



- Realization of the possibility of appearance of the head (Chitt) or tail (Patt) on the toss of a coin.
- Probability of appearing any one of the faces of coin tossed.
- Daily life related prospective with probability.

16.1 Introduction

In daily routine people are learned about interacting following dialogues with each other :

1. Today it is cloudy weather; there is humidity in air possibility it may rain.
2. My son has done good in written questions paper of practical be may successes.
3. This year may have some good event in our family.
4. Rabi crops in India are likely to be good this year, this may decrease the rate of fall in dearness.

Statements in above discussion have no surety of the result in the afore said statements, In first statement it is not sure whether it will rain or not. There is a clear situation of uncertainty. It may rain; may not rain. In the same way result of second statement is also ambiguous. It is not sure that my son will pass or not in the examination. He may pass, may not pass. Similarly there is no certainty in rest two statements. They can be true or not as per statement. Generally in such statement for cast of results or guess priority called predictions, are made as per the situation observed as per time. We have predictions of monsoon rainfall usually in May-June month of every year but it occurrence depend upon coincidence. Under statistics branch of Mathematics we study about the data of such statements under the topic "Statistics of Probability". In this chapter we will discuss about this topic.

16.2 Realization of appearance of the head or tail on the toss of a coin.

Do you know a cricket match starts with throwing up a coin which are call “toss of coin”. One who wins in the toss of coin gets right to have batting or fielding first. Before toss of coin one captain select head while tail is selected by other captain. Head is known as chit and tail by pat also. In short head and tail are represented by H. T. respectively.

After selection of Head and tail by both team captains, coin is tossed. Before toss of coin none of the captains is conferred that he will win the toss, guessing toss is a chance, because no one can have control on the result of coin tossed. So result of toss of a coin is always uncertain. Here we can assume (make a concept) that coin is independent in every respect, while tossing there in no other force except its weight, neither is affected by any other force and person tossing the coin is unbiased then possibility of head or tail on the top will be same in the toss. In other words getting a head or tail in the toss are equally likely. Such trail are called random. Head and tail are two result or outcomes of the trail.

Think tossing a coin is used in what activities ?
If two constituency in an Election have get same number of votes then their win or loose is decided by toss of coin.

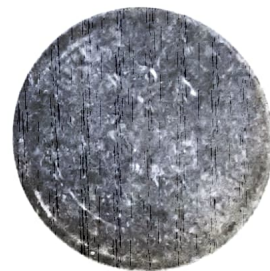
In the adjoining figure Head and tail of coin are shown, In any coin face with Ashoka Pillar image is called Head (Chitt) and its opposite face is called tail (pats).

Point to be note that a coin results a head a tail.
It is rare that coin balances in self on curved part.

Do it your own and write :

Activity 1 :

Toss a coin for 10, 20, 30, 40, 50, 75, 100 times case : note it in a table.



Head



Tail

No. of times coin tossed	Numbers of Head	No. of tail
10		
20		
30		
40		
50		
75		
100		

You can observe that number of head and number of tails are approximately equal, from this table we can conclude as we increase number of toss number head and number of tail approximately be equal.

Number of tosses of coin	Number of Heads	Number of tails
10	7	3
20	11	9
30	14	16
40	18	22
50	23	27
75	35	40
100	48	52

Above table is prepared by real experiments. If is found in 100 toss of coin number of head is 48 and number of tail is 52 students in a class can get repeat this experiment by a same type of coin. In these experiments it is a must that the coin is same in shape and size and of same value.

If we have such (same type of coins) coins then we can presume that each student is tossing same coin student shape form a new table of heads and tail number. If students have different type of coins then number of times head or tail will come can be affected, from such

experiments we can observe that the number of experiments (toss) increase number of heads and tail nearly equal. In such condition results of experiment are equally likely and random.

In any toss of coin appearance of head or tail have equal possibility.

