



- Area of trapezium.
- Relation between radius and circumference of circle.
- Area of a circular region.
- Volume and total surface area of a right circular cylinder
- Volume and total surface area of right circular cone.

18.1 Introduction

In the previous class we have studied to find the area of region enclosed by a parallelogram, by considering formula of a rectangular region with same base and same height as that of a rectangle. In the same way, for finding a formula for the area of region in the form of a trapezium we use the fact associated with the area of a parallelogram. Generally by experience we realize study of circles is harder than that of a rectangle. In the same way in comparison to a cuboid we face difficulties in the study of more familiar shapes like cone, cylinder etc.

In the study of these solid figure the actual difficulty comes when we talk of the surface area and volume of a cylinder, cone etc. The region is that the area of curved surface is a new concept. We shall see that for the above solid figures an equivalent plane surface area can be found on the base of which the area of curved surface can be obtained.

In this chapter we will develop formula for area of trapezium, relation between circumference and diameter of a circle, area of circular region, total surface area and volume of right circular cylinder and cone or we find a example which are very useful in our daily life because we interact with these shape in every steps of daily life.

18.2 Area of Trapezium field

Observe and Conclude

Observe adjacent figure of Trapezium ABCD

In this Quadrilateral $AB \parallel DC$ and AD and BC are non parallel sides. One of these two parallel side is called base of the Trapezium distance between these two parallel side is called height. Displayed in given figure height $AN = CM = h$.

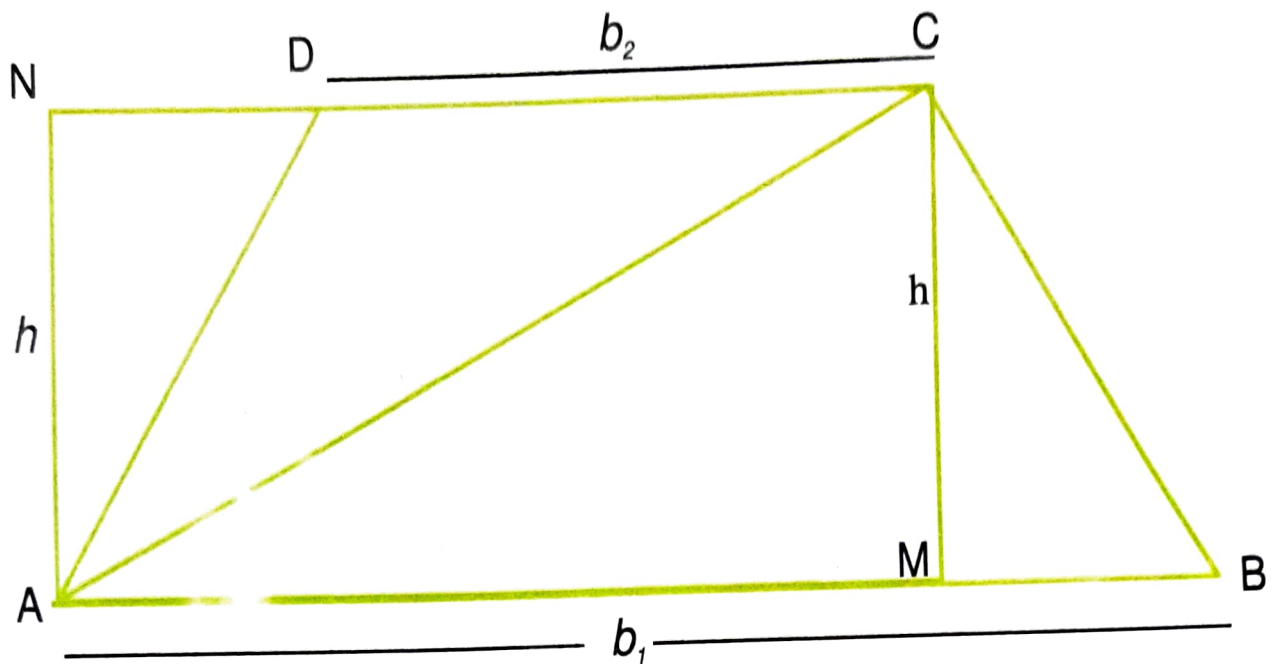


Fig. 18.1

We observe diagonal AC divides trapezium field $ABCD$ in two triangular field ABC and ACD .

Area of trapezium field $ABCD$ = Area of triangular field ABC + Area of triangular field ACD

Let $AB = b_1$ and $DC = b_2$

$$\text{Area of } \triangle ABC = \frac{1}{2} b_1 \times h$$

$$\text{and Area of } \triangle ADC = \frac{1}{2} b_2 \times h$$

$$\begin{aligned} \text{Therefore Area of trapezium field } ABCD &= \frac{1}{2} b_1 \times h + \frac{1}{2} b_2 \times h \\ &= \frac{1}{2} (b_1 + b_2) h \end{aligned}$$

$$\text{Area of trapezium field} = \frac{1}{2} (\text{sum of parallel sides}) \times \text{height}$$

Observe discuss and conclude

Second method

Draw and cut trapezium ABCD on a thick paper and another equal trapezium A'B'C'D'

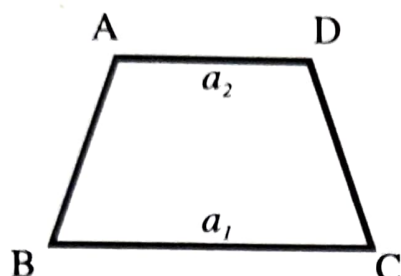


Fig. 18.2

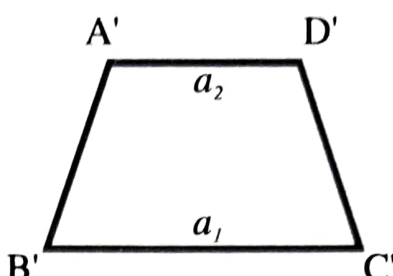


Fig. 18.3

Now put both together shown in given below fig (18.4) such that point C' lies on D and D' lies on C.

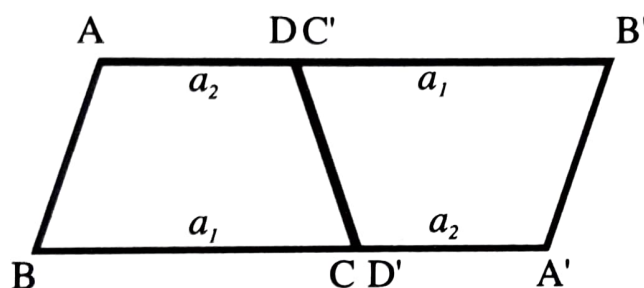


Fig. 18.4

Observe the above fig (18.4) and tell what is its shape ?

