

CONSTRUCTION OF QUADRILATERALS



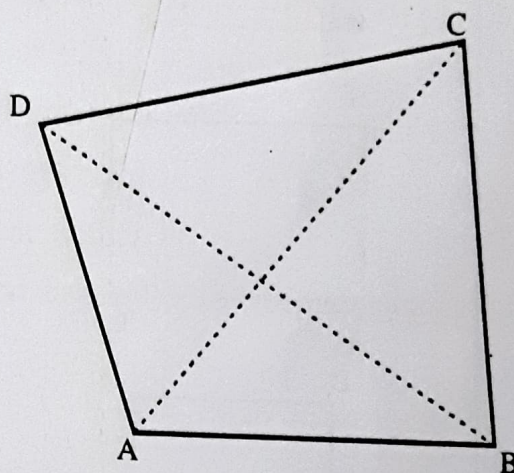
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Constructing a quadrilateral when :

- Four sides and a diagonal are given
- Three sides and both diagonals are given
- Two consecutive sides with included angle and two other angles are given
- Three sides and two included angles are given
- Four sides and one angle are given

10.1 Introduction

We have learnt about some geometrical figures as triangle, quadrilateral, circle etc in previous classes. We have also studied few characteristics of there. We can also construct a unique triangle if three of its measurements are given except only three angles are given. In this unit we will learn to construct a unique quadrilateral if any five specific combination of its measurements are known.

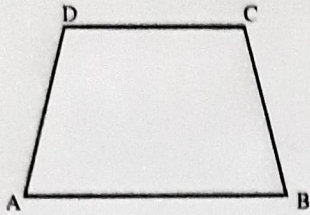
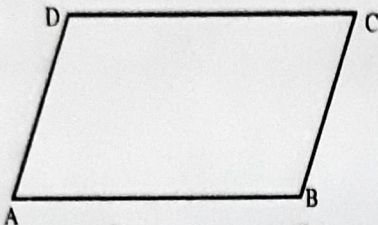
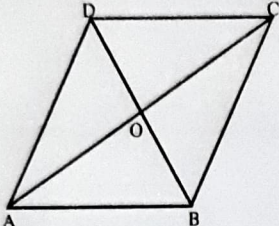
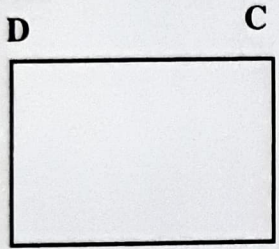
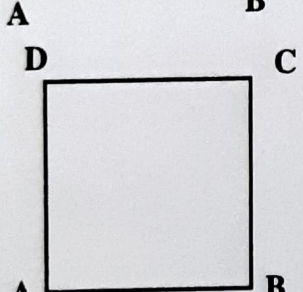
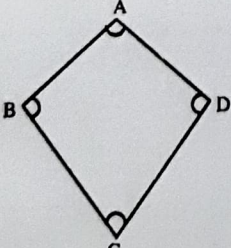
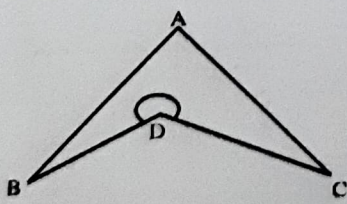


Do these

1. Complete the following table after viewing the figure of quadrilateral ABCD given above

Serial No.	Features of quadrilateral	Name of features of quadrilateral ABCD
(i)	Four vertices	A, B,
(ii)	Four sides	AB,
(iii)	Four Angles	, $\angle B$, $\angle C$ and $\angle D$
(iv)	Consecutive sides	DA and AB,
(v)	Diagonals	AC and
(vi)	Opposite angles	$\angle A$ and $\angle C$,
(vii)	Opposite sides	Opposite of side AB is side DC, opposite of side AD is

