

UNIT-14 TANGENTS OF A CIRCLE



- Secant, tangent and point of contact.
- Construction of a tangent on a circle from a given point, when the point is situated at the circle.
- Practical verification : The radius drawn from tangent and point of contact are perpendicular to each other.

14.1 Introduction

We must have paid attention to the wheels of cycle, bike, bus, truck etc., the various mode of transportation moving on the road. Their wheels are circular, which are moving to its destination touching the road. If you imagine a line aligned to the road (Fig. 14.1) then it will appear to you that this line passes through the wheel touching the circle.

Similarly, if a wheel is lying on the ground and a rod is placed in accordance with the fig. 14.2 then we will have the impression that a line has been drawn to intersect at two points of a circle.

Thus we find that a line passes either by touching the circle or passes through two different points of the circle. In mathematical language, first type of line is called the tangent and second is known as the intersecting line (secant) of the circle. In this unit we will study about the special properties related to the tangent of the circle and the intersecting line.

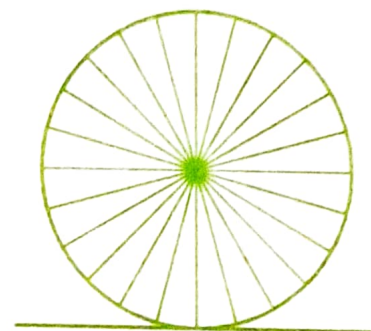


Fig. 14.1

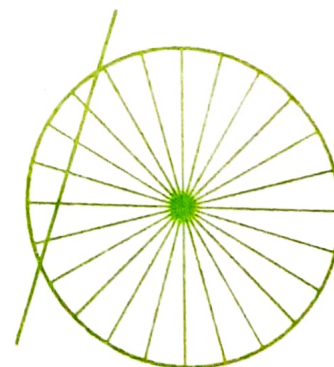


Fig. 14.2

14.2 Secant, tangent and point of contact

Try :

Draw a circle on your exercise book and draw different lines around the circle to tell that the different lines can intersect the circle upto how many maximum and minimum points ?

We find that if there is a circle and a line located on the same plane, then the following possibilities can be according to the figure of their conditions.

1. The line intersects the circle at two different points as shown in Fig. (i)

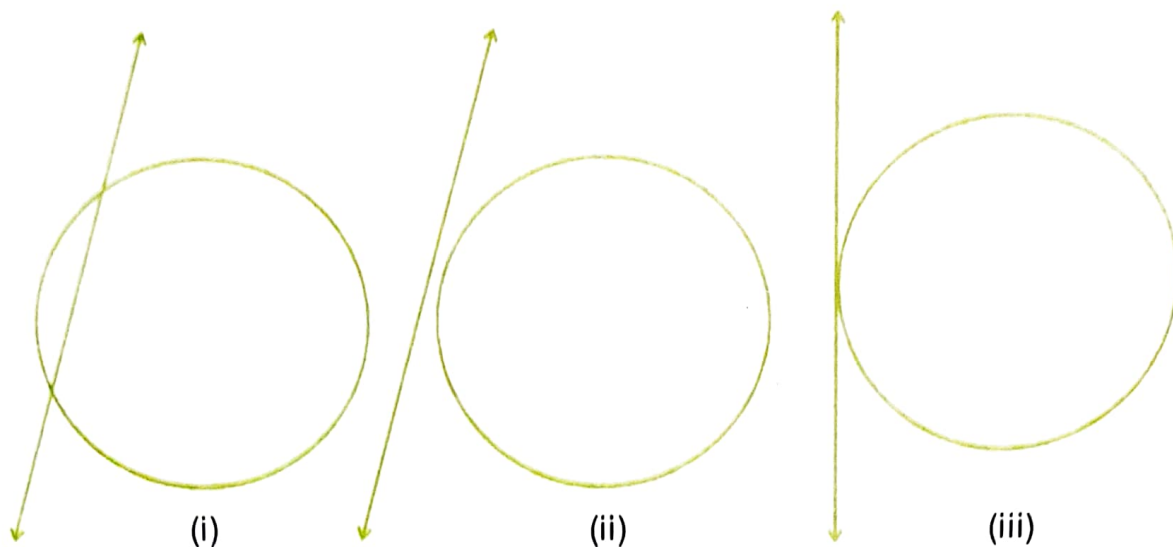


Fig. 14.3

2. The line does not intersect the circle as shown in Fig. (ii)
3. The line intersects the circle at only one point as shown in Fig. (iii)

In first situation the line is called an intersecting line of the circle.