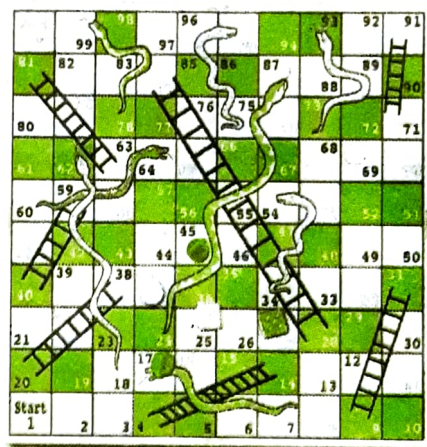
We known that there are many activities in our daily life, which do not change their result if repeated again and again, like what so ever type of triangle a student draw on his note book sum of all its interior angle will be 180° . Such experiments result does not affect by possibility. But result of toss of coin depend on possibility, whether all condition same in every respect.

Another experiment depending on possibility is throwing a die, which will be discussed next. Statistics of possibility study about these experiments. You can see results of random experiments changed if experiment is repeated again and again.

16.3 Possibility of topping up any face of dice in a throw of dice

Have you even played Ludo or snake ladder game or seen ? What items are used to play this game ?

In snake ladder game a square card board piece is pasted by a good quality of glazed paper sheet, which is fragmented into 100 equal squares. These fragments are numbered from 1 to 100 in serial. On this sheet there are ladders started from one square to other square. Similarly these are snakes which leads from one square to the other. To play the game there is a small shaped balanced shaped cube like solid called dice. This cube is homogenous in shape and its faces are numbered from 1 to 6. It is to be noted each face is numbered



by one number only. Sometimes dots are ignored or impression is observed equal to numbers 1 to 6. Atleast two players required to play the game. To play game each player throw the dice on the horizontal square plane one by one. Every player is allotted a slope (Gott) of distinct color (generally red, yellow, green, blue), which are by the player on the top face.

Whatever number observed in the throw of a dice, slopes of the player is forwarded to number of squares equal to number observed. There is a special rule in this game that if six digit is observed in a drown, player will be given one more chance to throw dice. If a player reaches to the bottom square of ladder gets a chance to move directly to the square on the top of

ladder. If a player reaches to the mouth of a snake square then his slope (Goti) moves back to the square at the tail of the snake. In this way game is played among competitive players. The player whose slope (Goti) reaches or crosses the number 100 square first is declared winner.



Try it :

Throw a homogenous dice and observe :

- How many faces of dice can be seen on the top simultaneously
- Throw a dice many times and see whether a particular number appear again and again ? Is there any number which do not appear ?

By throwing and observing a dice for 10, 20, 30, 40, 100, times never two faces can appear simultaneously.

If a face of dice tops up then it stops other faces to be seen on the top. At the same time in a homogenous dice throw, possibility of appearance of each face are equal. Like the throw of coin, outcome of a throw of a dice is also uncertain. It is also not sure that by following particular method of throws of a dice we can get a particular series of number shown on the top. Each time result can change and a particular number can be repeated as well.

But if we increase number of throw of dice and each time number appeared is noted, then it can be observed that number of times number 1, 2, 3, 4, 5, 6 turn up on the top reaches nearly equal.

Try it and write :

Activity 2

For the verification of above said fact throw a dice for 30, 60, 90 times and number observed on the top to be written in the following format :

Table

Number of times disc is thrown	Number of times following digit appeared on the top of dice					
	1	2	3	4	5	6
30						
60						
90						

You can observe it number of throw of a dice is high then number of times numbers 1, 2, 3, 4, 5, 6 appear on the top are equal nearly. In this activity more we increase the number of throws of a dice, in observations, number of times numbers topped up (1, 2, 3, 4, 5, 6) will be equal more accurately and exactly. So similarly to that of throw of coin (toss) throwing of dice can also be experienced as numbers (1, 2, 3, 4, 5, 6) have equal possibility of topping up. Clearly by observation of experiments all six outcomes of a throw of dice are equal.

In this way we conclude.