2. Fill in the blanks of the following table. With the characteristics of quadrilateral—

S.N.	Features of quadrilateral ABCD	Name and characteristics
1.		Trapezium —A quadrilateral whose one pair of opposite sides are
2.		Parallelogram —A quadrilateral with each pair of opposite sides are parallel, opposite sides of parallelogram are Opposite angles of parallelogram are
3.		Rhombus —A parallelogram whose all four sides are. The diagonal of rhombus intersect each other at
4.		Rectangle —Quadrilateral whose opposite sides are and every angle is Both diagonal of rectangle are
5.		Square —A quadrilateral whose all four sides are and every angle is Both diagonals of square bisect each other at right angle.
6.		Convex quadrilateral —A quadrilateral whose every angle is smaller than 180° . Every angle is smaller than in every quadrilateral of serial 1, 2, 3, 4, 5 and 6.
7.		Concave quadrilateral —A quadrilateral whose one angle is greater than 180° . The quadrilateral in serial no.7 has one angle D is greater than

Note :

In Geometry we study only properties of convex quadrilaterals.

Think, Discuss and Write

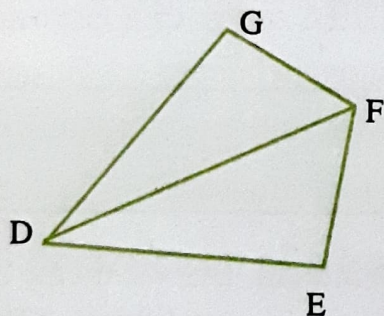
1. Can a parallelogram be a rectangle ?
2. Can a square also be a rectangle?
3. Can a square also be a rhombus ?

Exercise 10(a)

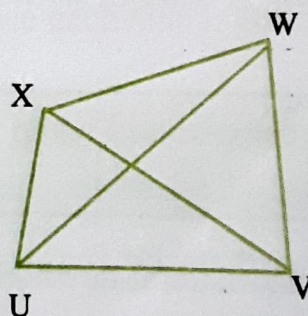
1. State whether True or False.

- (i) A quadrilateral has two diagonals.
- (ii) The four sides of a quadrilateral are always equal.
- (iii) The opposite sides of a quadrilateral are always equal.
- (iv) The square is a rhombus whose all four angles are right angle.
- (v) A rectangle is a parallelogram with all four angles are right-angle.
- (vi) Sum of all four interior angles of a quadrilateral is 360° .
- (vii) The diagonals of a parallelogram are always equal.
- (viii) The diagonals of a quadrilateral bisect each other.

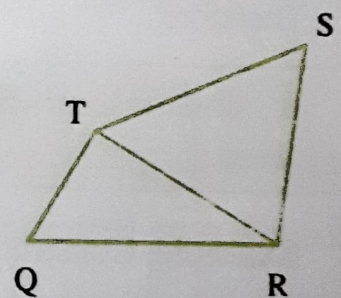
2. Write down the names of sides, vertices, angles and diagonals in the following quadrilaterals :



(i)



(ii)



(iii)

3. Three angles of a quadrilaterals are 75° , 95° and 110° respectively, find out the fourth angle.
4. If two angles of a quadrilateral are 60° and 120° and remaining two are equal, find out their value.
5. Draw the figures of following quadrilaterals :
 - (i) Trapezium
 - (ii) Convex quadrilateral
 - (iii) Concave quadrilateral
 - (iv) Kite

10.2 Construction of a quadrilateral when four sides and one diagonal are given :

Take four steek wooden splints. Join all the four splints together loosely with the help of nail. Now describe which geometrical figure you have made.

We can see this is a quadrilateral as shown in fig.—1.