Therefore, the intersecting line (secant) of any circle is that line which intersects the circle at two different points.

In third situation line is called a tangent of the circle.

Thus

Tangent of any circle is that line which intersects the circle at only one point.

14.3 Point of Contact

The point on the circle through which tangent of the circle passes, is called the point of contact. The point of contact is common in the circle and the tangent. In the Fig. 14.4, the line t is tangent; O the centre of the circle and P is the point of contact.

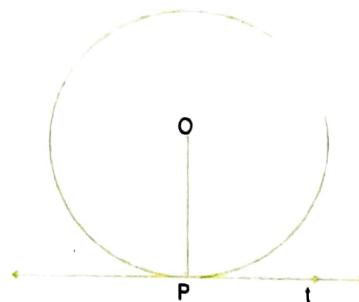


Fig. 14.4

We can visualize some examples related to a circle, tangent of a circle and point of contact around us. For example see the wheels of a train on the railway track which is represented by a figure in adjacent. Determine here the circle, tangent of the circle and point of contact.

Here the rim of the wheels of the train is a circle, track is the tangent of the circle and the point at which the wheel touches the track is the point of contact.

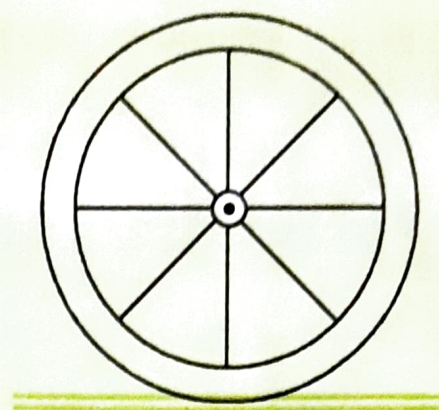


Fig. 14.5

14.4 Group of Intersecting Lines and Tangent

Think, Argue and Write

In the adjacent figure 14.6 'O' is the centre of the circle. P is a point on the circle. Distinguish between the tangents and intersecting lines in lines passing through the point P in the plane of the circle. Each one, except one of these lines, intersects the circle at a point other than the point P.

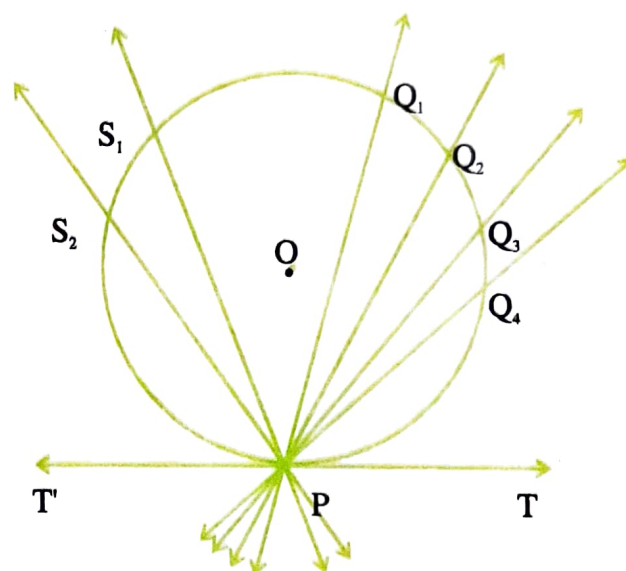


Fig. 14.6

Thus, there is a group of intersecting lines through P. Some of these intersecting lines intersect the circle again at the points Q_1 , Q_2 , Q_3 and Q_4 towards the right of P while some another intersecting lines intersect at the points S_1 and S_2 towards left of P.

There is only one line out of the lines that passes through P, which does not intersect the circle at any other point except P. This $T'PT$ is the tangent of the circle.

- (i) How many intersecting lines can be drawn through any point of a circle ?
- (ii) How many tangents can be drawn through any point of a circle.

Do these, Think and Conclude

According to adjacent figure draw a circle whose centre is O and draw parallel lines with the help of scale and ruler in which—

- Two lines do not intersect the circle
- Some lines be the intersecting lines of the circle and
- Two lines be the tangents of the circle.

