



- Median and Mode of unclassified data
- Frequency table and Histogram of Classified data

15.1 Introduction

In previous classes, we have read and we get different types of information from various sources like newspaper and magazines, advertisements through different means of information system such as Radio, Doordarshan (television) etc. These informations are related to each sector of life. The facts which are numerically collected from these informations are called data (figures). Each numerical fact is also called as observation. The data collected in the original form is cluttering (unordered). You have also read that to make these figures (datas) understandable or to achieve any objective conclusions from them, they have to be organized. For this the data have to be kept in ascending or descending order, you have also noticed that by graphical representations of data the objective conclusion can be easily obtained and understood. So far you have understood the notion of pictograph, bargraph, mixed (composite) bar graph, piegraph in the graphical representation of data and also have learned its representation and to get conclusion. Also you have learned to make frequency table of unclassified data and to get its mean. Now in this unit we will learn to find median and mode of an unclassified data, the graphical representation of classified data in the form of histogram and to get conclusion.

15.2 Measure of Central Tendency

The given figures contain a number which represents all the figures (data). This number is usually the number in the middle of the group or around that number. All figures are distributed around this number. Such tendency of figures (data) is called central tendency and that number (mean) around which all data are distributed is called measure of central tendency.

We know that measures of central tendency are of three kind. Mean, median and mode. We have read about mean in class-7. Here we will study about median and mode.

15.2.1 Median

Let the weights of 5 students of any class be 48, 54, 56 and 51 kg respectively. If weight of all these students are arranged in ascending order then serial of their weights will be 42, 48, 51, 54 and 56 kg. Similarly if they arranged in the descending order of their weights then the serial of their wts will be 56, 54, 51, 58 and 42 kg, in both situation student of weights 51 kg will stands in the middle of the row. In the case of ascending order of weight the weights of two previous students will be less than that of middle student, and the weights of two later will be more than that. Similarly when the student will stand in the descending order of the weight, the weights of two students previous to middle student will be more than that and weight of two later students will be less than that of middle one. In the given observations median is that value which exactly divides the observations in two parts :

Attention

If the data is arranged in ascending or descending order, then the value of middle term is median.

It also makes it clear that median divides the given data into two groups in which the value of each term of first group is less than that of median and value of each term of second group is more than the median, when the data are arranged in ascending order.

Think, Understand and Tell

If the data has been arranged in descending order, then whether the value of the terms previous to median will be higher than median or less ?

To Find Median

(a) When the data are unclassified

If the data are arranged in ascending or descending order and the number of data is odd, then the value of middle term is median and if number of data is even the average value of two middle term is a median.

After putting the data in ascending or descending order if the number (n) of data is odd then :

Median = the value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

if the number (n) of the data is even then

Median = $\frac{1}{2} \left[\text{value of } \left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \text{value of } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term} \right]$

Example 1. The data of heights of seven students in cm is as follows :

148, 151, 153, 145, 143, 152, 155. Find its median

Solution : The ascending order of data is

143, 145, 148, 151, 152, 153, 155.

Here number of data are 7 i.e., $n = 7$ (Odd)

Median = The value of $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

$$= \text{The value of term } \frac{7+1}{2}^{\text{th}} \text{ term}$$

$$= 4^{\text{th}} \text{ term}$$

$$= 151$$

Thus, median = 151 cm

Example 2. The score obtained in 10 matches by the school's basket ball team is as -10, 12, 8, 9, 11, 19, 13, 10, 20, 22. Find the median of match's score.

Solution : On arranging the match score in ascending order :

Ascending order is 8, 9, 10, 10, 11, 12, 13, 19, 20, 22

Here $n = 10$, the number of terms is even.

$$\text{Therefore, median} = \frac{\text{value of } \left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \text{value of } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

$$= \frac{\text{value of 5th term} + \text{value of 6th term}}{2}$$

$$= \frac{11 + 12}{2} = \frac{23}{2} = 11.5$$

$$\text{thus, median} = 11.5$$

(b) When data are unclassified but tabulated

In this type of data, the frequency of term is given. In such a situation, we first know the cumulative frequency. In order to determine the cumulative frequency of any term, the frequency of that term is added to the frequency of all the previous preceding terms. Note that the value of the cumulative frequency of the last term is equal to the sum of the frequencies of the total terms.

Example 3. Find the median of the following frequency distribution.

Term	2	4	6	8	10	12	14
Frequency	3	2	2	4	3	1	2

Solution : The cumulative frequency table of the data is as follows :

Term	Frequency	Cumulative frequency
2	3	3
4	2	$(3 + 2) = 5$
6	2	$(5 + 2) = 7$
8	4	$(7 + 4) = 11$
10	3	$(11 + 3) = 14$
12	1	$(14 + 1) = 15$
14	2	$(15 + 2) = 17$
Total	17	

Here $n = 17$ that is number of terms is odd.

$$\begin{aligned}\text{Therefore, Median} &= \left(\frac{n+1}{2} \right)^{\text{th}} \text{ term} \\ &= \left(\frac{17+1}{2} \right)^{\text{th}} = 9^{\text{th}} \text{ term}\end{aligned}$$

9th term will be in that class whose cumulative frequency is 11 and the value of the term having cumulative frequency 11 is 8.

Therefore, value of 9th term = 8

$\therefore$ median = 8

Example 4. The data of age (in years) for 24 students of a class are as follows :

Age (in years)	12	13	14	15	16
Student	4	5	4	6	5

Find the median of the data.

Solution : The cumulative frequency table of the above mentioned frequency distribution is as follows :

Age (in years)	Frequency	Cumulative frequency
12	4	4
13	5	$(4 + 5) = 9$
14	4	$(9 + 4) = 13$
15	6	$(13 + 6) = 19$
16	5	$(19 + 5) = 24$

Here $n = 24$, that is number of terms is even. To know the median of this first

to find $\frac{n}{2}$ th term and $\left(\frac{n+1}{2}\right)$ th term

$$\left(\frac{n}{2}\right)\text{th term} = \left(\frac{24}{2}\right)\text{th term.}$$

$$= 12\text{th term}$$

$$\left(\frac{n+1}{2}\right)\text{th term} = \left(\frac{24+1}{2}\right)\text{th term} = 13\text{th term}$$

$$\therefore \text{Median} = \frac{\text{value of 12th term} + \text{value of 13th term}}{2}$$

$$= \frac{14 + 14}{2} = 14 \text{ years}$$

(since both terms are in that column whose cumulative frequency is 13)

Group Discussion

1. Describe the ascending order of natural numbers 8, 12, 9, 15.
2. What is the descending order of 20, 24, 28, 19 ?
3. Describe the median of data 3, 4, 7, 9, 10, 13, 15.
4. What is the median of data 15, 12, 10, 9, 6 ?
5. The weights of five friends are 40 kg, 43 kg, 44kg, 46 kg, 47 kg, and 50 kg respectively. Find the median of their weights.