Two similar trapezium form a parallelogram

$$\begin{aligned}\text{Area of trapezium field} &= \frac{1}{2} (\text{Area of parallelogram}) \\ &= \frac{1}{2} (\text{base} \times \text{respective height}) \\ &= \frac{1}{2} (a_1 + a_2) \times \text{respective height}\end{aligned}$$

$$\text{Area of trapezium field} = \frac{1}{2} (\text{sum of parallel sides} \times \text{height})$$

Example 1. Parallel sides of a trapezium are 15 m and 8 m, distance between them is 12 m. Find the Area of trapezium.

Solution : Area of trapezium $A = \frac{1}{2}(b_1 + b_2) \times h$

Here, $b_1 = 15$ m, $b_2 = 8$ m and $h = 12$ m

$$\begin{aligned}\text{Therefore } A &= \frac{1}{2}(15 + 8) \times 12 \text{ m}^2 \\ &= 138 \text{ m}^2\end{aligned}$$

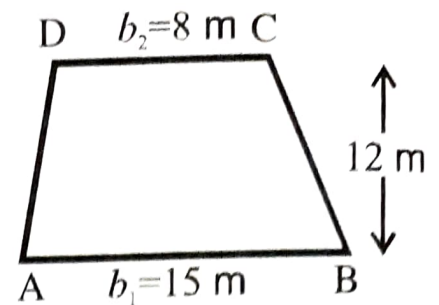


Fig. 18.5

Example 2 : The area of trapezium is 12 cm^2 whose height is 3 cm. If one of its parallel side is 3 cm. What is the length of another parallel side.

Solution : We know that for the area of trapezium

$$\text{Area} = \frac{1}{2}(b_1 + b_2) \times h$$

$$\text{Area} = 12 \text{ cm}^2 \quad h = 3 \text{ cm}$$

$$\begin{aligned}\therefore b_1 + b_2 &= \frac{2 \times \text{Area}}{h} \\ &= \frac{2 \times 12}{3} \text{ cm} = 8 \text{ cm}\end{aligned}$$

$$\text{But } b_1 = 3 \text{ cm}$$

$$b_2 = 8 \text{ cm} - 3 \text{ cm}$$

$$b_2 = 5 \text{ cm}$$

other side of trapezium is 5 cm

Exercise 18(a)

1. The parallel sides of a trapezium are 3 cm and 4 cm. The distance between them is 3 cm. Find the area of trapezium.
2. The area of trapezium is 36 cm^2 and 3 cm high. One of its parallel side is 9 cm. Find the length of other parallel side.

3. In the given trapezium ABCD, $AB \parallel CD$ and $AD \perp AB$, $AB = 8$ cm, $BC = DC = 5$ cm. Find the area.

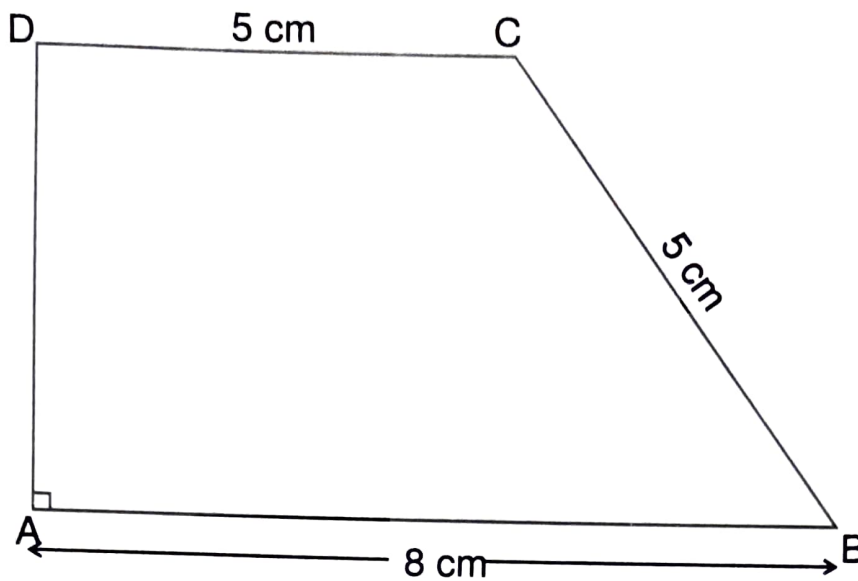


Fig. 18.6

4. The parallel sides of trapezium are 8 m and 6 m, and its height is 4 m. Find the area of trapezium.
5. The area and height of a trapezium are 240 m^2 and 15 m respectively. If one parallel side is twice the other, find their length.

18.3 Relation between diameter and circumference of the Circle

Try it

Observe the adjacent figure and give answer-

- (1) Which point is the centre of the circle ?
- (2) The name of radius of circle
- (3) Tell the diameter of circle.
- (4) Tell the relation between radius and diameter of the circle.
- (5) If r is the radius of a circle, express the diameter in terms of r .
- (6) If the diameter of a circle is D . What is the radius of circle ?

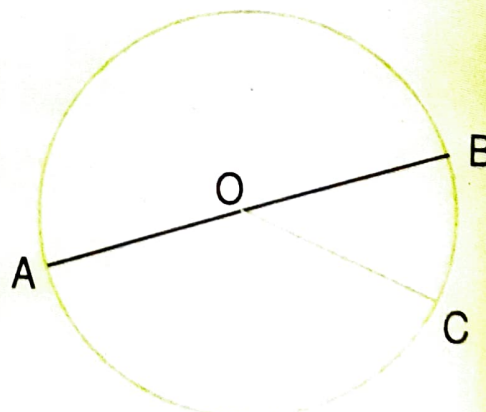


Fig. 18.7

Karim asks Blacksmith Gopi to frame the wheel of his horse carts whose diameter is 112 cm. What is length of iron plate required ?

Gopi can find the length of iron plate before making the frame.

Because there is a definite ratio between diameter and circumference of the circle.

Try these

Procedure

(i) Draw a circle of radius 3.5 cm on a hard paper with the help of compasses and pencil. Cut it with the help of scissor.

(ii) Measure the circumference of circle by a tape. Circumference = 22 cm

Diameter = 7 cm

Hence, the ratio of circumference and diameter is-

$$\frac{\text{circumference}}{\text{diameter}} = \frac{22}{7} = 3.14 \text{ (approximate)}$$

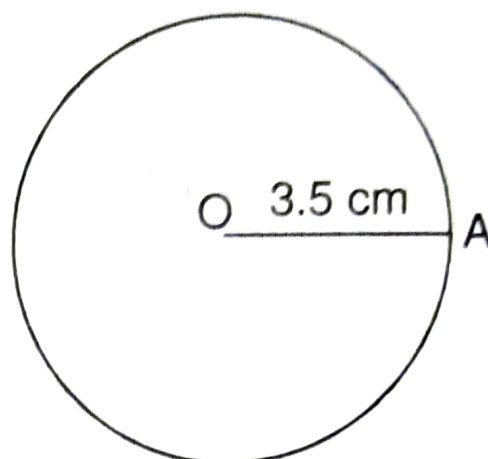


Fig. 18.8

Try these also

Similarly- draw three or more circles of different diameters on hard paper and cut. Measure the circumference and radius of the circles and fill in the table given below.