By measuring distance from the centre to each of these three types of line with the help of set square and by measuring radius of the circle, tabulate following on your exercise book.

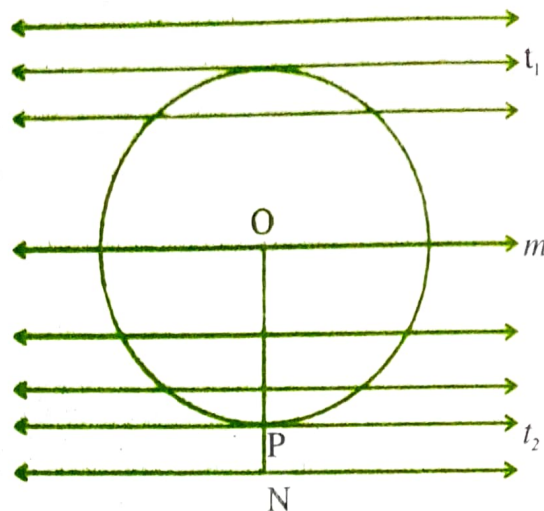


Fig. 14.7

Lines	Distance from the centre O (Let p)	Radius (Let r)
Line not intersecting the circle		
Tangent of the circle		
Intersecting line of the circle		

We will find that if p is the distance of any line of this group from centre O of the circle, then

- If $p > r$, then line does not intersect the circle.
- If $p = r$, then line be the tangent of the circle.
- If $p < r$ line be the intersecting line of the circle.

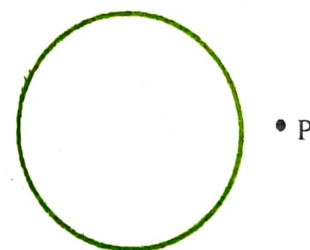


Fig. 14.8

Example 1. In the adjacent figure 14.8 there is the point P outside the circle. How many intersecting lines can be drawn from the point P.

Solution : Infinitely many intersecting lines can be drawn from the point P. (Think how ?)

Example 2. How many tangents are there and which are the point of contact in the Fig. 14.9.

Solution : Three; P, Q, R are the points of contact.

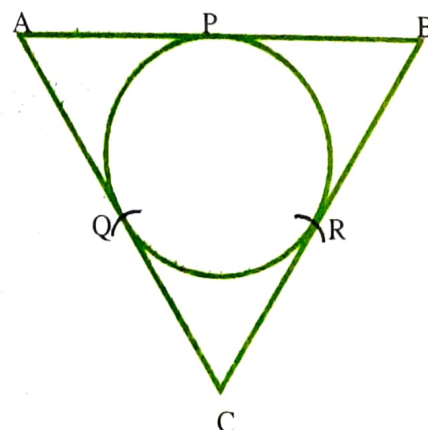


Fig. 14.9

Exercise 14(a)

1. In adjacent figure O is the centre of the circle and some line segments are drawn. Find out—

- (i) Two intersecting lines
- (ii) Two tangents
- (iii) One diameter
- (iv) One point of contact and
- (v) One chord

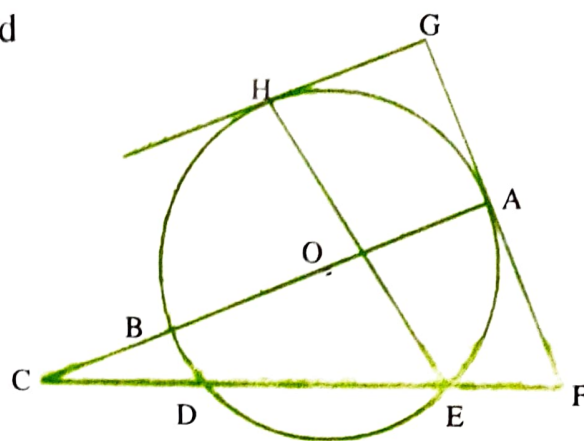


Fig. 14.10

2. If t is the tangent line of a circle with centre O and radius r which touches the circle at P . If Q be another point on the line t , then select true or false statement from the following statements :
 - (i) $OQ > r$
 - (ii) $OQ = r$
 - (iii) $OQ < r$
 - (iv) $OP < r$
 - (v) $OP = r$
 - (vi) $OP > r$
3. What is the difference between a chord and an intersecting line (Secant) ? Explain with help of a figure.
4. Draw a circle of radius 2.0 cm. Take a point P inside this circle. Find out is there any line can be drawn through P which touches the circle ?
5. What is difference between an intersecting line (Secant) and a tangent line of a circle ? Explain by drawing a figure.
6. Is a diameter be an intersecting line ?

