 90^\circ$ then prove that line l and m are mutually parallel. What will be value of $\angle 1 + \angle 3$?

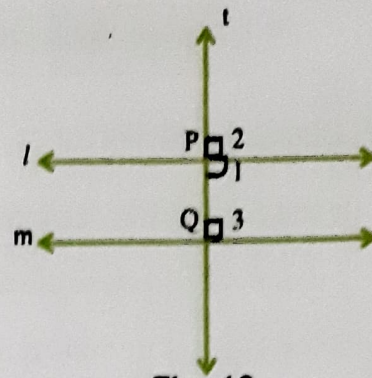


Fig. 16

3. ABCD is a trapezium in which $AD \parallel BC$. Line segments BL and CM are perpendicular to line AD . Show that $BL \parallel CM$. If $BC = BL$ then show that $BCML$ is a square.

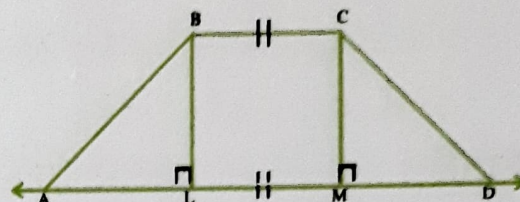


Fig. 17

4. ABCD is a square and L, M, N and O are mid points of side AB, BC, CD and DA respectively. Measure the angle and sides observe that shape $LMNO$ is also a square.

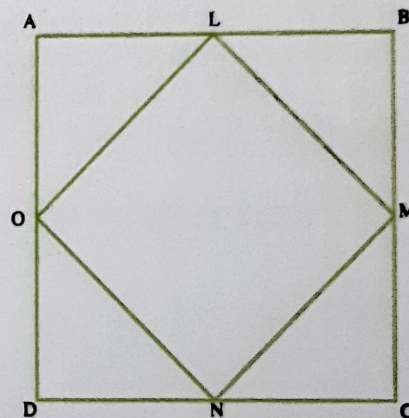


Fig. 18

5. In triangle ABC , angle B is a right angle, M is mid point of side AC , and perpendicular ML is drawn from M at AB . If MN is perpendicular to BC then show that the shape $LMNB$ is a rectangle.

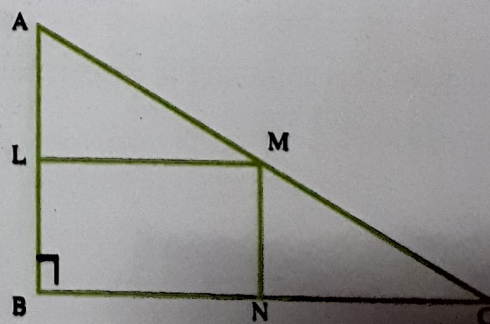


Fig. 19

6. In the adjacent figure $l \parallel m$, $p \perp m$ and $p \perp n$

(i) Is $m \parallel n$? why?

(ii) Is $l \parallel n$? why?

(iii) Is $p \perp l$? why?

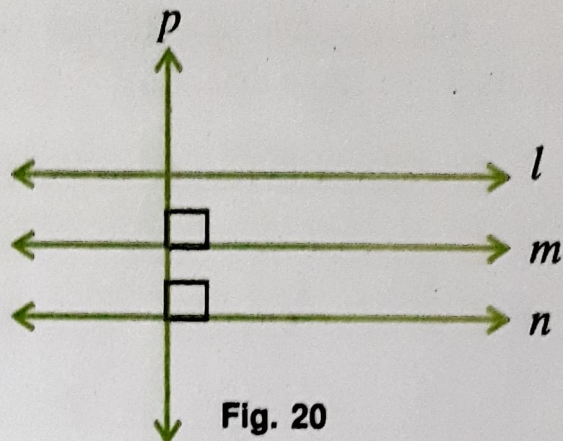


Fig. 20

9.2 Constructions

9.2.1 Dividing the line segment in two equal parts

Think, Discuss and Write

1. Can a point C be decided on a line segment $AB = 4$ cm, which divides it in two equal parts?
2. Can we find a point C on a drawn line segment $AB = 4.7$ cm which divides it in two equal parts.
3. Can a drawn line segment $AB = 7$ cm, divide into five equal parts?

Do the following :

In the given figure 1, a point M is taken on a line segment AB such that line segment $AM = MB$, we can see that point M divides line segment AB into two equal parts.