S.No.	Circumference of circle (measure in cm)	Diameter of circle (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
1			
2			
3			

We observe that ratio of circumference and diameter is approximately $\frac{22}{7}$

Hence, from above experiment we find two conclusions :

- (1) For every circle, the ratio of their circumference and diameters is equal for every measurement.
- (2) The value $\frac{\text{Circumference}}{\text{Diameter}}$ is same for every circle, that is 3.14.

In the Unani language, this ratio is expressed by the sign π . It is an irrational number. But for calculation, its value is taken as rational numbers approximately $\frac{22}{7}$

The value of π correct up to four decimal places 3.1416 which was discovered by Indian mathematician Aryabhata. Value of π can not be found exactly because it is an irrational number.

Note : Mathematicians had earlier known that irrespective of the size of a circle, ratio of diameter and circumference is same for all circles. Even then, it is proved later when a new concept of length of curve had been given. We will study their proof in higher classes. At present we have to accept the fact by means of experiment and not with the proof

So,

$$\frac{\text{Circumference}}{\text{Diameter}} = \pi$$

or $\text{circumference} = \pi \times \text{diameter}$

or $\text{circumference} = 2\pi r$ (r is radius of circle)

π is not a rational number

This fact was proved by German Mathematician Johanne Lambert in 1766.

Early mathematicians used value nearest to π . Early Greeks considered $\frac{22}{7}$ or 3.14 to be the value of π . Archimedees (3 C.E) proved that value of π lied between $3\frac{1}{7}$ and $3\frac{10}{71}$, superior nearest value was given by Indian Mathematician Aryabhata (476CE–550CE). He gave the following rules to calculate the value of π .

Add 4 to hundred, multiply by 8, Add 62000 the result would be divided by 20000 units. Hence, this gave the nearest value of π . That is

$$\frac{\{(100+4) \times 8\} + 62000}{20,000} = \frac{832 + 62000}{20,000}$$

$$= \frac{62832}{20,000} = 3.1416$$

Aryabhatta was the first Mathematician to give the nearest value of π to four decimal places. Now with the help of computer nearest value of π has been calculated correctly up to 5 lakh decimal places. Nearest value of π upto 20 decimal places is as following

3.14159265358979323846

From here the value of π would be considered $\frac{22}{7}$ or 3.14 unless stated otherwise therefore considering π as sign, relationship between radius, circumference and diameter is as follows :

(1) For a circle circumference C and diameter d $C = \pi d \quad \text{and} \quad d = \frac{C}{\pi}$ (2) For a circle circumference C and radius r, $C = 2\pi r \quad \text{and} \quad r = \frac{C}{2\pi}$
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Note : Since we are using nearest value of π in all the numerical problems, hence result, that is the values of C or r is the nearest values only, whether it is mentioned or not.

Example 3. Find the circumference of a circle whose-radius is 21 cm.

Solution :

$$\begin{aligned} \therefore \text{Circumference} &= 2\pi r \\ r &= 21 \text{ cm} \\ \text{Circumference} &= 2\pi \times 21 \text{ cm} \\ &= 2 \times \frac{22}{7} \times 21 \text{ cm} \\ &= 132 \text{ cm.} \end{aligned}$$

Therefore circumference = 132 cm

Example 4. A circular running track has to be constructed such that one round of 110 metre race can be completed. What would be diameter of the track ?

Solution : Circumference of circle = 110 metre

$$\text{diameter} = \frac{\text{Circumference}}{\pi}$$

$$\text{diameter} = \frac{110}{\pi} \text{ metre}$$

$$= \frac{110}{\frac{22}{7}} \text{ metre}$$

$$= \frac{110 \times 7}{22}$$

$$= 35 \text{ metre}$$

Diameter of the track = 35 metre

Example 5. Find the diameter of the circle whose circumference is 15.7 cm.

Solution : We know that $C = \pi D$

Here $C = 15.7 \text{ cm}$

$$D = \frac{C}{\pi} = \frac{15.7}{\frac{22}{7}} \times 7 \text{ cm}$$

$$\frac{109.9}{22} \text{ cm} = 5 \text{ cm. (Approx.)}$$

Example 6. A piece of wire which was initially in the shape of rectangle of length 8.9 cm and breadth 5.4 cm is moulded in the circular form. Find the radius of this new formed circle.

Solution : Length of wire = Perimeter of rectangle
 $= 2 \times (8.9 + 5.4) \text{ cm}$
 $= 28.6 \text{ cm}$

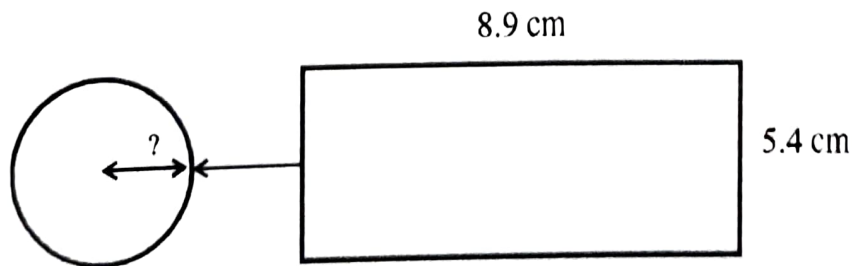


fig 18.9

Now wire is changed in circle

$$C = 2\pi r$$

$$\text{or, } r = \frac{C}{2\pi}$$

$$\text{Here } C = 28.6 \text{ cm}$$

$$\begin{aligned}\text{or, } r &= \frac{28.6 \times 7}{2 \times 22} \\ &= \frac{286 \times 7}{2 \times 22 \times 10} \\ &= 4.55 \text{ cm}\end{aligned}$$

Try it

Complete the following table :

SN	Radius of circle	diameter of circle	Circumference of circle
1	—	7 cm	—
2	1.4 cm	—	—
3	—	—	88 cm
4	—	1.82 cm	—
5	16.1 cm	—	—

Exercise 18 (b)

1. An iron wire is used to make 8 rings of same radius. If the radius of one ring is 22.75 cm, then how long wire do we need.
2. A thin rope is to be wrapped on a hockey stick. If the diameter of the stick is 4.9 cm and has to be wrapped 250 rounds, then how long rope is needed.