14.5 Tangent and Radius Drawn from the point of contact are mutually perpendicular (Experimental verification)

Do these, Think and Conclude

Draw three circles with different centres and radius. Nominate the centre of all circles by O for convenience.

First take a point P at a circle. Draw a line PT from P with help of scale such that PT passes through only one point of the circle. The obtained line PT touches the circle at the point P. Draw the line segment OP and measure $\angle OPT$ and find the value of $90^\circ - \angle OPT$.

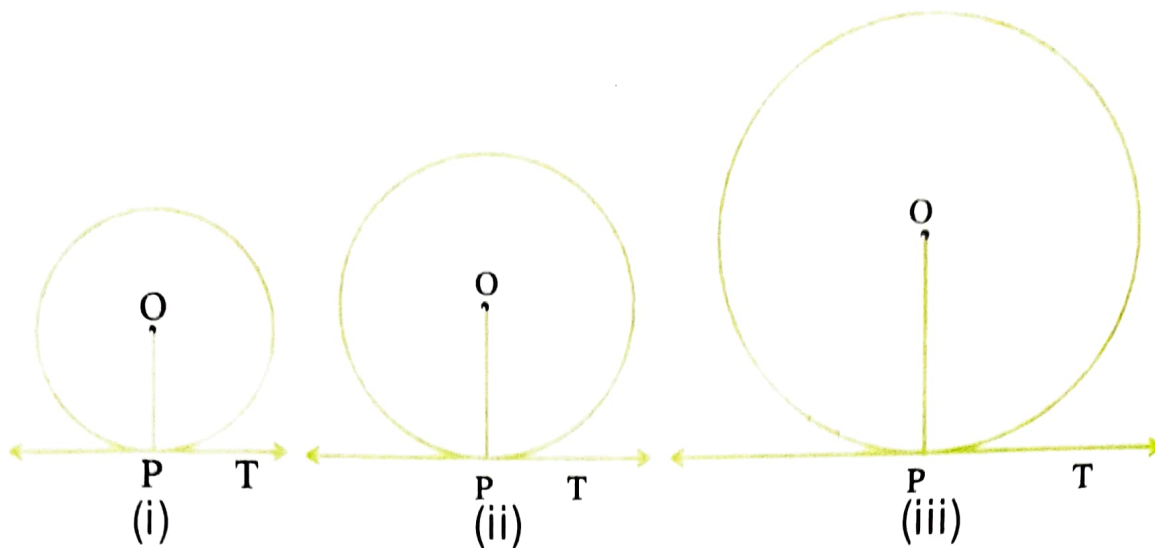


Fig. 14.11

Repeat this process for other two circles and tabulate the results so obtained in following manner :

Serial Number of circle	$\angle OPT$	$90^\circ - \angle OPT$
(i)		
(ii)		
(iii)		

We'll see that the value of $90^\circ - \angle OPT$ is zero or almost zero in each situation.

Thus we conclude that

In circle, the tangent line at any point and radius passing through the point of contact are mutually perpendicular.

Tell by Thinking :

From the extreme point P of the radius OP can there be more than one normal drawn on OP ?

14.6 To construct a tangent line from a given point on a circle when the point is situated at the circle

Given : There is a circle of centre O. A point P is situated on the circle.

Objective : To draw a tangent line passing through the point P which touches the circle at P.

Construction :

1. Draw the line segment OP.
2. Draw a normal PX from the point P to the line segment OP.
3. Extend XP till the point T. Thus XT became the tangent line of the circle which touches the circle at the point P.