Similarly look the figure

2. Point L, M and N divides line segment AB into four equal parts. Similarly we can divide any line segment into 2, 2^2 , 2^3 ... equal parts.

How can a line segment be divided into five, six, seven or given number of equal parts with help of compass and ruler?

Draw mutually parallel lines l , m and n and look to the following figure 3. Draw two slant lines p and q .

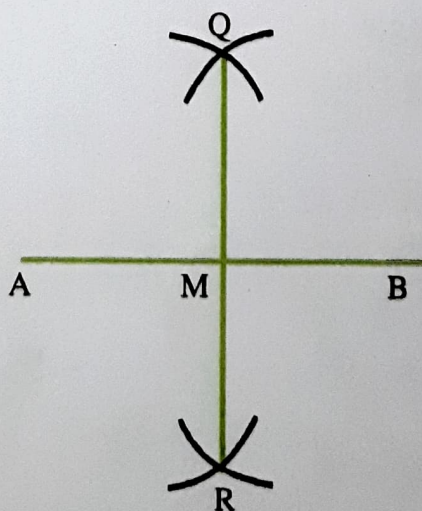


Fig. 1

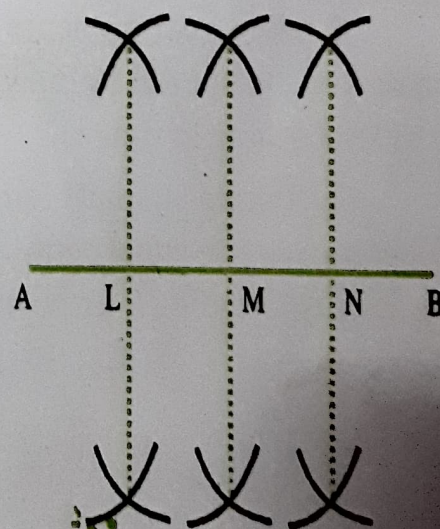


Fig. 2

If intersection $AB = BC$ then we know that intersection $PQ = QR$. Similarly look to the figure 4. We know that if $AB = BC = CD$ then $PQ = QR = QS$.

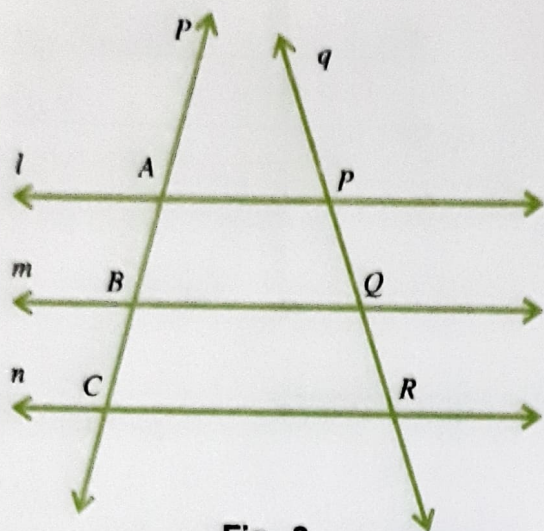


Fig. 3

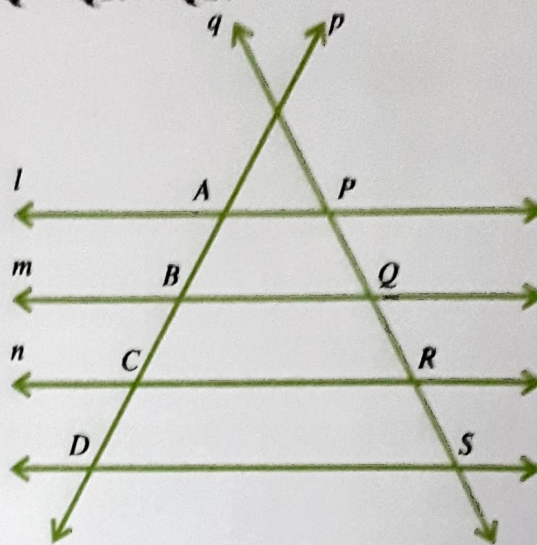


Fig. 4

9.2.2 To divide the given line segment into five equal parts.

Construction 1

Dividing line segment AB into five equal parts.