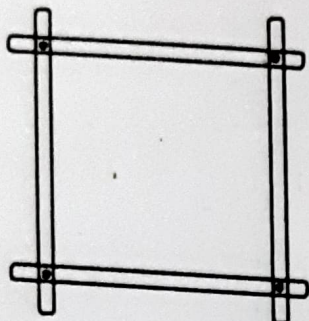


Fig.—1

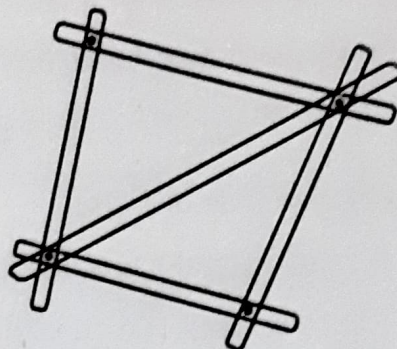


Fig.—2

Now push or pull any of two splints. You can see the shape of quadrilateral is changed. So quadrilateral take different shapes when it is made of only four splints. No stable quadrilateral is formed.

Now join any two opposite angles with another splint. Does it give the typical shape of quadrilateral ?

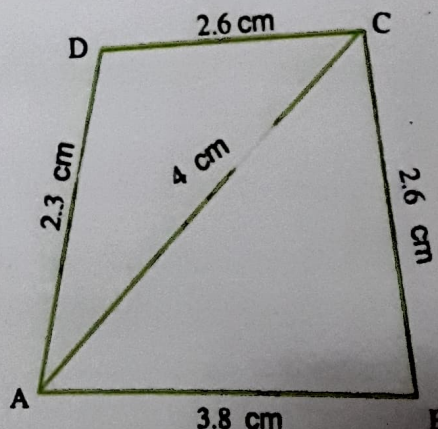
Now in this situation a typical shape of quadrilateral is formed. Fig.—2.

Therefore

A quadrilateral can be constructed when four sides and one diagonal are given.

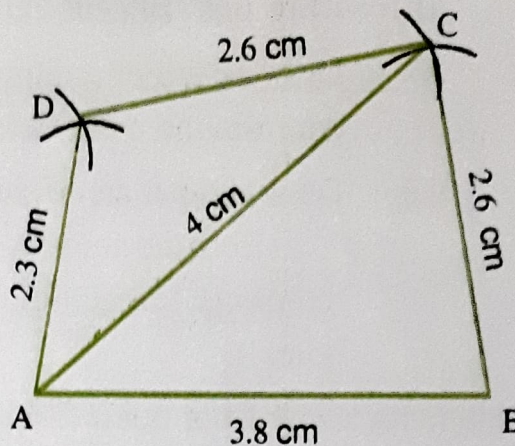
Example. Construct a quadrilateral in which $AB = 3.8$ cm, $BC = 2.6$ cm, $CD = 2.6$ cm, $AD = 2.3$ cm and diagonal $AC = 4.0$ cm.

Solution : Draw a rough sketch of quadrilateral. Mark all the measurements in this rough sketch, it is easy to see that this quadrilateral is made of $\triangle ADC$ and $\triangle ABC$ whose each side is known. The construction of these two triangles shall automatically complete the construction of quadrilateral.



Construction :

- (i) Draw line segment $AB = 3.8$ cm.
- (ii) Draw an arc of radius 2.6 cm. from the point B and other arc of radius 4 cm from point A .
- (iii) Both arc intersect each other at the point C . Draw sides AC and BC .
- (iv) Draw an arc 2.3 cm from point A and other from point C of radius 2.6 cm. Both arc intersect at point D .
- (v) Draw sides AD and CD . The constructed quadrilateral $ABCD$ is the required quadrilateral.