Exercise 15(a)

1. The age (in years) of 7 teachers of a school is 25, 57, 32, 23, 42, 30, 47. Find medians of the data.
2. The height in cm of 8 girls of any class is 140, 142, 135, 133, 137, 150, 148 and 138 respectively. Find the median of data.
3. Find the median from following table :

Term	3	4	6	8	12
Frequency	2	5	4	5	3

4. The data of numbers of shoes size of 40 students are given in following table :

Shoes size	4	5	6	7	8
Number of students	6	5	8	15	6

Find the median.

15.2.2 Mode

The age (in years) of 10 students of a class are 13, 14, 13, 15, 12, 13, 15, 16, 13, 13, respectively. We may put these data as following manner.

Age (in years)	12	13	14	15	16
Frequency	1	5	1	2	1

We see that 5 out of 10 students are such that their age is 13 years. So we can say that mode of data is 13 years, since mode means data with highest frequency.

In the given group of data, the highest occurring term is called a mode. That is the term whose frequency is highest is called a mode of that group.

Necessity of Mode

The necessity of knowledge of mode is very much in the trade and industry. For producers of goods on large scale or traders, the importance of mode is very much. In any factory that size of vest will be produced more whose demands will be more. The seller also stores that size of vest and other readymade cloths in his shop, which will have more sales.

Similarly in a shoes shop more shoes will be of that size which is the size of feet of most persons.

Mode can be known by observation only.

Method to find mode (when data are unclassified).

Example 5. Find the mode of following scores of 10 students (out of 15). 9, 10, 12, 10, 13, 12, 10, 12, 10, 9.

Solution : The frequency of 10 is highest. Therefore mode of this is 10.

Example 6. Find the mode of following frequency distribution.

Term	12	16	20	24	28
Frequency	4	10	14	20	6

Here it is clear that frequency of 24 is highest and is 20. Therefore it's mode is 24.

Remark : Sometimes there are such type of frequency distribution in which terms with highest frequency are more than one. In such situation we find mode by grouping or (syndication) method. The study of this method is not required in this class.

Exercise 15(b)

- Find the mode of 12, 12, 13, 12, 10.
- Find the mode of following data :
 - 13, 14, 10, 12, 11, 12, 13, 20, 18, 12, 10, 12.
 - 19, 25, 36, 38, 20, 18, 38, 3, 38, 22, 38, 38.
- Find the mode of following distribution :

Term	18	22	26	30	34	38
Frequency	3	5	11	3	9	2

15.3 Classification of Data :

We know that when the number of data is very large then for more effective and statistical calculation of numerical content, it is represented in more appropriate form. If we make a frequency table for each observational data, then it will be very long. For convenience, we arrange these data into classes or class intervals. See the following example.

The marks obtained by class 8th student in mathematics subject in paper II, are as follows :

22, 26, 15, 7, 10, 32, 40, 40, 25, 28, 16, 15, 35, 25, 25, 16, 20, 42, 45, 48, 10, 8, 26, 8, 1.

Method of Constructing Class

1. At first, we find the largest and smallest numbers from the data, and then find their difference. This difference is called range. From above example, the largest number is 48, and the smallest number is 1. Hence the range of data is $48 - 1 = 47$.
2. Now it is clear that range 47 is to be divided into many classes. For above data, construct the classes 0-10, 10-20, 20-30, 30-40, 40-50. These are known as class-intervals. To determine each class, there are two numbers. For example in the class interval 10-20 the lower limit is 10 and upper limit is 20. Similarly in 20-30, 30-40 etc. The first numbers are respectively 20 and 30 which are lower limits and the second number are 30 and 40 respectively which are upper limits. To make classes, we must careful that these should not overlap. For example, 0-10, 10-20, 20-30 etc. classes have been made. This cannot be as 0-10, 8-18, 20-24 etc.

We see that the upper limit of first class-interval is the lower limit of the next coming class-interval. For example for 10-20, 20-30 class, the upper limit of 10-20 is 20, which is the lower limit of 20-30.

The difference of upper limit and lower limit is called the class-interval or class-difference or class measure. For example, the class difference of 10-20 is $20 - 10 = 10$.

3. After determining class, we make the tally mark to find the frequency. It is notable that in class 0-10, we take 0 or more than 0 and less than 10. But upper limit 10 is not included but it is taken in next interval 10-20.

Now make four columns as :

Serial No. (1)	Class (2)	Tally Mark (3)	Frequency (4)
1	0-10	IIII	4
2	10-20	IIII I	6
3	20-30	IIII III	8
4	30-40	II	2
5	40-50	IIII	5

Table-1

- Informations in numerical form are called an observation
- When the observed information are very high then data are classified into classes these classes are called class-interval.

Attention

1. Classes donot overlap each other.
2. There should be no gap (vacancy) in between the classes.
3. The size of classes should be same.
4. Each data must fall in some class.
5. The number of classes in any table should be at least 5 and not more than 15.

Example 7. Answer the questions to see the table given below :

Serial No. (1)	Class (2)	Tally Mark (3)	Frequency (4)
1	5-10	I	11
2	10-15		4
3	15-20	II	12
4	20-25	III	18
5	25-30	IIII	14
6	30-35		3
7	35-40		10

- I. How many classes are there in this table ?
- II. What is the range of data in column-2 ?
- III. What is the lower limit and upper limit in class 25-30 ?
- IV. What is the class interval in the class 30-35 ?
- V. How many tallies are there in class 35-40 ?
- VI. Which class has frequency 10 ?
- VII. The number 15 will be put in which class 10-15, 15-20 ?