In a homogenous dice throw numbers 1, 2, 3, 4, 5, 6 shown on the faces have equal possibility of appearance :

A group activity of throwing dice can also be performed. For this divide students of the class into 5 small groups. Instruct to throw dice 30 times by each group. It will be better if only one person in a group throw the dice, and others be observation carefully write the number of times a number topped up the dice, Note the observation in the following table :

Group	Number of dice throw in a group	Group wise cumulative number of Number of times following number appeared					
		1	2	3	4	5	6
1	30						
2	30						
3	30						
4	30						
5	30						
	Total = 150						

In this table above note carefully the cumulative number sum, after observation we will find very small difference in cumulative number of the numbers 1, 2, 3, 4, 5, 6. In other words it can be said that cumulative number sum of these numbers is equal nearly. In the above said activity we should be careful about the above and size of dice given to each group they must be same in all respect so as it gives a feeling that every group is throwing same dice.

Uptill this discussion it is clear to the student that there are only six outcomes possible by throw of a disc as 1, 2, 3, 4, 5, 6 and each of them are equally likely i.e. possibility of each out come is same.

Group discussion

1. How many faces are there in a coin ?
2. Which face of coin is head and which face is tail ?
3. How many face can appear together on the throw of a coin ?
4. A cricket match starts by throwing what ?
5. How many faces of homogeneous dice have ?
6. If a dice is thrown how many faces appear on the top ?

Exercise 16(a)

1. Following table gives the number of heads and tails when a coin is tossed many number of times. Find the gaps of the table in your exercise book.

Number of times the coins is tossed	Number of times (Chit) Head of topup	Number of times (Pat) tail topup
20	12	–
30	–	17
–	22	18
–	32	28
n	m	–

2. A disc is thrown many number of times and the number of times and number appeared is given in the following table. Fill in the blanks of the table in your exercise book.

Number of times disc is thrown	Number of times following numbers on the dice appeared					
	1	2	3	4	5	6
15	2	3	4	1	1	–
30	4	3	5	6	–	4
45	7	8	8	–	5	5

60	8	9	10	11	–	12
90	13	–	17	12	14	18
–	22	18	25	20	16	19

3. A homogenous (or fair) dice is thrown 48 times and possibility of appearance of each face is same. Find how many times numbers 1, 2, 3, 4, 5, 6 each will appear on the top ?
4. A homogenous dice is thrown 54 times and number of times even number occur on the top is 25. Find the number of times odd numbers will occur ?

16.4 Trail of two or three Coins thrown simultaneously.

Now observing the situations of head or tail appearing if two coins are thrown (tossed) simultaneously you will find :

- (1) Both coin has Head on the top
- (2) First coin has head on the top and second coin has tail on the top
- (3) First coin has tail on the top and second coin has head on the top.
- (4) Both coin has tail on the top.

If Head represented (H) and tail by (T) then above four situations can be represented as HH, HT, TH, TT.

Whenever two coins are tossed simultaneously, one of these results will be observed surely. No other result can be observe in any case. It is to be noted that all the four results HH, HT, TH, TT are equally likely. It is also to be noted HT, TH are distinct. Now throw (toss) three coins simultaneously, and carefully observe the results.

First coin can result head or tail.

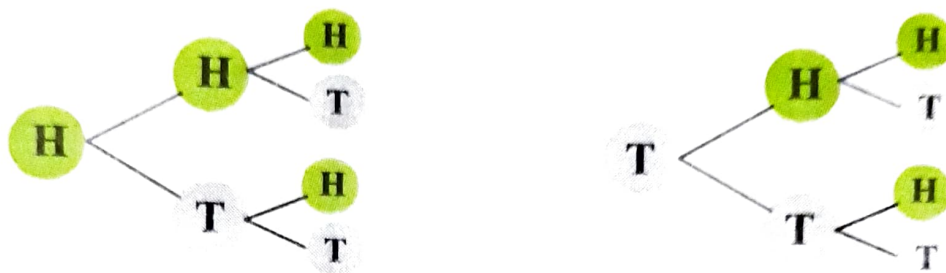
H **T** {HT}

Corresponding the Head in first coin second coin can result head or tail similarly, corresponding to Tail in first coin second coin can result head or tail.