3. Diameter of a cycle wheel is 77 cm. How many rotations would wheel make to cover 2.42 km.
4. A race track has to be constructed, which spans 1 km in 8 rounds. Determine the diameter of track up to nearest meter.
5. 10 ring have to be made out of 66 cm long silver line. What would be diameter of every ring.
6. Distance of Moon from Earth is approximately 384000 km. Find the circumference of moon's trajectory around the Earth. It is circular in shape.
7. The ratio of radii is 2 : 3 find the ratio of their circumference.
8. There are two ways of entering and existing a circular lawn. One through radius other through diameter. If both ways have difference of 16.4 metre, then find the diameter and circumference of lawn.
9. Circumference of Equator is 40040 km. If 7000 km above the surface of Earth a satellite revolves constantly then what distance it has to cover to complete one revolution.

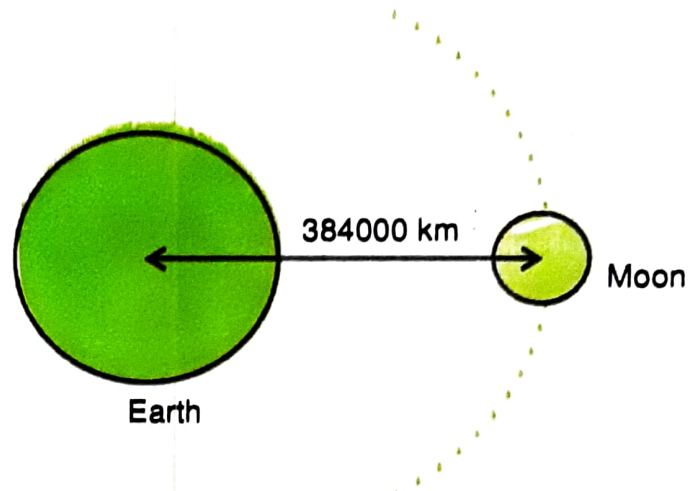


Fig. 18.10

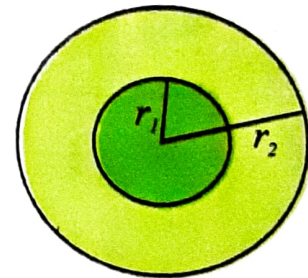


Fig. 18.11

18.4 Area of circle

Do these and conclude

Activity 1

Draw a circle on a thick paper divide its circumference into 16 equal parts. Draw the radii and observe that circular area has been divided in to 16 radii as shown in the figure. Label these radii from number 1 to 16. Cut away every part.

Divide these unit radians in two groups equally. Arrange these two groups up and down alternately as shown in the following figure

It somehow looks like a rectangle, but its not a perfect rectangle, why ?

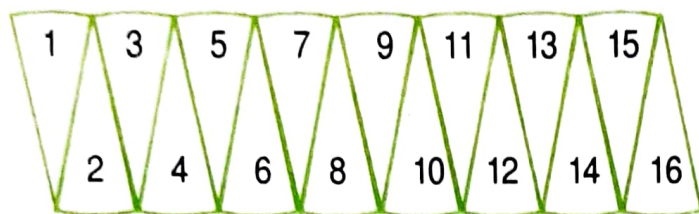


Fig. 18.13

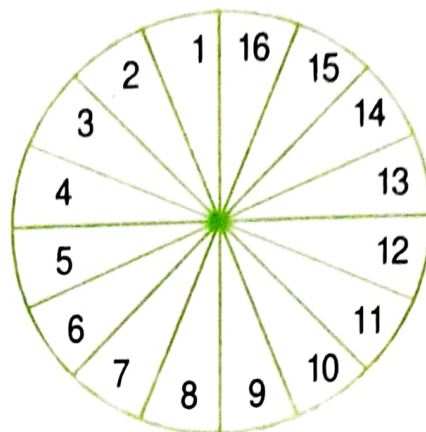


Fig. 18.12

Similarly if more radian units are cut and arranged as fig 18.13 then we can obtain figure closer to rectangle. Hence circular area could be closer to rectangular area.

Length of the rectangular area is half of the circumference of the circle and breadth is equal to its radius. Why ?

Let the radius of circle	=	r cm
Circumference of circle	=	$2\pi r$ cm
$\frac{1}{2} \times (\text{Circumference of circle})$	=	πr cm
or Length of rectangular area	=	πr cm
Breadth of rectangular area	=	r cm
Area of rectangle	=	$\pi r \times r \text{ cm}^2 = \pi r^2 \text{ cm}^2$
Therefore area of circle	=	$\pi r^2 \text{ cm}^2$
Area of circle	=	$\pi \times (\text{radius})^2$
	=	πr^2

Area related simple problems

Example 7. A cow tugged at a point in a green lawn can graze up to 10.5 metre distance. How much area can be grazed ?

Solution : Radius of circular area can be grazed by cow is 10.5 cm.

$$\begin{aligned}
 \text{Radius} &= 10.5 \text{ m} \\
 &= \frac{21}{2} \text{ m} \\
 \text{required area} &= \pi \times \frac{21}{2} \times \frac{21}{2} \text{ m}^2 \\
 &= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \text{ m}^2 \\
 &= \frac{693}{2} \text{ m}^2 \\
 &= 346.5 \text{ m}^2
 \end{aligned}$$

Example 8 : The area of floor of a circular pump house is 55 m^2 . Evaluate the inner radius of the room upto two decimal places, where it is given $\pi = \frac{22}{7}$

Solution : Let the radius = $r \text{ m}$

$$\begin{aligned}
 \therefore \pi r^2 &= 55 \\
 \frac{22}{7} \times r^2 &= 55 \\
 r^2 &= \frac{55 \times 7}{22} \\
 &= 2.5 \times 7 = 25 \times 0.7 \\
 r &= 5 \times \sqrt{0.7} \\
 &= 5 \times 0.8366 = 4.18
 \end{aligned}$$

Therefore inner radius = 4.18 m.

Example 9. Calculate the area of circular field of radius 2.1 cm

Solution : We know that

Area of a circular field = πr^2

Here $r = 2.1 \text{ cm}$ and we have taken $\pi = \frac{22}{7}$

$$\begin{aligned}
 \text{Area of circular field} &= \frac{22}{7} \times 2.1 \times 2.1 \text{ cm}^2 \\
 &= 13.86 \text{ cm}^2
 \end{aligned}$$

Example 10. Area of a circular field is 154 cm^2 . Find the radius of circular field.