- VIII. How many tally marks are in table ?
- IX. How many total number of frequency are there in the table ?
- X. Which class has the highest frequency ?

Solution :

- I. There are 7 class total 5-10, 10-15.... 35-40 in the table.
- II. Range of the data is $40-5 = 35$.
- III. In the class 25-30, lower limit is 25 and higher limit 30.
- IV. The class interval in the class 30-35 - $35-30 = 5$
- V. In the class 35-40 there are 10 tallies.
- VI. The frequency of 35-40 is 10.
- VII. The number 15 will be added in class 15-20.
- VIII. There are 72 tally marks in the table (in total).
- IX. In table total frequencies is 72.
- X. The class 20-25 has highest frequency.

Exercise 15 (c)

1. Make classes with class range (interval) 5 in the following numbers :
 - (i) 5, 11, 26, 24, 21, 10, 9, 8, 7, 11, 25, 21, 17, 14, 16, 11, 13, 17
 - (ii) 22, 36, 42, 37, 40, 19, 23, 27, 20, 36, 40, 25, 24, 36, 23, 20.
2. The score in mathematics of 22 students in a class are as follows. Distributing the data with class interval 10, draw the frequency table.
67, 52, 54, 66, 88, 82, 67, 54, 50, 50, 66, 67, 50, 50, 48, 55, 56, 67, 88, 67, 78, 83.

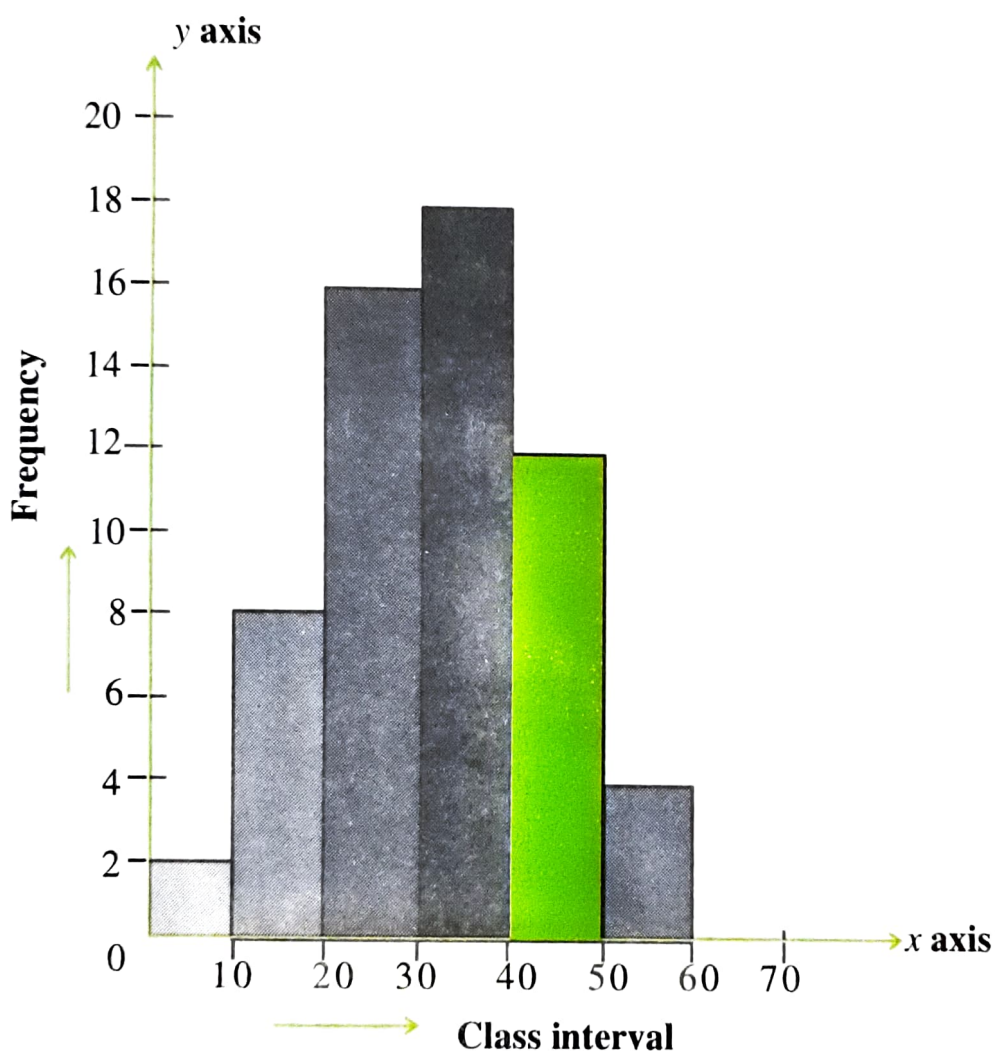
15.4 Histogram

So far we have been familiar with making graph with unclassified data by the means of bar graph. By this method we have represent the data by bar. But if the data are classified and frequency distribution is given, then it may also be represented by graphs. Such type of representation is known as histogram.

In this type of representation rectangles are drawn taking base as class intervals on x-axis. The breadth of rectangles are proportional to class difference. The height of rectangles are proportional to the related class frequency.

Let us consider the classified frequency distribution of marks of 60 students in science out of 60