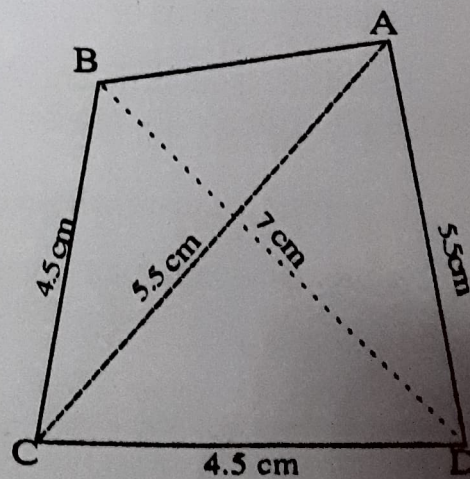
Exercise 10(b)

1. Construct a quadrilateral $ABCD$, given that $AB = 4.0$ cm, $BC = 6.0$ cm, $CD = DA = 5.2$ cm and $AC = 8$ cm.
2. Construct quadrilateral $ABCD$ where $AB = 4.4$ cm, $BC = 4$ cm, $CD = 6.4$ cm, $DA = 2.8$ cm and $BD = 6.6$ cm. Measure the side AC .
3. Construct a quadrilateral $PQRS$ where $PQ = 3$ cm, $QR = 5$ cm, $QS = 5$ cm, $PS = 4$ cm and $SR = 4$ cm. Measure the side PR .
4. Construct a rhombus $ABCD$ whose each side is 4.5 cm and one diagonal is 6.0 cm. Measure the second diagonal.
5. Construct a parallelogram $ABCD$, whose $AB = 3.6$ cm, $BC = 4.2$ cm and $AC = 6.5$ cm. Measure the remaining two sides and write down the measurement in your answer book.

10.3 Construction of a quadrilateral when three sides and two diagonals are given.

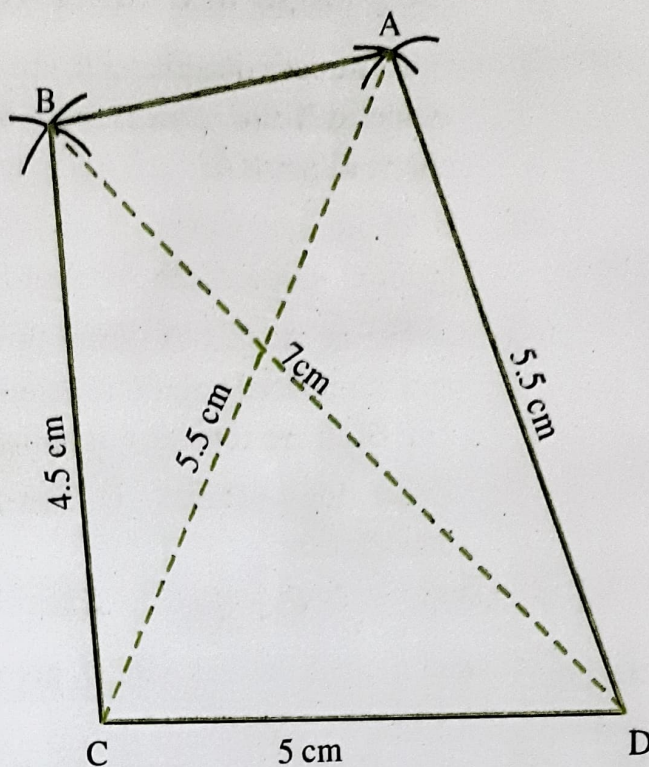
Example : Construct a quadrilateral $ABCD$ where $BC = 4.5$ cm, $CA = AD = 5.5$ cm, $CD = 5$ cm and $BD = 7$ cm.

Solution : Draw a rough sketch and mark all the given measurements. Construct $\triangle ADC$ and $\triangle BDC$. Quadrilateral $ABCD$ is easily formed after joining the points A and B . There is no need to show diagonals in the construction of quadrilateral $ABCD$, why ?



Construction :

- (i) Draw line segment $CD = 5.0$ cm.
- (ii) Draw an arc of radius $CB = 4.5$ cm from the point C.
- (iii) Draw another arc of radius $DB = 7$ cm from the point D. Both arcs intersect to each-other at the point B.
- (iv) Now from point C draw an arc of radius $CA = 5.5$ cm to the right side of point B.
- (v) Taking point D as a centre draw an arc of radius $DA = 5.5$ cm. This arc will cut the previous arc drawn in steps (iv) at the point A.
- (vi) Join DA, AB and BC. ABCD is the required quadrilateral.