We know that the area of circular field has been following :

$$A = \pi r^2$$

$$\therefore r^2 = \frac{A}{\pi}$$

$$\therefore r = \sqrt{\frac{A}{\pi}}$$

$$\text{Let here } A = 154 \text{ cm}^2. \quad \pi = \frac{22}{7}$$

$$\text{Then } r = \sqrt{154 \times \frac{7}{22}} \text{ cm} = 7 \text{ cm}$$

Therefore radius of circle is 7 cm.

Example 11. Radius of a circular field is 56 cm. field is encircled with 7 metre road. Calculate area of the road.

Solution : Radius of smaller circle = 56 m
Radius of bigger circle = $(56 + 7) \text{ m}$
= 63 m

$$\text{Now area of smaller circle} = \frac{22}{7} \times 56 \times 56 \text{ m}^2$$

$$\text{Area of bigger circle} = \frac{22}{7} \times 63 \times 63 \text{ m}^2$$

$$\begin{aligned} \text{Area of the road} &= \frac{22}{7} \times 63^2 - \frac{22}{7} \times 56^2 \\ &= \frac{22}{7} \times (63^2 - 56^2) \text{ m}^2 \\ &= \frac{22}{7} \times (63 + 56) \times (63 - 56) \text{ m}^2 \\ &= \frac{22}{7} \times 7 \times 119 \text{ m}^2 \\ &= 2618 \text{ m}^2 \end{aligned}$$

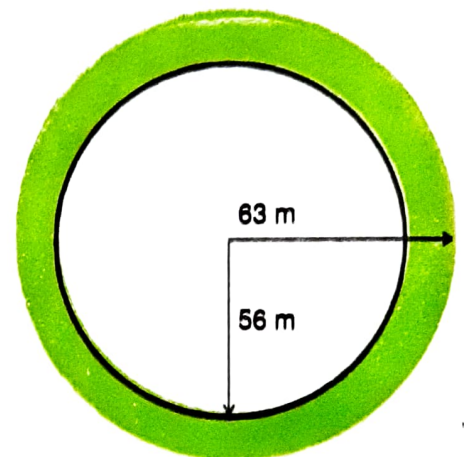


Fig.18.14

Try it :

Fill in the blanks-			
S.No.	Circumference of circle	Radius of circle	Area of circle
1	—	3.5 cm	—
2	—	—	154 m ²
3	3.14 m	—	—
4	—	—	12.58 m ²
5	—	2.5 m	—

Exercise 18(c)

- Find the area of a circle whose diameter is 14 dm.
- A circular cardboard has area $9\frac{5}{8}$ dm². Calculate its diameter.
- Blacksmith Dayaram makes the biggest circular plane Pan from the squared iron sheet of side 28 cm. Calculate the area of Pan. How much sheet is left ?

Hint—Diameter of pan is equal to side of square.

- The cost of painting a semi-circular signboard at 15 paise per cm² is ₹49.50. If the inner radius of semicircle is 14 cm, then calculate the width of signboard.

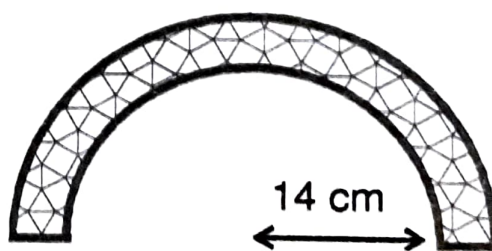


Fig. 18.15

- A square and a circle have same perimeter. If both have perimeter 44 cm, then whose area is greater, and by how much?

6. The area of a circular field is hundred times than that of another circular field. Find the ratio of their circumference.
7. A plastic rectangular sheet measures 36×24 cm. 864 circular buttons of 1 cm diameter are cut out of it. Calculate the area of remaining sheet.
8. A circular plate has diameter 28 cm. State the diameter of the smaller plate, whose area is half the area of bigger plate.

18.5 Volume and total surface area of right circular cylinder.

18.5.1. Right circular cylinder

Pictures of some objects are given below. Look at these and name the similarly objects found around us.

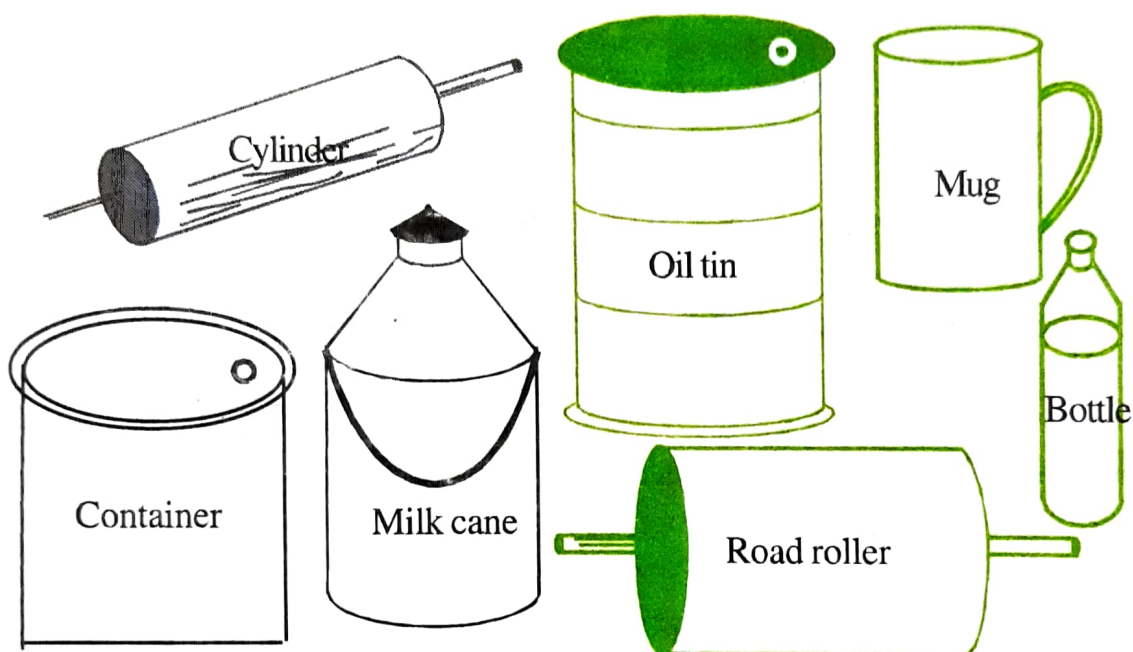


Fig 18.16

Main parts of these objects are similar in shape and these are in cylindrical shape.

Do it

Experiment 1

Cut circular discs from a thick cardboard of equal radius. Stack these cardboard discs exactly above each other and see if the formed shape is cylindrical or not. We see that shapes formed like these are cylindrical.