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	2	8	16	18	12	4	60



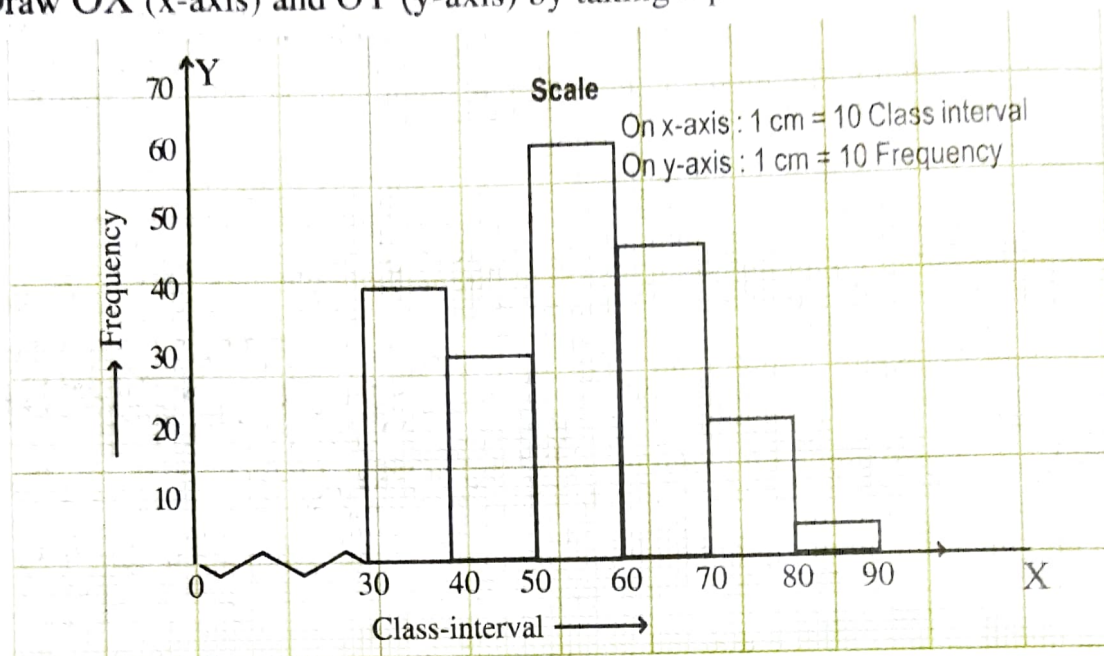
In this diagram the class intervals have been represented in horizontal axis. The length of bar shows the frequency of class-intervals and there is no gap between the bars because there is no gap in between the class interval.

Such type of graphical representation of data is called histogram.

Example 8. Draw the histogram on the basis of following frequency distribution :

Class interval	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	40	30	60	45	20	5

1. Draw OX (x-axis) and OY (y-axis) by taking a point O on graph paper

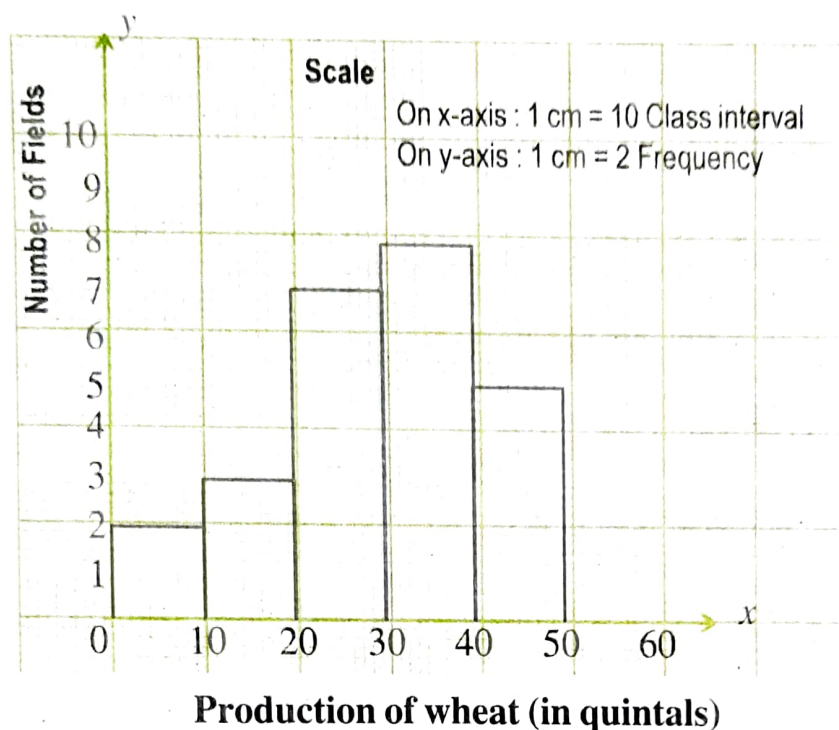


(The zig-zag line on x-axis is used to show that we are not demonstrating the numbers from 0 to 30.)

2. On x-axis taking an appropriate scale 1 cm (10 small boxes) = 10 class interval represent all the classes 30-40, 40-50....80-90.
3. Taking an appropriate scale 1 cm = 10 frequency on x-axis. Show all the frequencies 5,10, 30,30,....60.
4. On x-axis first class is 30-40 whose class interval is 10 make it one side of the rectangle. Frequency of this class on y-axis is 40, make it the height of the rectangle as a second side. Now draw a rectangle on the class 30-40.
5. Similarly draw a rectangle taking one side as a distance between the class 40-50 and second side of rectangle as its frequency 30. The rectangle of class 30-40 and 40-50 will be adjacent to each other.
6. Similarly draw rectangles for all other classes. This will be the required Histogram.

Example 9. In a village, the production of wheat per hectare (in quintals) in some fields and the number of fields is given. Answer the question to see the histogram.

Production of Wheat per hectare (in quintals)	0-10	10-20	20-30	30-40	40-50
Number of fields	2	3	7	8	5



- What is the class-interval of the classes in the table ?
- How many number of fields with production less than 10 quintals are there in the Histogram ?
- In how many fields the production of wheat is more than 19 and less than 30 quintals ?
- Which is the class of least production ?
- Which is the highest production class ?