Given : Line segment AB

To Construct :

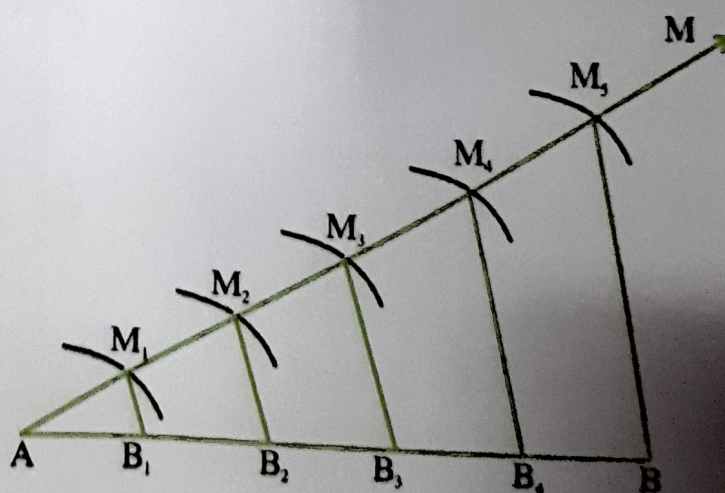
Dividing line segment AB into five equal parts.

Steps of Construction :

1. Construct an acute angle at point A on the line AB by drawing a line AM (According to figure).

2. Taking a small length on a compass, cut five equal parts on the line segment AM starting from point A. Mark these as AM_1 , M_1M_2 , M_2M_3 , M_3M_4 , and M_4M_5 .

3. Join the last point M_5 with point B.



4. Draw parallel lines from the points M_4, M_3, M_2 and M_1 , parallel to line M_5B , which meet at line segment AB at respectively points B_4, B_3, B_2 and B_1 . Points B_1, B_2, B_3 and B_4 divide the line segment AB into five equal parts.

Verification : Measure every line segment $AB_1, B_1B_2, B_2B_3, B_3B_4$ and B_4B_5 and verify that $B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

We observe that all line segment are equal.

Alternative method

Dividing the line segment into five equal parts.

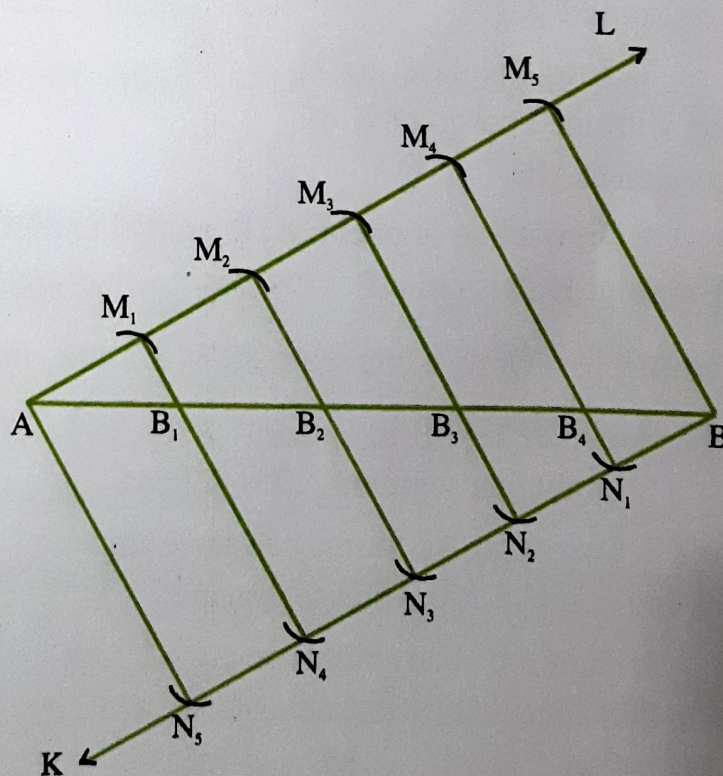
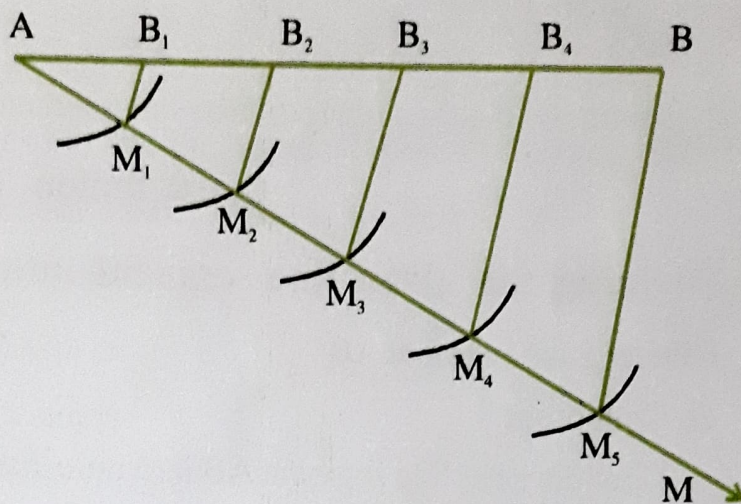
Steps of Construction :