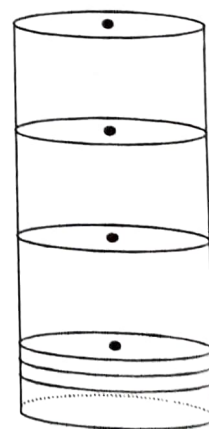


fig 18.17

Radius of circular plane cross section is called radius of cylinder. Find out the radius of cylinder in the following figure.

The line passing from the centre of circular plane cross section is known as axis. Find the name of axis from the given figure.

Bottom circular surface of the cylinder is known as base.

How many bases are there in the cylinder ?

If cylinder is turned up side down, its top becomes its base. The length between the bases of the cylinder is known as length or height (h) of the cylinder.

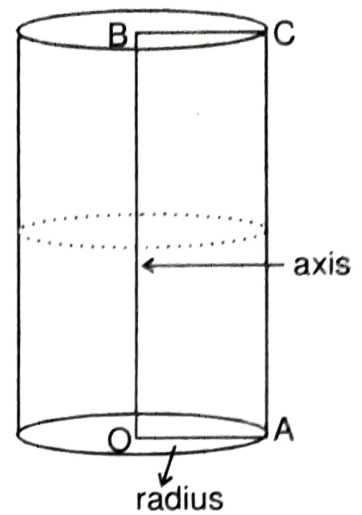


Fig 18.18

If the axis of the cylinder is perpendicular to its every cross section. Then the cylinder is called right circular cylinder.

Here we will call right circular cylinder, only cylinder.

Experiment 2

Take a rectangular thick sheet of paper. Fold the sheet by its width forming a cylinder as shown in figure. Notice that length of rectangular sheet is same as the height of the cylinder and the width of the rectangular paper is equal to the diameter of the cylinder.

Similarly if we fold the sheet by its length, what would be its height and perimeter if a rectangular sheet is rotated about one of its side then what shape would be formed ?

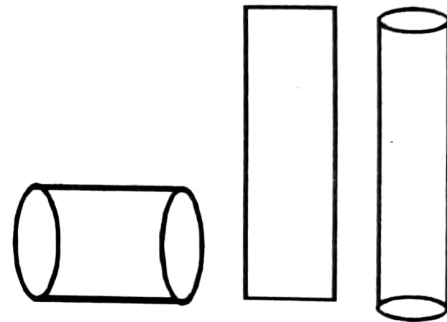


fig 18.19

We notice that rotating a rectangular sheet by its side about it forms a cylindrical shape.

Hence

The solid formed by rotating a rectangle about its one arm is known as right circular cylinder. The side about which the rectangle is rotated is known as cylinder height and the other side is the radius of base.

Above explanation of right circular cylinder presents two different but similar images in our mind. Hollow cylinder and solid cylinder.

18.5.3 Total Surface area of right circular cylinder

Look at the given figure. Total surface of a cylinder is found of its curved surface area and its plane circular areas. Circular planes of cylinder are circles of same radius

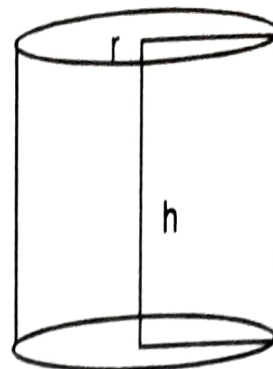


Fig. 18.20

Now make a hollow cylinder of paper. After joining with tape draw a line along its height. Cut the line with scissor and open it. Notice that cylinder gets converted into rectangular area.

Also notice that length of the sides of rectangular pieces are $2\pi r$ and h .

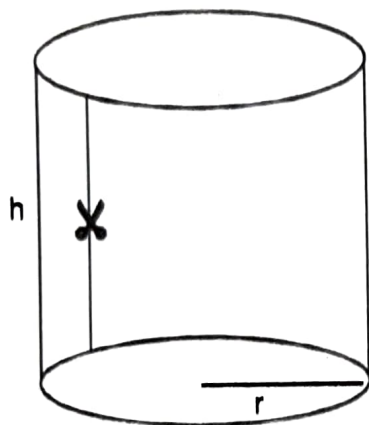


Fig 18.21

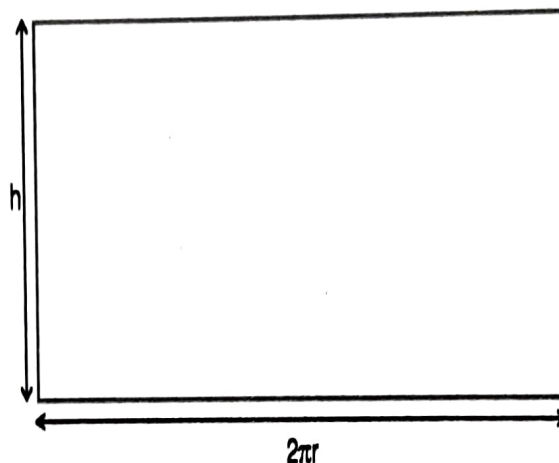


Fig 18.22

Hence curved surface area $= 2\pi r \times h = 2\pi rh$

Total surface of cylinder $= \text{Curved surface of cylinder} + \text{Area of both plane surface}$

Area of both plane surface

$$2\pi rh + 2\pi r^2 = 2\pi r (h + r) = 2\pi r (h + r)$$

Curved surface of cylinder $= 2\pi rh$

Total surface of cylinder $= 2\pi r (h + r)$

Example 12. A cylinder has height 18 cm and radius 10.5 cm. Find the volume, curved surface area and total surface area of cylinder.

Solution : Here $r = 10.5$ cm, $h = 18$ cm.

$$\text{Curved surface area} = 2\pi rh$$

$$= \left(2 \times \frac{22}{7} \times 10.5 \times 18 \right) \text{ cm}^2$$

$$= 1188 \text{ cm}^2$$

$$\text{Area of base} = \pi r^2$$

$$= \left(\frac{22}{7} \times 10.5 \times 10.5 \right) \text{ cm}^2$$

$$= 346.5 \text{ cm}^2$$

$$\text{Total surface area} = \text{curved surface area} + 2 \times \text{Area of base}$$

$$= (1188 + 2 \times 346.5) \text{ cm}^2$$

$$= (1188 + 693) \text{ cm}^2$$

$$= \mathbf{1881 \text{ cm}^2}$$

Try it

Fill in the following table about cylinder

S.N	Radius	height	curved surface	Total surface	
1	7 cm	7 cm	—	—	
2	7 cm	—	—	—	
3	—	15 cm	1320 cm ²	—	

Exercise 18 (d)

1. A square sheet whose side is 25 cm is folded to form a cylinder. Find the curved surface area of cylinder.
2. A right circular cylinder drum had diameter 55 cm and height 120 cm. How many litre water can contain it?