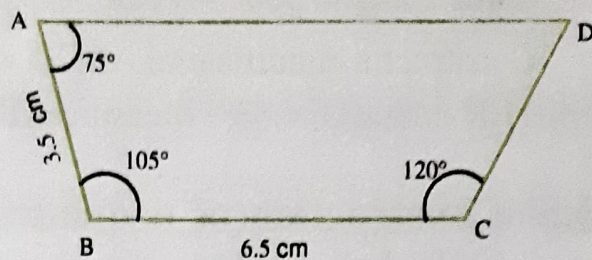
Exercise 10(c)

1. Construct the quadrilateral, given $AB = 3.8$ cm, $BC = 3$ cm, $AD = 2.3$ cm, $AC = 4.5$ cm and $BD = 3.8$ cm. Know the length CD after measuring it.
2. Construct quadrilateral ABCD, where $BC = 7.5$ cm, $AC = AD = 6$ cm, $CD = 5$ cm and $BD = 10$ cm. Find out the measurement of fourth side after measuring it.
3. Construct a parallelogram whose one side is 4.4 cm and both diagonals are 5.6 cm and 7.0 cm respectively. Find out the measurement of other side.
(Hint : Diagonals of a parallelogram bisect each other).
4. Construct a quadrilateral ABCD, where $AB = 3.0$ cm, $CD = 3.0$ cm, $DA = 7.5$ cm, $AC = 8.0$ cm and $BD = 5.5$ cm.
5. Construct a square whose diagonal is 6.4 cm.
(Hint : Diagonals of a square bisect each-other at right angle.)

10.4 Construction of quadrilateral when two adjacent sides, one included angle and other two angles are known.

Example. Construct a quadrilateral ABCD where $AB = 3.5$ cm, $BC = 6.5$ cm, $\angle B = 105^\circ$, $\angle A = 75^\circ$ and $\angle C = 120^\circ$.

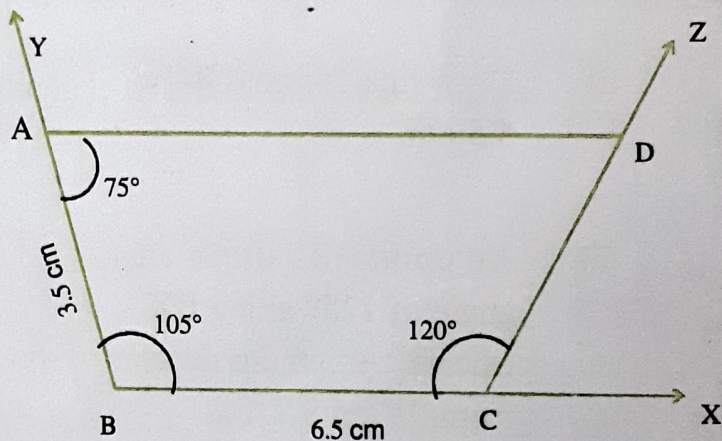
Solution : Draw a rough figure and label it with the given measurements.



See the rough fig. and draw a line segment $BC = 6.5$ cm. make an angle of 105° at point B. Cut $BA = 3.5$ cm from the side of angle of B. At the point A make an angle of 75° and from point C make an angle of 120° . The sides of both angle intersect at point D. ABCD is required quadrilateral.

Steps of construction :

1. Draw line segment $BC = 6.5$ cm.
2. Make $\angle YBC = 105^\circ$ at the point B.
3. With B as centre and radius $AB = 3.5$ cm, draw an arc to cut YB at A.
4. At point C, construct an $\angle BCZ = 120^\circ$.
5. Draw AD in such a way to make $\angle DAB = 75^\circ$ at A and will cut ray CZ at D. ABCD is a required quadrilateral.