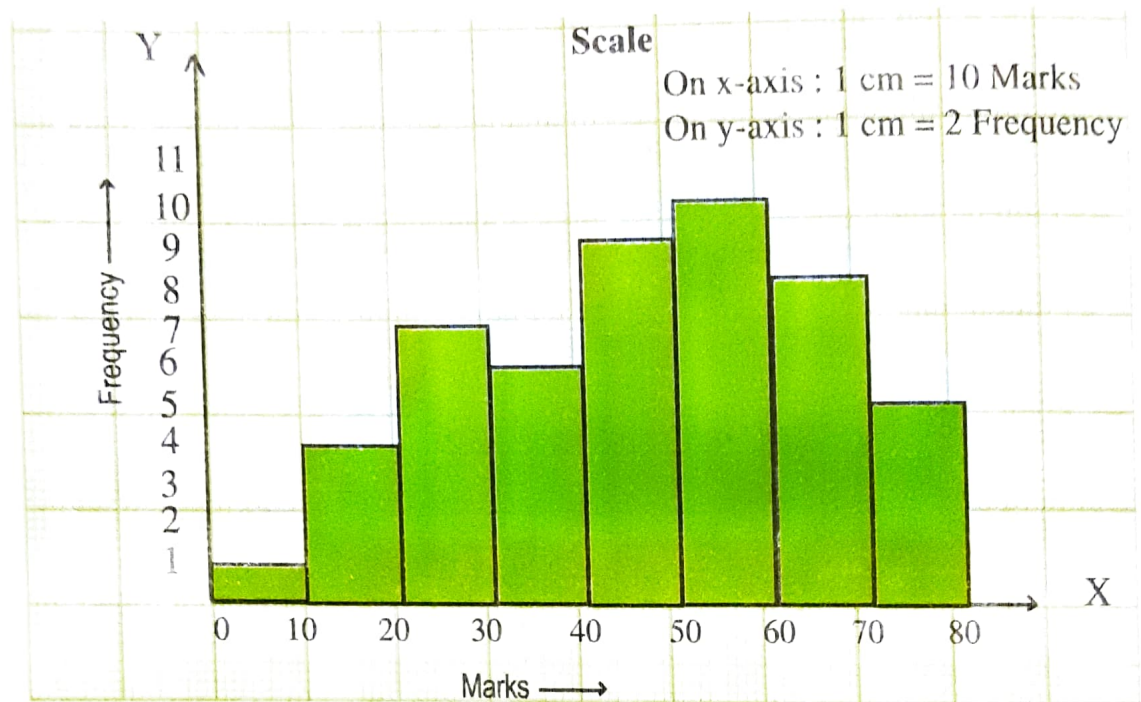
Solution:

- In table classes are 0-10, 10-20.....etc. therefore class interval is 10.
- In 2 fields wheat production is less than 10 quintals.
- In 7 fields, the production of wheat is more than 19 and less than 30 quintals.
- Class 0-10 is the class of lowest production
- Class 30-40 is the class of highest production

Example 10. Score earned by 50 students of any class have been displayed by given histogram.

Answer the following questions by looking the histogram.

- (i) What is the class extension (range) ?
- (ii) How many students have earned the score less than 10 ?
- (iii) How many students have earned the score more than 40 and less than 50 ?
- (iv) Which class interval has highest frequency and how many students are in that class ?
- (v) If 40 marks is essential to pass then how many students fail ?



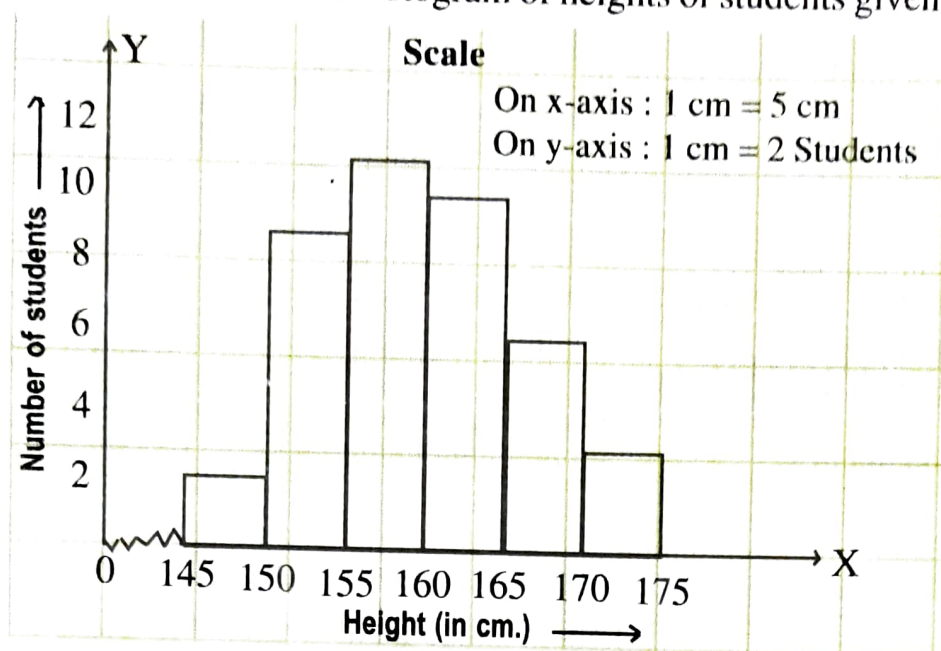
Solution

- (i) In the histogram, the class interval (range) is 10 in class intervals 0-10, 10-20, etc.
- (ii) The students having score less than 10 are in the class intervals 0-10 whose height 1 is equal to its frequency. Therefore 1 student have earned score less than 10.
- (iii) The number of students having score more than or equal to 40 and less than 50 is 9.
- (iv) The frequency of class-interval 50-60 is highest. It has 10 students.
- (v) The students having score less than 40 come in the classes of class interval 0-10, 10-20, 20-30 and 30-40. Therefore failed students are $1 + 4 + 7 + 6 = 18$.

Histogram is the graphical representation of classified data. These rectangles are drawn by making base as class-interval and height as their corresponding frequency.

Exercise 15(d)

1. Answer the questions to see the histogram of heights of students given below.



- Which height group has maximum students ?
 - What is the number of students of height 160-165 cm ?
 - How many students are there with height less than 160 cm ?
 - In which class interval there are least students ?
 - How many students are there with height 155 cm or more ?
2. The scores of 60 students are given in the following frequency distribution table :
Express these data by histogram.

Score	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Students	2	4	5	8	12	15	9	5

3. The number and their age of workers working in a small industry is given in the following given table.

Age (in year)	25-30	30-35	35-40	40-45	45-50	50-55
No. of workers	40	60	80	70	60	50

Represent these data by histogram.