3. A roller has diameter 70 cm and length 2 m, then how much meter² will it move in 50 rotation.
4. What would be cost of digging 3 m diameter 14 m deep well at the rate 30 rupees per cubic meter ?
5. A 17 cm long and 7 cm wide rectangle rotated about its breadth forms a cylinder. Find its curved surface area.
6. If the radius of base of a right circular cylinder is 7 cm and height 14 cm, then find its total surface area.
7. A right circular cylinder has curved surface area 1320 cm². If height of cylinder is 15 cm, find the radius of its base.

18.6 Volume and total surface of right cylindrical cone

We see various objects like hat of joker, ice cream cone, conical wrap to hold street food etc. Objects like these have shape like right circular cone. Its base is circular and its curved surface is conical.

Name more object of right circular conical shape of your surrounding.

Take a cardboard of shape right-angle triangle. Rotate it about any perpendicular side. Is the formed solid a cone or not ?

Look at the given figure. In this figure, right angle triangle VOD is rotated about side VO gives different positions VD_1 VD_2 VD_3 . They are forming a curved surface. Side OD forms circular plane area on one round.

Hence,

If right-angled triangle is rotated about one of its perpendicular sides then the solid formed is known as right circular cone.

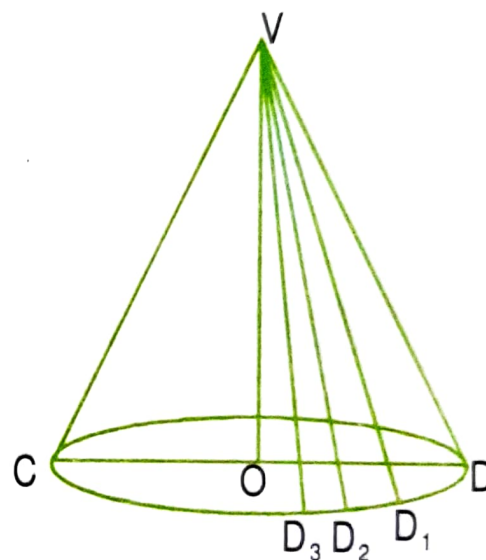


Fig. 18.23

In the following figure V is the vertex of the cone VO is height of the cone and the line joining the vertex and any point on the circular base is known as slant VD or VC height l .

In right triangle VOD

$$VD^2 = VO^2 + OD^2$$

or, $l^2 = h^2 + r^2$

or, $l = \sqrt{h^2 + r^2}$

Slant height of cone = $l = \sqrt{h^2 + r^2}$

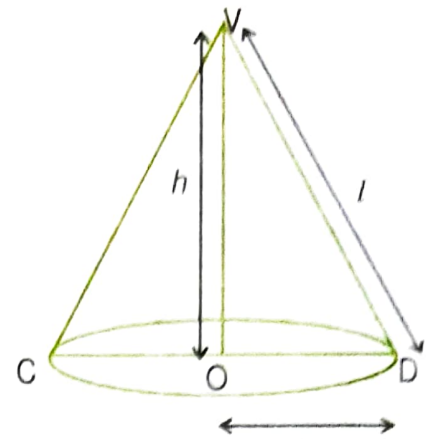


Fig. 18.24

18.6.2 Total surface area of right circular cone

Look at the given figures its total surface area is made up of two surfaces

- (1) Circular plane surface
- (2) Curve surface or slant surface

Area of circular plane surface of cone = πr^2
(r is radius of circle)

Now take out cone from maths kit, consider the radius of its r and slant height l. Draw a circle of radius l on paper.

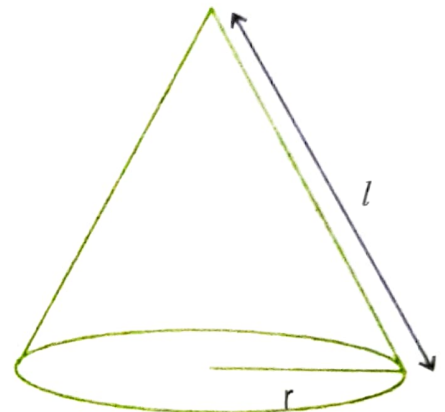


Fig. 18.27

What is the circumference of base of cone ? We know that circumference is $2\pi r$. Draw the circle of radius OA. With the help of scissor from O to A put it on cone such that point O is placed on vertex V and point A on point C. Wrap the circle around the cone one time cut of the remaining part of the circle with scissor. Now spread the portion of circle wrapped on cone on a paper and notice that it is sectoral as shown in figure.

Measure the arc and circumference of the spread portion with thread and between the two lengths

We see that both lengths are same therefore

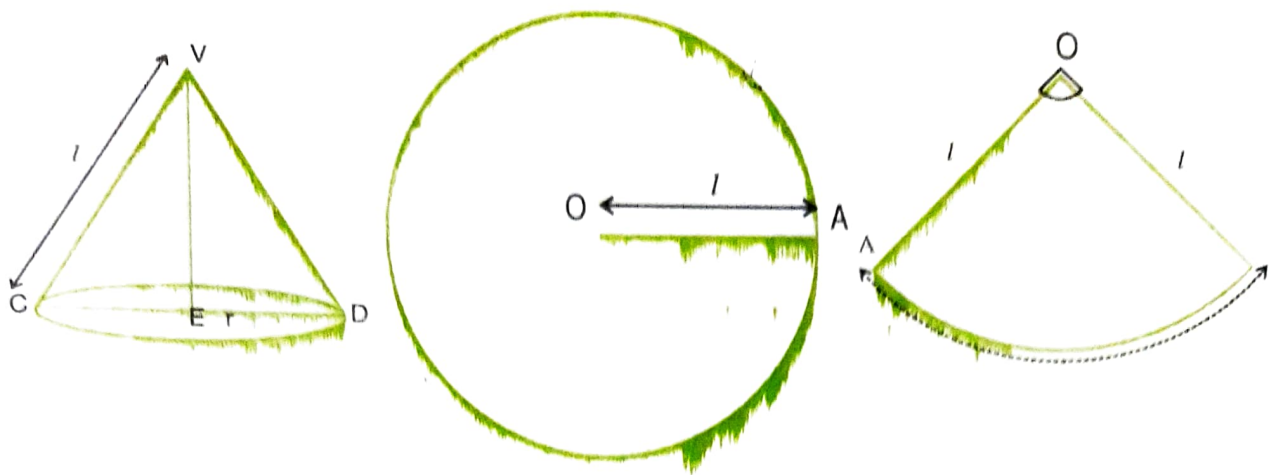


Fig 18.28

Length of arc of sector is equal to the circumference of the cone.