Try it :

Construct a quadrilateral in which $AB = 3.5$ cm, $BC = 5$ cm., $\angle A = 80^\circ$, $\angle B = 105^\circ$ and $\angle D = 85^\circ$.

Exercise 10(d)

1. Construct a quadrilateral ABCD in which, $AB = 5.5$ cm, $BC = 3.7$ cm, $\angle A = 60^\circ$, $\angle B = 105^\circ$ and $\angle D = 90^\circ$.
2. Construct a rectangle whose sides are 4.5 cm and 6 cm. Measure its both diagonal.

3. Construct quadrilateral PQRS where $PQ = 3.5$ cm, $QR = 6.5$ cm, $\angle P = \angle R = 105^\circ$ and $\angle S = 75^\circ$.
(Hint : $\angle Q = 360^\circ - (105^\circ + 105^\circ + 75^\circ) = 75^\circ$)
4. Construct a quadrilateral ABCD where $BC = 5.5$ cm, $CD = 4.1$ cm, $\angle A = 70^\circ$, $\angle B = 110^\circ$ and $\angle D = 85^\circ$. Measure AB and DA.

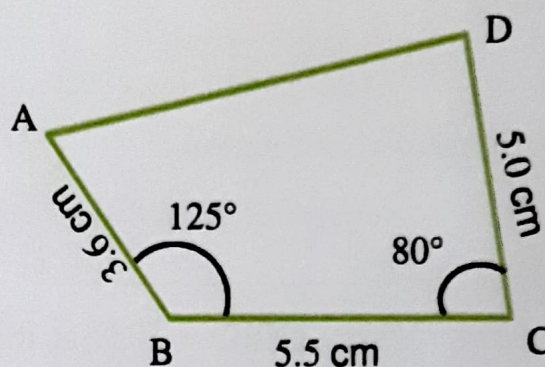
10.5 Construction of a quadrilateral when three sides and two included angles are given.

Example. Construct a quadrilateral in which $AB = 3.6$ cm, $BC = 5.5$ cm, $CD = 5.0$ cm, $\angle B = 125^\circ$ and $\angle C = 80^\circ$.

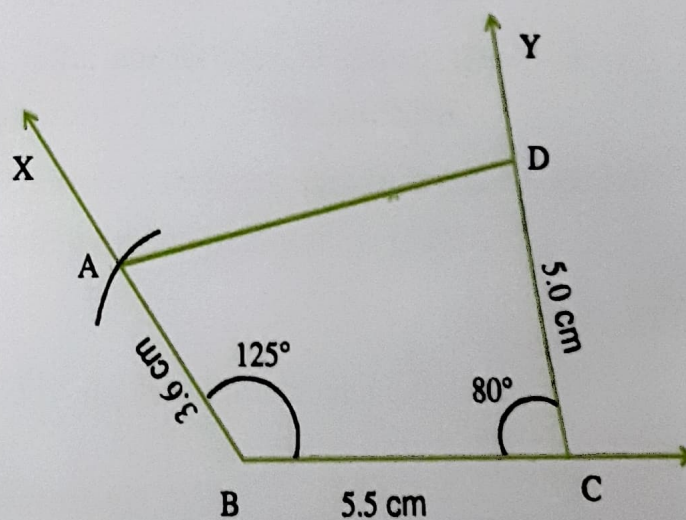
Solution : Here is a rough sketch that would help in deciding steps of construction.

Steps of Construction :

- (i) Draw line segment $BC = 5.5$ cm.
- (ii) At point 'B' draw an angle of 125° along BX. Locate 'A' 3.6 cm away from 'B' on BX.
- (iii) Make $\angle BCY = 80^\circ$ at point C. Taking point C as a centre draw an arc of radius $CD = 5.0$ cm on CY to locate the point D.
- (iv) Join AD. It is required quadrilateral.