4. Explain by histogram the frequency distribution given below :

Measurement	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	8	15	20	16	6

Efficiency Practice 15

1. Find the median of the following data :

(i) 23,20,22,19,17,22,14,16,15

(ii) 2,4,6,8,10,12,14,16,18

(iii) 1,2,3,4,5,6,7,8,20,10

2. Find the median from the following table

Term	4	7	10	12
Frequency	5	8	10	1

3. Age of 40 students (in years) is given in the following table. Find the median

Age (in years)	10	12	14	16	18
No. of Students	4	8	12	6	10

4. Find the mode of the following data :

(i) 1,3,5,5,,5,4,6

(ii) 2,4,6,3,4,3,4,4,7,2

(iii) 5,7,9,8,7,9,7,7,3,5,2

5. Following is the time (in second) to wash their hand by the students of a school in a competition organised on world handwash day :

20,25,20,17,15,13,17,19,20,21,25,20,15,23.

Find the mode of the data.

6. The score of 60 students are shown in a histogram given below :

Answer the following questions by looking the histogram :

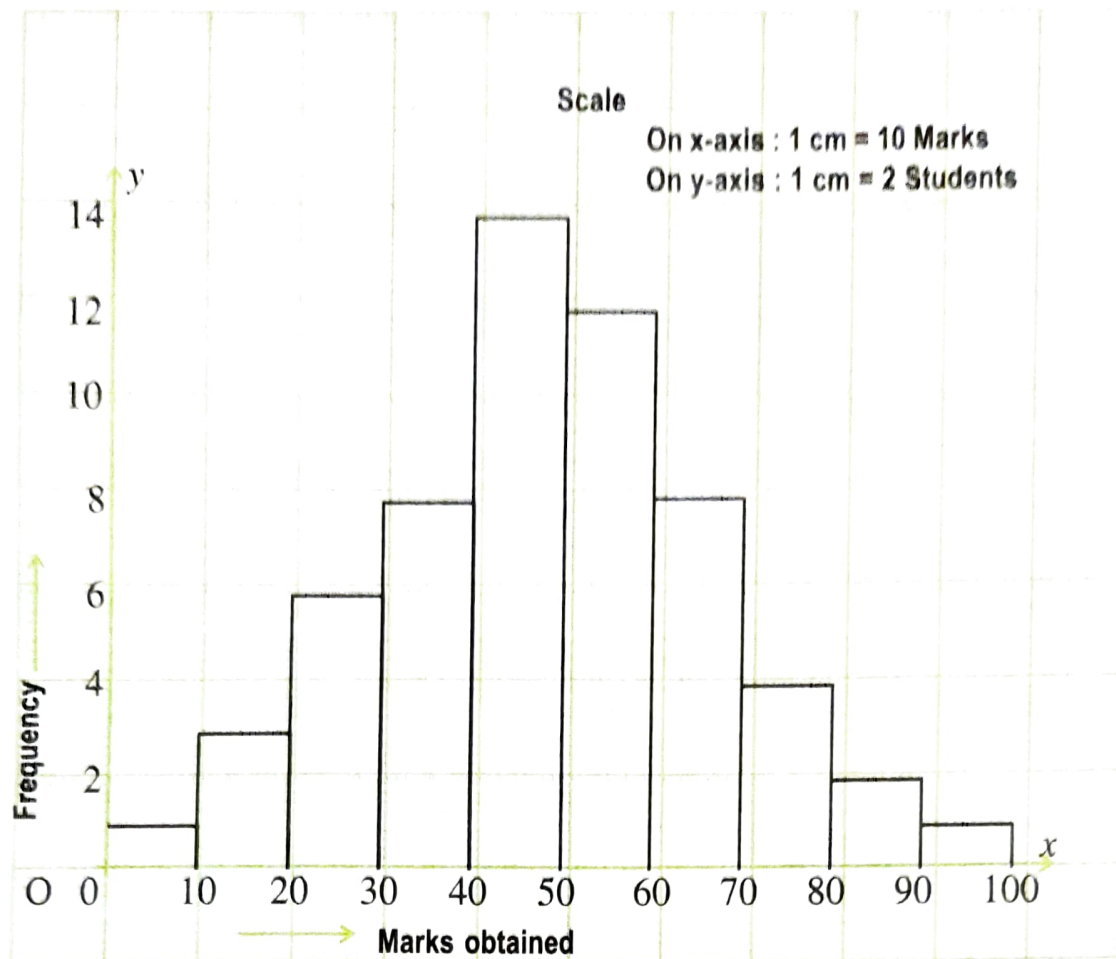
(i) What is the class range ?

(ii) How many students have obtained the score below 40 ?

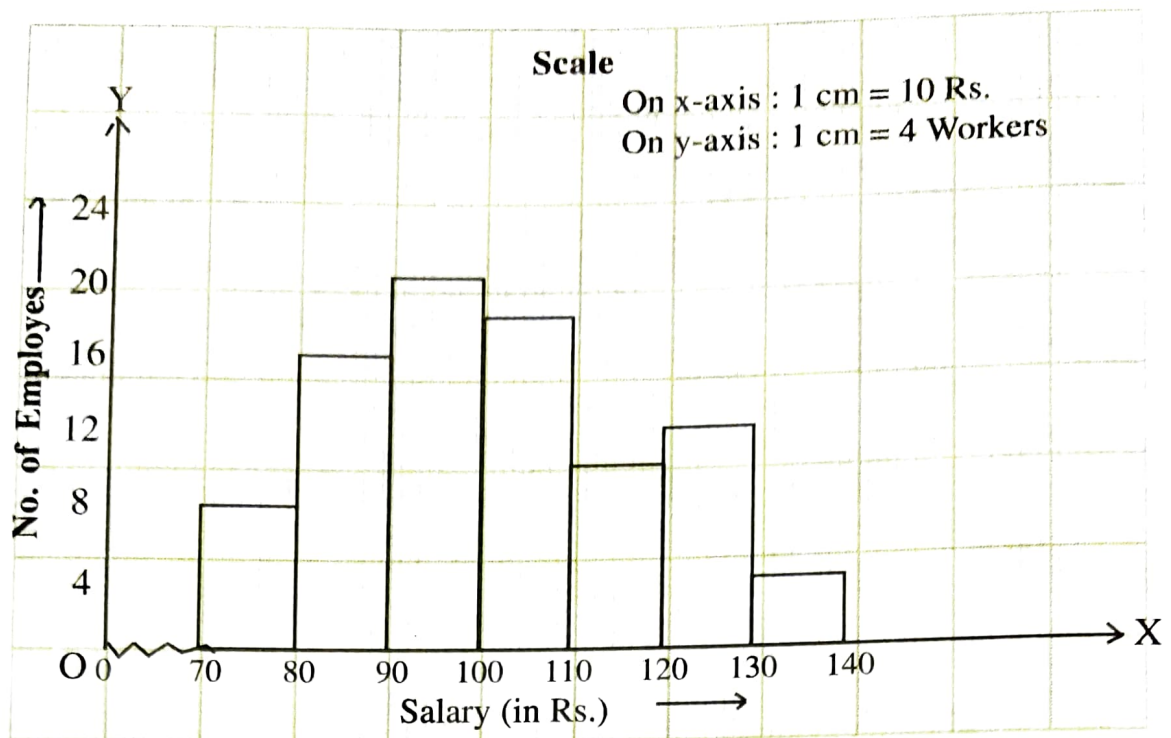
(iii) How many students have score between 50-60 ?