1. Draw a ray AL forming an Acute angle at the end point A of line segment AB .

2. Draw a line BK parallel to ray AL from the other end point B .

3. Taking a certain radius cut arcs on AL . Marking five equal segments $AM_1, M_1M_2, M_2M_3, M_3M_4$ and M_4M_5 .

4. Similarly, by taking radius if same distance, mark five equal segments on ray BK , shown as $BN_1, N_1N_2, N_2N_3, N_3N_4, N_4N_5$.



5. Join the last marked point M_5 with B and N_5 to A. Similarly M_4 to N_1 , M_3 to N_2 , M_2 to N_3 and M_1 to N_4 .

6. These lines, cut the line segment AB at four respective points B_4 , B_3 , B_2 and B_1 . Similarly points B_1 , B_2 , B_3 and B_4 divides line segment AB into five equal parts.

Construction 2

9.2.3 Dividing the given line segment into given ratio.

Given : Line segment AB

To Construct :

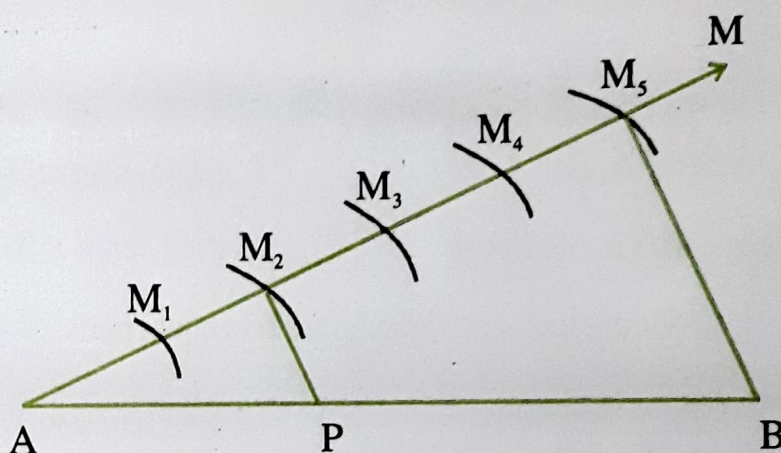
Dividing the line segment AB into ratio 2 and 3.

Steps of Construction :

1. Draw a ray AM making an acute angle at point A of line segment AB.

2. Sum of ratio 2 and 3 is 5. Hence mark five equal parts on ray AB starting from point A at AM_1 , M_1M_2 , M_2M_3 , M_3M_4 and M_4M_5 .

3. Join the last mark point with the end point B of line segment AB.



4. Draw a line segment M_2P parallel to M_5B from point M_2 , which cuts line segment AB at point P. This point P divides the line segment AB into ratio 2 : 3.

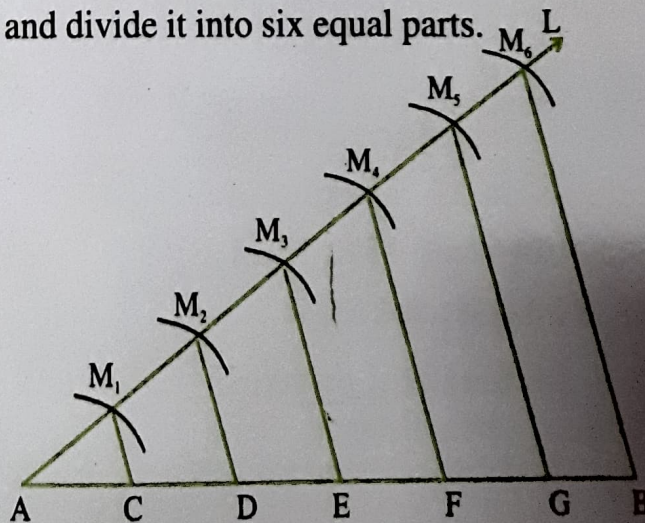
Example 1. Draw a line segment of 8.2 cm. and divide it into six equal parts.