Rough figure



Exercise 10(e)

1. Construct a quadrilateral ABCD in which $AB = 4.2$ cm, $BC = 3.6$ cm, $CD = 4.8$ cm, $\angle B = 30^\circ$ and $\angle C = 150^\circ$. Measure AD.
2. Construct a quadrilateral PQRS in which $PQ = 3.5$ cm, $QR = 2.5$ cm, $RS = 4.1$ cm, $\angle Q = 75^\circ$, $\angle R = 120^\circ$. Measure RS.
3. Construct a quadrilateral ABCD where $AB = BC = 3$ cm, $AD = 5$ cm, $\angle A = 90^\circ$, $\angle B = 105^\circ$.
4. Construct a quadrilateral PQRS in which $\angle Q = 135^\circ$, $\angle R = 90^\circ$, $QR = 5$ cm, $PQ = 9$ cm and $RS = 7$ cm.

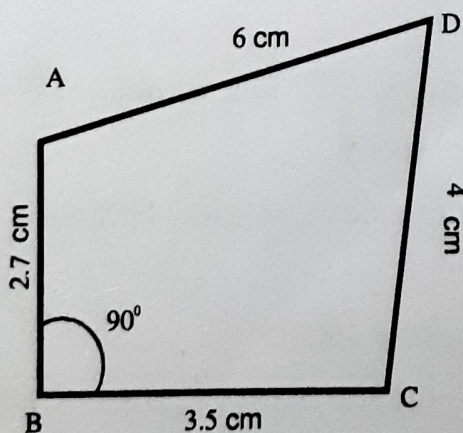
10.6 Construction of a quadrilateral when four sides and one angle is given

Example : Construct a quadrilateral in which $AB = 2.7$ cm, $BC = 3.5$ cm, $CD = 4$ cm, $AD = 6$ cm and $\angle B = 90^\circ$.

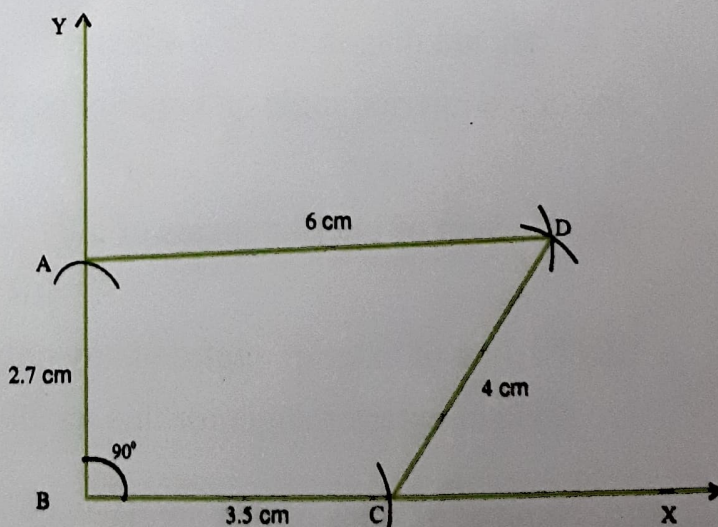
Solution : Draw a rough sketch. With the help of it construct quadrilateral.

Steps of Construction

- (i) Draw $BC = 3.5$ cm.
- (ii) Make $\angle YBC = 90^\circ$ at B.
- (iii) Draw an arc of radius $AB = 2.7$ cm from centre B on ray YB which cuts on A.
- (iv) From A draw an arc of radius $AD = 6.0$ cm, towards the side where point C is situated.



Rough figure



- (v) From C draw an arc of radius $CD = 4$ cm which will cut previous arc of step (iv) at D.
- (vi) Join DA and CD. This is required quadrilateral ABCD.