(iv) How many students are there with score more than 50 ?

(v) How many students are there with score more than 10 and less than 30 ?



7. 30 students obtained the following marks in mathematics of a class are 25,22,28,30,35,25,20,42,45,48,41,42,25,23,35,36,37,38,30,21,25,23,28,25,25,24,23,48,29,30,5. Tabulate the frequency of data with class interval 5. Start first class interval with 20.
8. The histogram of daily wages of employees working in a factory is given. Answer the following question on the basis of histogram.
- How many number of employees are there having wages between ₹ 70-80 ?
 - How many employees are there having wages more than ₹ 90 and less than ₹ 120 ?
 - How many employees get daily wages less than ₹ 100 ?
 - How many employees get daily wages more than or equal to ₹ 80 ?
 - How many employees are there with highest daily wages ?



9. On surveying 50 families the following data were obtained for each family having living room. Find the mode

No of rooms	1	2	3	4
No. of families	12	24	8	6

10. Find the median from the following table

Score	33	34	35	36	37
Frequency	2	3	4	5	1

11. Find the mode of following frequency distribution :

Term	21	22	23	24	25
Frequency	2	3	5	1	2

12. The median of first five prime numbers is

(a) 6.2 (b) 5.6 (c) 5.0 (d) 3.0

We have learnt

1. Measure of central tendency is one which represents all the data. This number is usually the number in the middle or around the given data. All the data are distributed around this number.
2. Measures of central tendency are of three type, that is mean, median, mode.
3.
 - (i) If unclassified data are arranged in ascending or descending order then the value of middle term is median.
 - (ii) When the data are in odd number then median is $\frac{n+1}{2}$ the term where n is the number of terms.
 - (iii) When the data are in even number then

$$\text{median} = \frac{\text{value of } \left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \text{value of } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

where n is the number of terms

4. To find the median of unclassified but tabulated data first the cumulative frequency of term is known. In order to the cumulative frequency of any term the frequency of that term is added to the frequency of all previous terms :
5. The terms occurring highest times in the given data is called the mode. That is the term with highest frequency is mode.
6. To present the data in a more appropriate form, and for more effective and statistical calculation, these data are arranged in putting in groups or classes. Classes are also known as class interval.
7. Histogram is the graphical representation of classified data in which class interval are taken on horizontal axis and frequencies on vertical axis. These rectangles are made by taking class interval as a base and corresponding frequency as height.

ANSWERS

Exercise 15 (a)

1. 32. years, 2. 139 cm., 3. 6, 4. 7 number

Exercise 15 (b)

1. 12, 2. (i) 12, (ii) 38, 3. 26

Exercise 15 (c)

1. (i) 0-5, 5-10, 10-15, 15-20, 20-25, 25-30, (ii) 15-20, 20-25, 25-30, 30-35, 35-40, 40-45

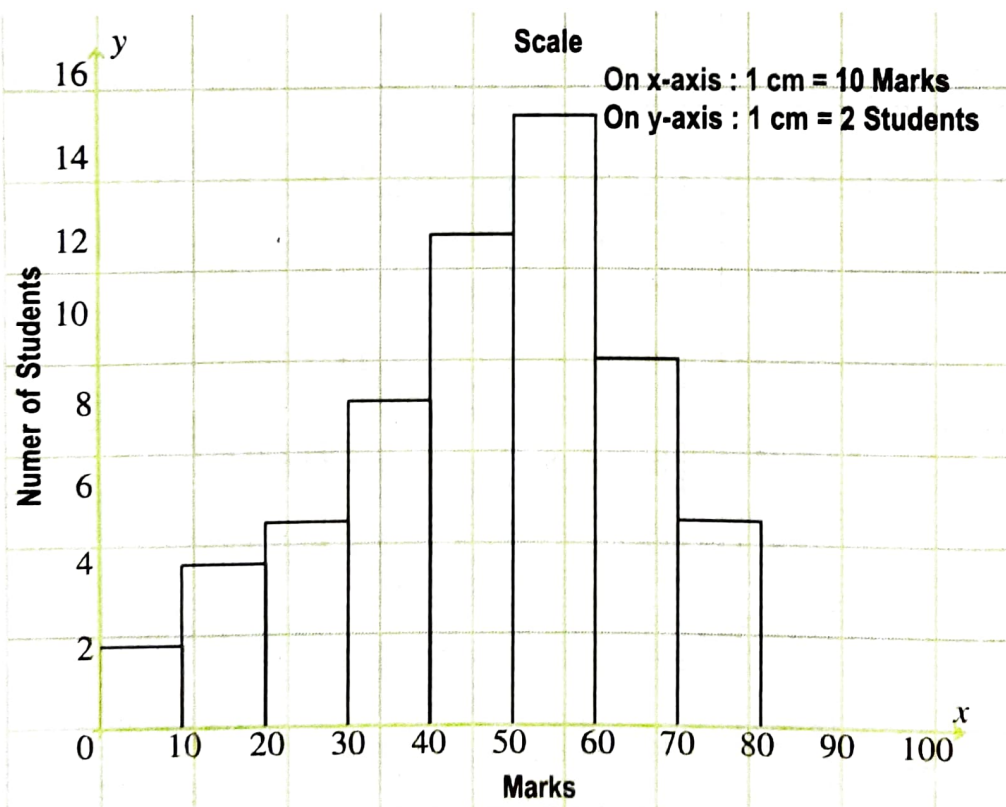
2.