(HH, HT, TH, TT)

Now corresponding outcomes of first and second coin HH, HT, TH, TT, third coin can result head or tail.



In this way there are outcomes if three coins thrown (tossed) simultaneously.

In other words if three coins are tossed simultaneously, possible outcomes are HHH, HHT, HTH, HTT, THT, TTH, TTT.

Here possibility of each outcome HHH, HTH, HHT, THH, is equal.

Try it and write

Activity 3

Toss two coins simultaneously for 10, 20, 30 times and write down the outcomes as per following table format.

Number of times two coins tossed simultaneously	Number of times HH appeared	Number of times HT appeared	Number of times TH appeared	Number of times TT appeared
10				
20				
30				

You can observe as the number of times tossing is increased No. of times HH, HT, TH, TT appearance becomes a equal nearly or possibility of occurring these four outcomes is same nearly.

Activity 4

Throwing three coins simultaneously and result obtained to be noted in the table below :

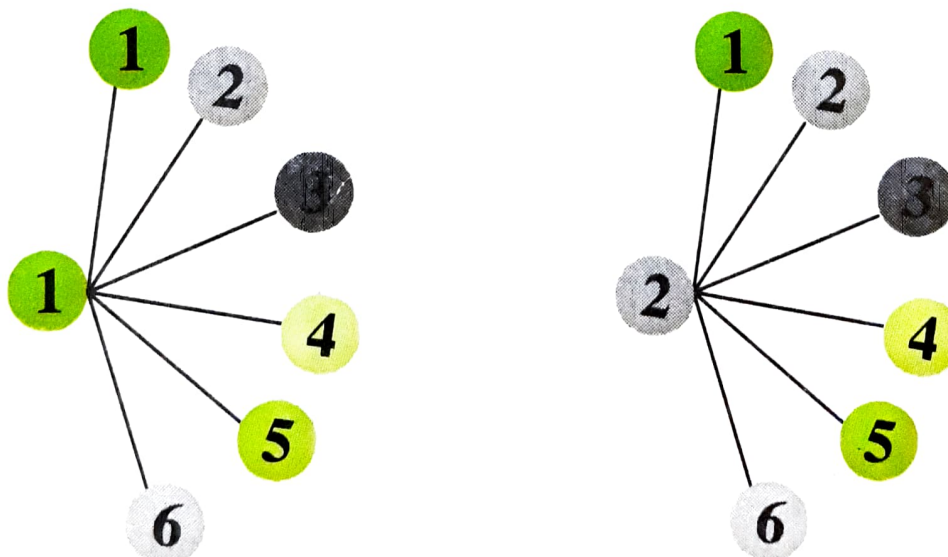
Number of times three coins tossed simultaneously	Number of times result appeared on the top							
	HHH	HHT	HTH	THH	HTT	THT	TTH	TTT
20								
40								
60								
80								