Steps of Construction :

1. Draw line segment $AB = 8.2$ cm.

2. Draw a ray AL forming an acute angle from end point A of the line segment.

3. Draw arc of any radius on ray AL and divide it in six equal parts, and mark as AM_1 , M_1M_2 , M_2M_3 , M_3M_4 , M_4M_5 and M_5M_6 .



4. Join point M_6 with point B.

5. Draw parallel lines along BM_6 from points M_5, M_4, M_3, M_2 on line segment AB, which cut it at G, F, E, D and C respectively.

Therefore points C, D, E, F and G divide line segment AB into six equal parts.

Example 2. Draw a line segment of 8 cm. Divide it into ratio 3 : 5.

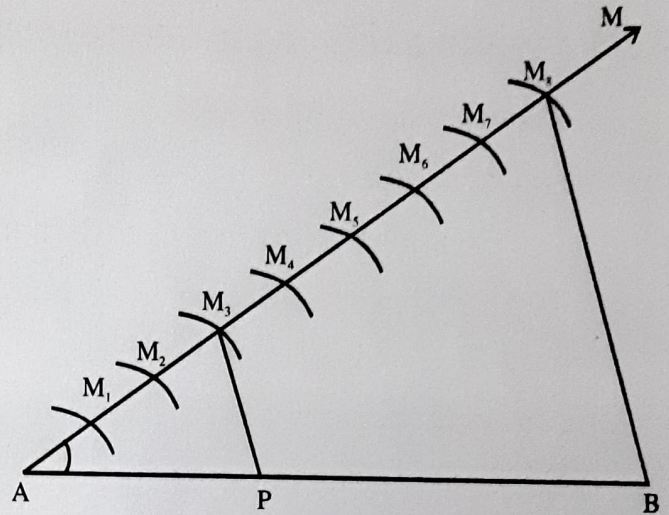
Steps of Construction :

1. Draw a line segment $AB = 8$ cm.
2. Draw ray AM , forming acute angle from end point A of the line segment AB.

3. The segment has to be divided into ratio 3 : 5, hence the sum of ratio $3 + 5 = 8$. Segments are to be made with arc of any radius on ray AM starting from point A. Mark these as $M_1, M_2, M_3, M_4, M_5, M_6, M_7$ and M_8 .

4. Join point M_8 with point B.

5. Draw M_3P from M_3 parallel to M_8B which cuts AB at P. Point P divides line segment into ratio 3 : 5.



Note : These construction can be constructed easily using alternate method of construction.

Exercise 9(c)

1. Draw line segment AB of 10 cm. and divide it into 5 equal parts with the help of ruler. Measure the length of each segment.
2. Divide the line segment AB of 8 cm into ratio 2 : 3. Measure the length of obtained line segment obtained and verify that their ratio is 2 : 3.
3. Draw line segment AB of 8 cm. Find the $\frac{3}{5}$ part of the segment from the end point A through construction.
4. Draw a line segment of 8.4 cm. Find a point P through the construction such that $AP = \frac{2}{5} AB$.

We have Learnt

1. Two lines parallel to a line are also mutually parallel.
2. Two lines perpendicular to a line are mutually parallel.
3. A given line segment can be divided into equal parts with the help of compass and ruler.
4. Given line segment can be divided into any given fixed ratio.

SPECIAL

Euclid (300 B.C.)

Euclid is known as father of Geometry. The Geometry which we study is known as Euclid's Geometry. He wrote a prodigious epic named Elements which is consisted of literature on Congruency, Parallel lines, Identities, Circles, Inscribed and Circumscribing polygons, Ratio. Algebraic equation solved by Geometrical methods, similarly of polygons solid geometry with elements are translated in different languages. In our country Jagannath Pandit translated in Hindi in 1719.

Some people say Euclid was born in Greece while some people believe that he was born in Egypt.

ANSWERS

Exercise 9 (a)

3. (i) three, (ii) three; 4. $\angle 2$, $\angle 4$, $\angle 6$ and $\angle 8 = 150^\circ$; $\angle 3$, $\angle 5$ and $\angle 7 = 30^\circ$;
5. $\angle ABC = 122^\circ$, $\angle DBK = 30^\circ$, $\angle BAC = 28^\circ$; 6. (i) yes $p \parallel q$ because corresponding angles are equal, (ii) $\angle 2 = 63^\circ$.

Exercise 9 (b)

2. 180°