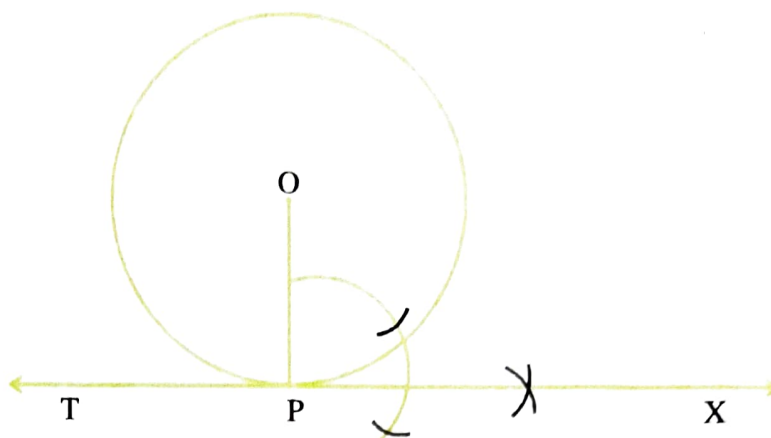


Fig. 14.12

Example 3. In the adjacent figure TPT is the tangent of a circle of centre O and PQ is diameter passing through P. Also PR is a chord of the circle and Q is joined with R. If $\angle RPT = 40^\circ$, then what is the value of $\angle PQR$?

Solution : Since PT is the tangent at P of the circle and OP is the radius through P,

Therefore $\angle OPT = 90^\circ = \angle QPT$

$$\text{or, } \angle QPR + \angle RPT = 90^\circ$$

$$\text{or, } \angle QPR = 90^\circ - \angle RPT$$

$$= 90^\circ - 40^\circ$$

$$= 50^\circ$$

Since $\angle PRQ$ is an angle of semicircle, therefore $\angle PRQ = 90^\circ$

Now in ΔPQR

$$\angle PQR + \angle QPR + \angle PRQ = 180^\circ$$

$$\text{or, } \angle PQR + 50^\circ + 90^\circ = 180^\circ$$

$$\text{or, } \angle PQR + 140^\circ = 180^\circ$$

$$\text{or, } \angle PQR = 180^\circ - 140^\circ$$

Therefore, $\angle PQR = 40^\circ$

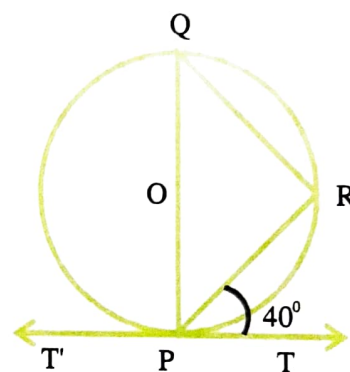


Fig. 14.13

Example 4. Draw a circle of radius 2.5 cm, whose centre is O. Draw two radii OA and OB of this circle such that $\angle AOB = 120^\circ$. Draw the tangents through the points A and B of the circle which intersects each other to a point P. Measure PA and PB.

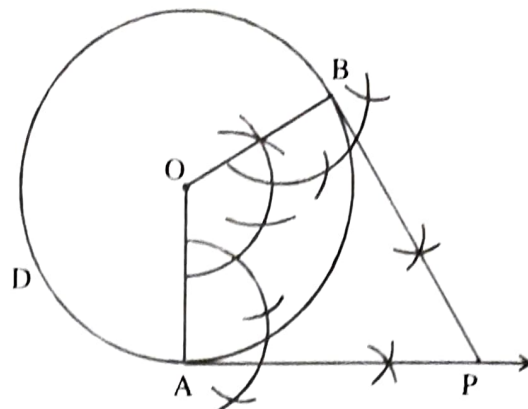


Fig. 14.14

Solution :

Construction : Draw a circle of radius 2.5 cm. whose centre is O. Draw a radius OA. Draw another radius OB making an angle 120° of line segment OA at the point O. Now draw the normals on radii OA and OB through points A and B respectively which intersects each other to the point P. Now measure PA and PB which is almost 4.3 cm.

Exercise 14(b)

1. 'O' is centre of the circle in adjacent figure. PQ is the tangent of the circle and P is the point of contact. What is the value of $\angle OPQ$?