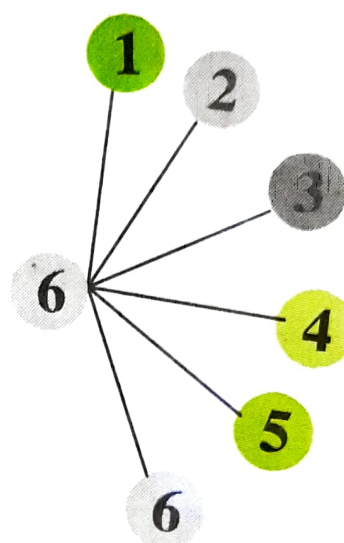
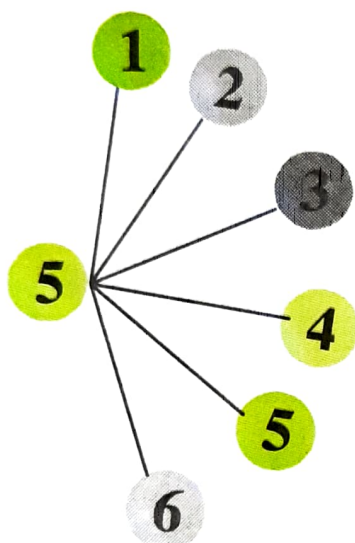
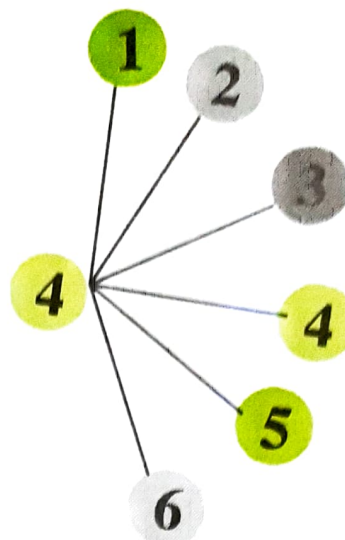
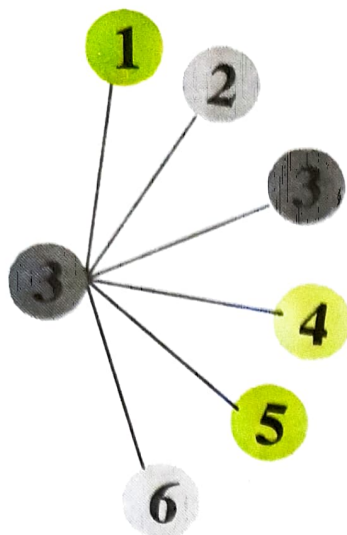
Here also, you can see as the number of times tossing increases than the number of appearance of HHH, HHT, HTH, THH, HTT, THT, TTH, TTT becomes nearly equal. It means possibility of occurrence of all above 8 outcomes is same nearly.

16.5 Experiment of throwing two dice simultaneously

When we throw a homogeneous dice there 1, 2, 3, 4, 5, 6 coming on the top were equally likely. Now throw two dice simultaneously you can see corresponding to any number appeared on the first dice second dice can represent 1, 2, 3, 4, 5, 6 each.

It can be understood by following figure :





In this way you can see there are 36 possible outcomes if two dice are thrown simultaneously. All these outcomes are distinct and possibility of each outcomes is same.

36 outcomes are as follows :

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

be careful (1, 2) and (2, 1) are distinct similarly (5, 6), (6, 5) are different etc.

Try it

By observing above results answer the following questions :

- (i) What will be the minimum or maximum total of the numbers appeared on the top when dice are thrown simultaneously ?
- (ii) How many event are there in which number appeared in both of the dice is same ?
- (iii) In what number of event both dice show only odd number ?
- (iv) In what number of event both dice show only even numbers ?
- (v) In what number of event one dice show even number and other dice show odd number.

Numerical measurement of possibilities :

We have seen if a coin is tossed only two results can be obtained. Similarly if two coins are tossed simultaneously only four results can be obtained. If three coins are tossed simultaneously we can obtain only eight results.

In all above said conditions every result obtained in particular is an event.

- **Example :** If a coin is tossed then getting a head on the top is an event, similarly getting a tail on the top is also an event. Symbolically getting H an event and getting T is another event group of these results of random experiment $\{H, T\}$ is called a sample space which is normally represented by S i.e.

$$S = \{H, T\}.$$

Here both results or event are called elements of sample space.

- Similarly when two coins tossed simultaneously then four possible outcomes HH, HT, TH, TT are four distinct event set of these four outcomes is sample space for the experiment throwing two coins simultaneously. So sample space is as

$$S = \{HH, HT, TH, TT\}$$

- Again when three coins are tossed simultaneously then eight possible outcomes of the random experiment forms sample space is

$$S = \{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$$

in the above random experiment all events are equally likely and if an event occurs it automatically stops others to happen.

For example : If a coin is tossed then either head or tail occurs it is not possible that both head and tail occur on the top of coin. Here we can see in a throw of coin total two events H or T can occur and possibility of each event is same, so it can be said possibility of event. H is $\frac{1}{2}$ and possibility of event T is $\frac{1}{2}$.

This is the numerical measurement of any event to happen.