Exercise 10 (f)

- Construct quadrilateral ABCD in which $AB = BC = 3$ cm, $AD = CD = 5$ cm and $\angle ABC = 120^\circ$.
- Construct quadrilateral ABCD in which $AB = 2.8$ cm, $BC = 3.1$ cm, $CD = 2.6$ cm, $DA = 3.3$ cm and $\angle A = 60^\circ$.
- Construct a rectangle whose sides are 4.2 cm and 2.5 cm. Measure the diagonal of it.
- Construct a rhombus whose one angle is 75° and one side is 5.2 cm.
- Construct a square whose one side is 5.0 cm.

Efficiency Practice 10

- Construct quadrilateral ABCD from the given following measurement :
 - $AB = 25$ cm, $BC = 7.5$ cm, $CD = 10$ cm, $DA = 7.5$ cm, $BD = 6.5$ cm.
 - $AB = 4$ cm, $BC = 3$ cm, $CD = 6$ cm, $\angle B = 135^\circ$, $\angle C = 60^\circ$
 - $BC = 4$ cm, $\angle C = 120^\circ$, $CD = 5$ cm, $\angle BDA = 26^\circ$, $\angle A = 64^\circ$.
- Construct a square ABCD whose side is 4 cm. Measure AC.
- Construct a rectangle ABCD where $AB = 4$ cm and $AC = 6$ cm. Measure AD.
- Construct a parallelogram ABCD whose sides are 5.8 and 6.2 cm and diagonal is 7.3 cm. Measure its other diagonal.
- Construct a rectangle ABCD in which $AB = 5$ cm, $AC = 6$ cm. Measure $\angle BAD$.
- Construct a quadrilateral ABCD in which $AB = 4.1$ cm, $BC = 4.5$ cm, $CD = 3$ cm, $AD = 3.7$ cm and diagonal $AC = 4.2$ cm.
- One of the interior angle of a regular polygon is 108° . The number of its sides are :

(i) 5	(ii) 6	(iii) 7	(iv) 8	NTS 2007
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(HINT : Sum of interior angles of polygon of sides $n = (2n - 4) \times 90^\circ$)

OR

HINT : Sum of exterior angles of any polygon $= 360^\circ$

$\therefore$ Value of exterior angle for this regular polygon $= 180^\circ - 108^\circ = 72^\circ$

$\therefore$ Number of sides of this regular polygon $= \frac{360^\circ}{72^\circ} = 5$.

Think, Discuss and Write

$(2n-3)$ independent measurements are needed to determine the construction of any polygon, where n stands for number of sides in this statement. How many independent measurements are needed to construct triangle, rectangle and pentagon respectively base on this statement.

We have learnt

Construct of a quadrilateral when measurements of a quadrilateral are given as following :

- (i) Four sides and one diagonal are known.
- (ii) Three sides and two diagonals are known.
- (iii) Two adjacent sides one included angle and other two angles are known.
- (iv) Three sides and two included angles are known.
- (v) Four sides and one angle is known.

ANSWERS

Exercise 10 (a)

1. (i) True, (ii) False, (iii) False, (iv) True, (v) True, (vi) True, (vii) False, (viii) False;
2. (i) sides DE, EF, FG and GD; vertices D, E, F and G; angles $\angle D$, $\angle E$, $\angle F$ and $\angle G$; Diagonals DF and EG (ii) sides UV, VW, WX and XU; Vertices U, V, W and X; angles $\angle U$, $\angle V$, $\angle W$ and $\angle X$; Diagonals UW and VX; (iii) Sides QR, RS, ST and TQ; vertices Q, R, S and T; angles $\angle Q$, $\angle R$, $\angle S$ and $\angle T$; diagonals QS and Rt.; 3. 80° , 4. 90° and 9° .

Exercise 10 (f)

3. 4.9 cm

Efficiency Practice 10

2. 5.7 cm, 3. 3 cm; 7. (i) 5