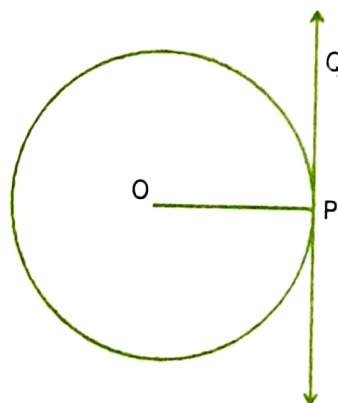


Fig. 14.15

2. Draw a circle of radius 3.0 cm. Take a point P on the circle. Draw the tangent through P of the circle. Write the construction.
3. There is a circle with centre O. A point P is given on the circle. Tell how many tangents can be drawn through P on the circle. Explain the answer by drawing a figure.
4. Draw a circle of radius 3.0 cm. and centre O. Draw two radii OA and OB of this circle such that $\angle AOB = 125^\circ$ with the help of protractor and scale. Draw two tangents through points A and B. If both tangents intersects each other at the point P then find $\angle APB$.

Example 5. In the adjacent figure PT is the tangent at a point P of a circle with centre O, PQ is a diameter and PS and PR are two chords of the circle such that $RP \perp PS$. If the $\angle QOS = 62^\circ$, then find—

- (i) $\angle QPS$ (ii) $\angle SPT$
 (iii) $\angle QOR$ (iv) $\angle RPT$

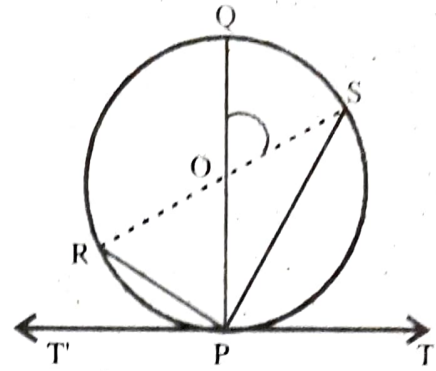


Fig. 14.16

Solution :

(i) Join OS and OR, Now $\angle QOS = 62^\circ$
 therefore $\angle QPS = \frac{1}{2} \angle QOS = \frac{1}{2} \times 62^\circ = 31^\circ$ -----(i)

(ii) $\angle QPT = 90^\circ$

or, $\angle QPS + \angle SPT = 90^\circ$

or, $\angle SPT = 90^\circ - \angle QPS$

or, $\angle SPT = 90^\circ - 31^\circ$
 $= 59^\circ$ -----(ii)

(iii) Since $RP \perp SP$

therefore $\angle RPS = 90^\circ$

or, $\angle RPQ + \angle QPS = 90^\circ$

or, $\angle RPQ = 90^\circ - \angle QPS$

or, $\angle RPQ = 90^\circ - 31^\circ$
 $= 59^\circ$

therefore $\angle QOR = 2 \angle RPQ$

$= 2 \times 59^\circ$

$= 118^\circ$

(iv) $\angle RPT = \angle RPQ + \angle QPT$

$= 59^\circ + 90^\circ$

$= 149^\circ$

Exercise 14(c)

- Write down the true/false statement on your practice book of the following statements.
 - The tangent of the circle and the radius draw from the point of contact are perpendicular to each other.
 - The secant of a circle intersect that circle more than two points.
 - The tangent of a circle only intersects the circle at two points.
 - The perpendicular drawn from centre to any chord of a circle, bisects that chord.
- Draw a circle of radius 2.5 cm. taking O as a centre. Take a point P on this circle. Draw the radius OP. Draw a normal (perpendicular) PT at the point P of line segment OP. Is PT a tangent of the circle ?

- In the adjacent figure O is centre of the circle.

Fill in the blanks in following by figure on your exercise-book :

- Line-segment AB is the of the circle.
- Line segment OR is the of the circle.
- Line CD is the of the circle.
- Line PQ is the of the circle.

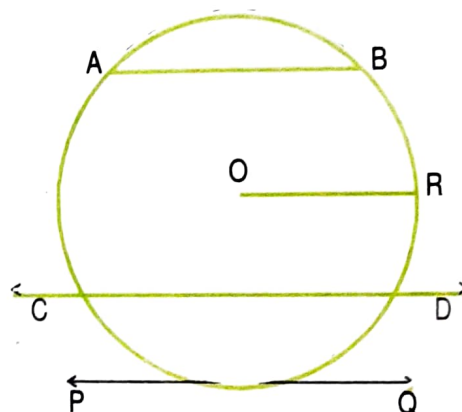


Fig. 14.17

- Draw a circle of radius 2.5 with centre O. Take two points A and B on this circle so that $\angle AOB = 60^\circ$. Draw the tangents AP and BP from the point A and B respectively. Write by measuring $\angle APB$.