- Similarly if two coins are tossed simultaneously, then in this random experiment four results HH, HT, TH, TT are found and possibility of happening each is same. So the possibility of happening of any of these results or event is $\frac{1}{4}$.
- Again in a throw of three coins simultaneously. Random experiment has 8 possible results are equally likely so possibility of occurring any of these events is $\frac{1}{8}$, means HHH, HHT, HTH, THH, TTH, THT, HTT, TTT each has possibility.

Conclusion :

- In a throw of a coin probability of each event = $\frac{1}{2}$
- In a throw of two coins simultaneously probability of each event = $\frac{1}{4}$
- In throw of three coins simultaneously probability of each event = $\frac{1}{8}$

- Now think of throwing a dice when a dice is thrown then six equally likely results are possible. Here also results possible is called event and set of all these possible results of this random experiment is called sample space.

$$\text{So } S = \{1, 2, 3, 4, 5, 6\}$$

Here also occurring of each number 1, 2, 3, 4, 5, 6 on the top of dice is equally likely i.e. same, there probability of each event to happen

$$\text{Probability of an event} = \frac{1}{6}$$

$$\text{Probability of an event } P = \frac{\text{Number of results in favour of the event (E)}}{\text{Total possible results (S)}}$$

In this way when two dice thrown simultaneously, then there are 36 events (equally likely) given below.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

All are the members of its sample space and numerical probability of each = $\frac{1}{36}$

Conclusion :

Probability of each event when a dice is thrown = $\frac{1}{6}$

Probability of each event when two dice thrown simultaneously = $\frac{1}{36}$

- Remark :**
- (1) All events discussed above are called simple event
 - (2) To measure the possibility of happening an event is called probability.

16.6 Relation of possibility in day to day life :

Possibilities have deep relationship with our daily life. We do any work to meet and certain goal. For example : you are studying in class 8 for what ? Clearly our every activity in our daily life is related to some goal. Our objective is to meet our goals and be successful in achieving our target, but is it possible always ? Sometimes not. Sometimes we succeed sometimes not. Outcome of each work end with success or failure it is not that we always fail or always succeed. Sometimes success, sometimes failure, sometimes repeated success or sometimes repeated failures happen in our lives. Our whole life is full of uncertainties from morning till night so many events happen in our lives for there we can guess or possibility can be expressed; but it will be true or not, one cannot be sure about it. Today total Economic world depends on possibilities. Statistics of possibility is used in agriculture, commerce, economic world, medicine, biological world weather and other sectors.

Daily hike or fall in per unit price of share, inflation rate, Agricultural and industrial production and distribution etc. all depends on possibilities. Budget of any country depends upon these possibilities. Establishment of any new industry also depends upon possibilities. In any sovereign state results of general elections also based on possibilities.

Results of all type of surveys are based on possibilities. You can do any work but its success is not in your hands. You can be successful or fail. From here starts the world of possibilities. Numerical measurement of success or failure in our daily life activities is being studied under “Statistics of possibility”. Generally it is known as probability.

Suppose there are 50 seats in a competitive exam and 50,000 applicant appear in the exam, it is assumed that possibilities of getting selected is same to each applicant. So numerical measurement of selection for each applicant is $\frac{50}{50,000}$ or $\frac{1}{1000}$. In other words, probability of each student getting selected is $\frac{1}{1000}$.

Example : If a company invites 10 lakh shares and 50 lakh persons applied for it then what will be possibility of getting the share for each applicant.

Solution : Possibility of getting a share $= \frac{10,00,000}{50,00,000}$
 $= \frac{1}{5}$

Statistics of probability is highly used in Insurance companies ; share market, planning etc.

Group discussion

1. What results can be obtained when throw two coins simultaneously.
2. What are total number of events if three coins are tossed simultaneously ?
3. How many doublets are there in sample space in a simultaneous throw of a pair of dices ?
4. What is the total number of elements in the sample space of simultaneous throw of a pair of dice ?
5. Describe any two sectors associated with probabilities.