Serial No.	Class	Tally mark	Frequency
1	45-55		8
2	55-65		2
3	65-75		7
4	75-85		3
5	85-95		2

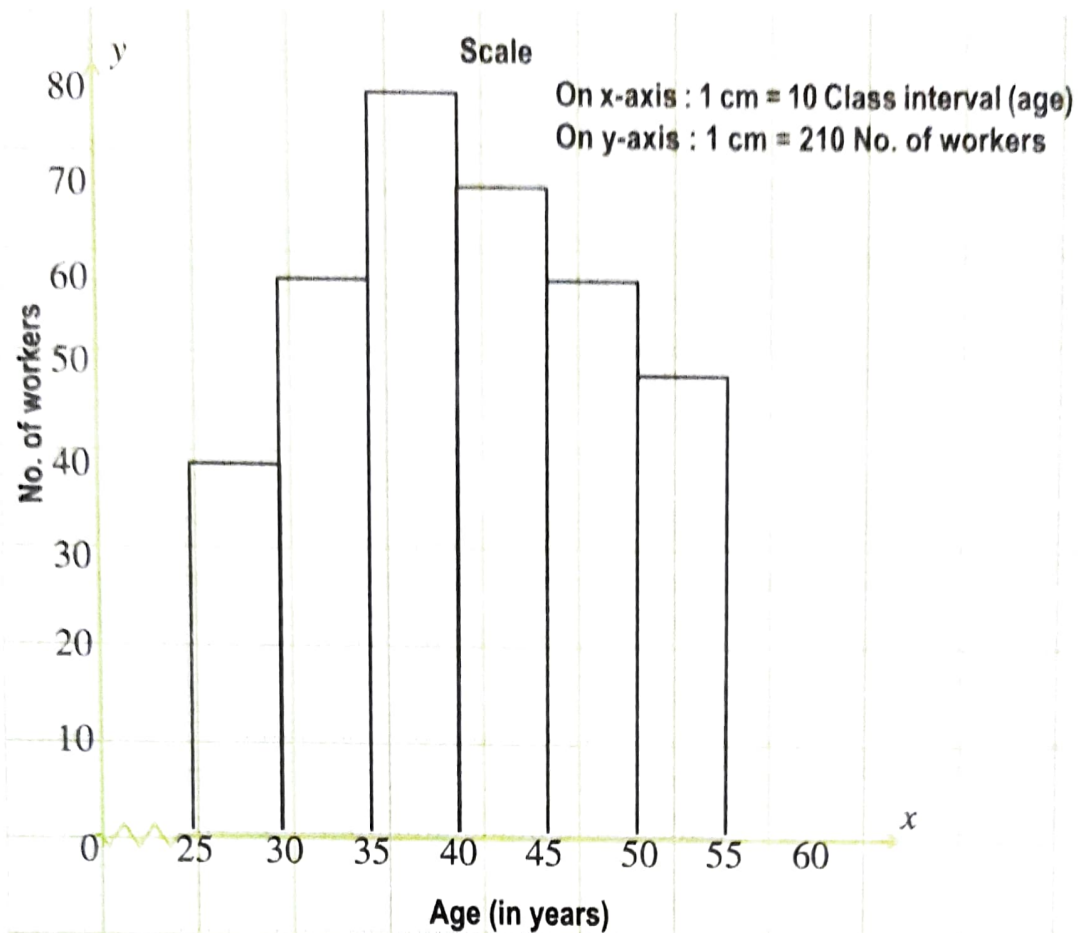
Exercise 15 (d)

1. (i) 155-160, (ii) 10, (iii) 22, (iv) 145-150 cm., (v) 30 $\lceil \underline{n} \rceil$

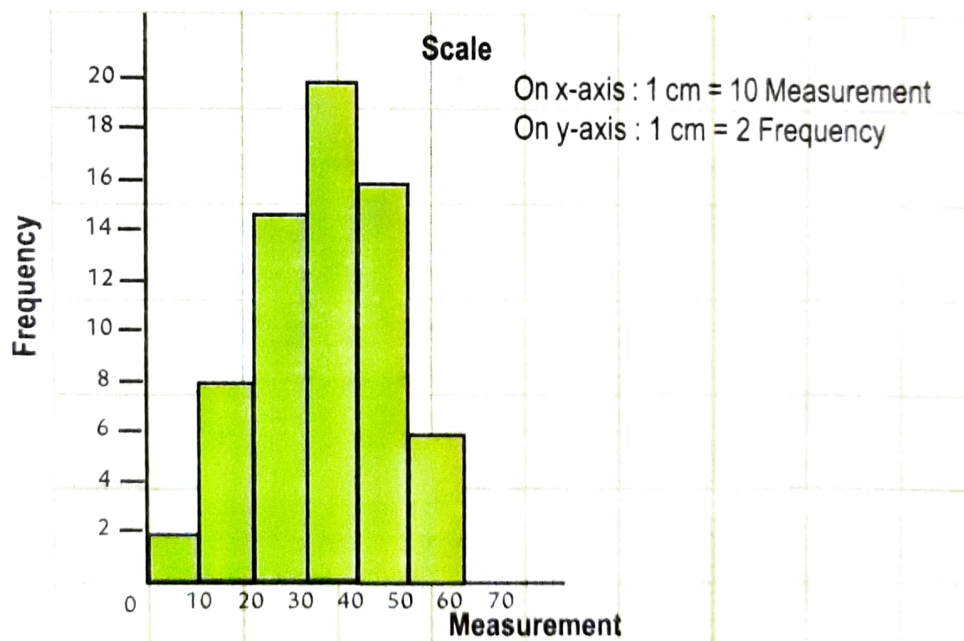
2.



3.



5.



Efficiency Practice 15

1. (i) 19, (ii) 10, (iii) 5.5; 2. 7; 3. 14; 4. (i) 5, (ii) 4, (iii) 7; 5. 20 seconds; 6. (i) 10, (ii) 18, (iii) 12, (iv) 27, (v) 9; 8. (i) 8, (ii) 48, (iii) 44, (iv) 80, (v) 4; 9. 2 rooms; 10. 35; 11. 23; 12. (c) 5.0.