- In the adjacent figure PQ is a diameter of a circle and PR and QS are tangents of the circle at P and Q respectively. Is $PR \parallel QS$? Give reason in favour of your answer.

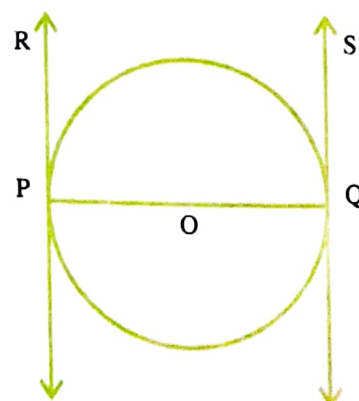


Fig. 14.18

6. In the adjacent figure there is two concentric circles with centre O. (Both circle have same centre O). A chord AB of the bigger circle is a tangent of the smaller circle at P. Is it true to say that AB is bisected at point P ? Answer with reason.

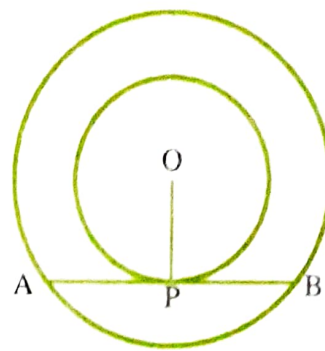


Fig. 14.19

Efficiency Practice 14

Write down the true (right) option in question 01.

1. In the adjacent figure O is the centre of circle. Its two radii OP and OQ are such that $\angle POQ = 120^\circ$. Tangents are drawn from the points P and Q of the circle, which intersects each other at P. The value of $\angle PTQ$ is—

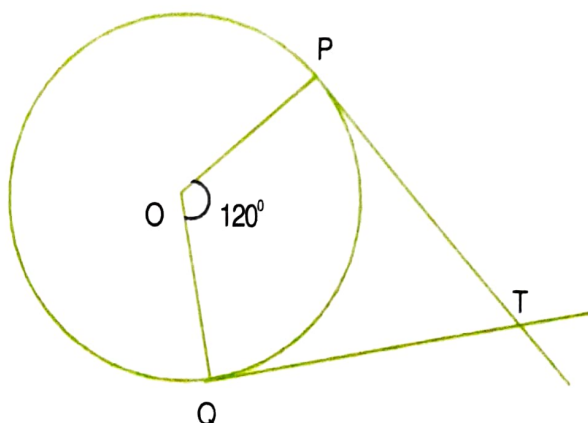


Fig. 14.20

- (i) 60°
- (ii) 120°
- (iii) 90°
- (iv) 100°
2. The length of two parallel chords on one side of the centre of a circle is 6 cm. and 8 cm. If they be 1 cm. apart, then the diameter of the circle will be—
- (i) 14 cm
- (ii) 10 cm
- (iii) 8 cm
- (iv) 5 cm
- (N.T.S. 2006)

We have Learnt

1. The secant of a circle is the line that intersects that circle at two points.
2. The tangent of a circle is that line which intersects that circle to only one point.

3. The point on a circle through which the tangent of the circle passes is called the point of contact of the tangent.
4. If p be the distance of a line from the centre O of a circle and r be the radius, then
 - (i) If $p > r$, then the line does not intersect the circle.
 - (ii) If $p = r$, then the line be the tangent of the circle.
 - (iii) If $p < r$, then the line be the intersecting line of circle.
5. The tangent and radius drawn through the point of contact are perpendicular to each other.

ANSWERS

Exercise 14(a)

1. (i) AC and FC (ii) HG and GF, (iii) BOA, (iv) Point of Contact A or H, (vi) HE or DE. 2. (i) True (ii) False (iii) False (iv) False (v) True (vi) False, 3. Student explain by construction, 4. Not possible

Exercise 14(b)

1. 90° , 4 $\angle APB = 55^\circ$

Exercise 14(c)

1. (i) True, (ii) False, (iii) False (iv) True.

2. Tangent line, 3. (i) Chord (ii) Radius (iii) Intersecting line, (iv) Tangent line, 4. 120° , 5. Parallel 6. Yes.

Efficiency Practice 14

1. (i) 60° (ii) 10 cm.