Exercise 16(b)

- Two coins were tossed simultaneously. If HH; HT, TH occurred 9,8,12 time respectively then find number of times TT occurred.
- A coin tossed for 1000 times and results head is 455 times. What is the percentage of occurring of tail ?
- Two coins are tossed simultaneously for 400 times and it was observed that
two heads 90 times
one head 210 times
No head 100 times

Find percentage of occurrence of each event.

- Frequencies of results as numbers 1,2,3,4,5,6 in 1000 times throw of a dice are given in the table. Find the percentage of occurrence of 1,2,3,4,5 and 6.

Result	1	2	3	4	5	6
Frequency	180	150	160	140	180	190

Find percentage occurrence of numbers 1, 2, 3, 4, 5 or 6 each.

- Two dice are thrown simultaneously and total of the number on the top is taken. Write following events as sets
 - sum is an even number
 - sum is a multiple of 3
 - sum is less than 4
 - sum is more than 10.
- Three coins are tossed simultaneously then write the following events in the form of sets :
 - No head appears
 - Only one head appeared
 - at least two heads appears
 - all three are heads.
- Write set form of the event, getting even number on both dice in simultaneous throw of pair of dices.
- Write set form of the event getting odd numbers on each of the dice in a simultaneous throw of pair of dice.
- To dices are thrown simultaneously write the set of the event where sum of the numbers appeared on the dice is odd.

10. Two dice thrown simultaneously then find the set of the event with sum of the numbers on the dices is not divisible (prime)
11. A lottery has 100 prizes while 100000 tickets were sold. What is the probability of win of an individual buyer of the lottery.

We have learnt:

1. In any experiment, happening or not happening of a event, its possibility and related data is studied under “statistics of probability”.
2. In a throw of any coin either head or tail (put) is shown on the top and possibility of each is same.
3. In a throw of a homogenous dice possibility of topping up of numbers, 1,2,3,4,5,6 is same.
4. In a throw of a coin probability of getting head or tail is equal to $\frac{1}{2}$.
5. In a throw of a homogenous dice probability of getting each number 1,2,3,4,5,6 is same and equal to $\frac{1}{6}$.
6. Probability has deep bonding and effect in our daily life. From morning till night there are numerous events about which we guess or preestimate of their probability to happen or not. But it is not sure that our guess will be true or false.

Special

We have seen that statistics of probability has uncertain results. Its study started with an interesting event. In 1654 there was French gambler named Chevaliar de More was suffering from strange problem. His problem was related to win of a gambling. He wants to know how he starts to play or how to play so as his probability for win gets increased. For his problem he met a great scientist and mathematician friend Blaise pascal (1623-1662). Pascal got interest in Chevaliar and he started study of it. Pascal discussed about the topic to his friend Pierre de Fermat (1601-1665). By serious efforts of these two gentle men mathematics has started a new branch. This branch was later got famous as theory of probabilities.” where we study only mathematical measurement of possibility.

Some names of other mathematicians who worked on this important work :

- Jerome Cardan (1501-1576)
- J.Bernoulli (1654-1705)
- Pierre Laplace (1749-1827)
- A.A. Markov (1856-1922)
- A.M. Kolmogorov (born 1903)



ANSWERS

Exercise 16(a)

1. Rowwise 8, 13, 40, 60; n-m 2. Rowwise 4, 8, 12, 10, 16, 120; 3. 8 times 4. 29

Exercise 16(b)

1. 11; 2. 54.5%; 3. 22.5%, 52.5%, 25% ; 4. 18%, 15%, 16%, 14%, 18%, 19% 5. (i) { (1,1) (1, 3) (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6) } (ii) { (1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6) } (iii) { (1, 1), (1, 2), (2, 1) } (iv) { (5, 6), (6, 5), (6, 6) } 6. (i) { TTT } (ii) { HTT, THT, TTH } (iii) { HHT, HTH, THH, HHH }, (iv) { HHH }; 7. { (2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6) }; 8. { (1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) }; 9. { (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5) } 10. { (1, 1) (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1) (4, 3), (5, 2), (5, 6), (6, 1), (6, 5), } 11. $\frac{1}{1000}$