Now fold the sector in such the way that it is divided into sixteen sections as shown in figure. Carefully cut the section and separate them. Arrange these sections according to given figure. 8 sections up and 7 sections down alternately in one line and cut the sixteenth part in two equal parts and arrange them at the ends of the arrangement as the figure given below.

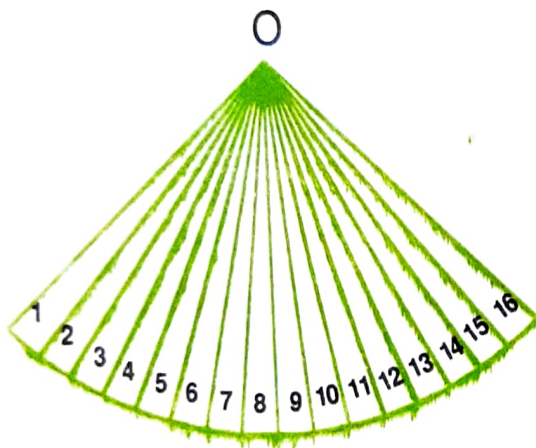


Fig 18.29

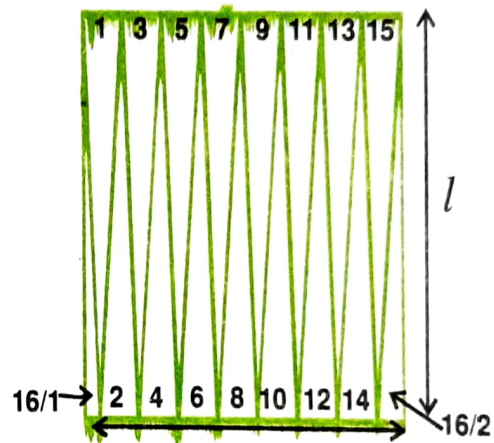


Fig. 18.30

Tell

- (1) What is length of this rectangle ?
- (2) What is width of this rectangle ?

Now we see that length of this rectangle is $\frac{2\pi r}{2} = \pi r$ and width is l .

Therefore slant surface of cone

$$= \text{Area of rectangle}$$

$$= \pi r \times l$$

$$= \pi r l \text{ unit square}$$

Total surface area of cone

$$= \text{slant surface area} + \text{Area of base}$$

$$= \pi r l + \pi r^2$$

$$= \pi r (l + r) \text{ unit square.}$$

Therefore,

$$\text{Total surface area of cone} = \pi r (l + r)$$

Example 14. A cone has base radius is 3.5 cm and height is 12 cm. Find the volume and curved surface area of cone.

Solution : Height of cone $h = 12 \text{ cm}$

Radius of cone $r = 3.5 \text{ cm}$ or $\frac{7}{2} \text{ cm}$.

$$\text{Slant height } l = \sqrt{h^2 + r^2} \text{ (formula)}$$

$$= \sqrt{12^2 + \frac{7^2}{2}}$$

$$= \sqrt{144 + \frac{49}{4}}$$

$$= \sqrt{\frac{625}{4}}$$

$$= \frac{25}{2}$$

$$= 12.5 \text{ cm}$$

$$\text{Slant surface area} = \pi r l \text{ (formula)}$$

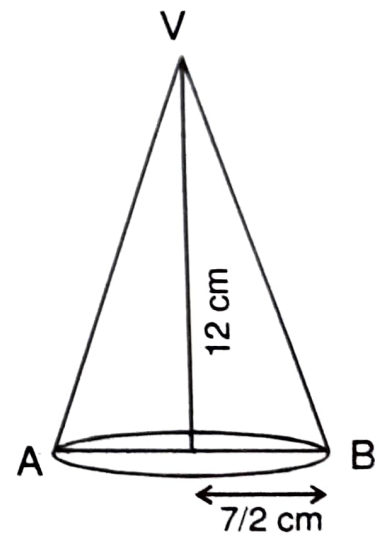


Fig 18.31

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2}$$

$$= 137.5 \text{ square cm}$$

Example 15. 15 metre height conical tent has base circumference of 44 metre. How much canvas is required to make this tent. Find the volume of air contained by the tent.

Solution : Circumference of base $= 2\pi r$

$$\therefore \text{According to question, } 2 \times \frac{22}{7} \times r = 44$$

$$\therefore r = \frac{44 \times 7}{2 \times 22}$$

$$\therefore r = 7 \text{ m}$$

$$\text{Slant height} = \sqrt{h^2 + r^2}$$

$$= \sqrt{15^2 + 7^2}$$

$$= \sqrt{225 + 49}$$

$$= \sqrt{274} \text{ m.}$$

$$\therefore \text{Area of canvas} = \text{Slant Surface area}$$

$$= \pi r l$$

$$= \frac{22}{7} \times 7 \times \sqrt{274}$$

$$= 22\sqrt{274} \text{ m}^2$$

Group discussion

1. A cone has slant height 5 cm and height 3 cm. Find its radius of base.
2. What shape is formed if right angle triangle is rotated about its sides other than its hypotenuse.

Exercise 18(e)

1. Find the curved surface area of the cone of the cone given in figure where $VO = 15$ cm and $OB = 8$ cm.

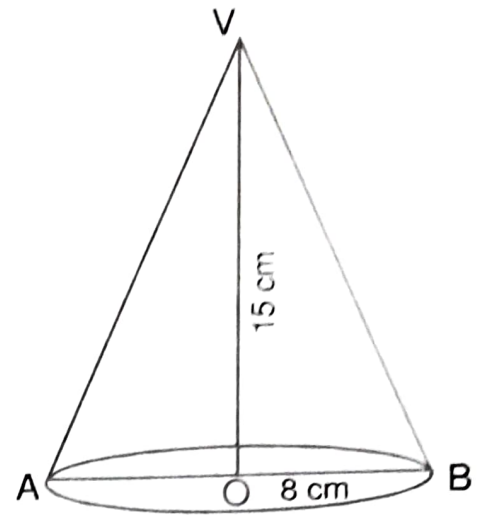


Fig 18.32

2. A cone has height 12 cm if radius of base is 5 cm, find its curved surface area.
3. For the construction of conical tank 264 square metre canvas is required. If the slant height of cone is 12 metre. Find its height.
4. A joker has conical hat. If it is made by 840 square cm cloth and circumference of hat is 56 centimetre. Find the slant height of hat.
5. If a right circular cone has base radius 3 cm and height 4 cm, then find its total surface area.
6. A right circular cone has total surface area $301\frac{5}{7}$ square metre and its base radius is 6 metre. Find the height of cone.

Efficiency Practice 18

1. Fill in the blanks of following statements :
- (i) Area of trapezium =
 - (ii) Radius of a circular is r cm circumference = and its area
 - (iii) A cylinder has base radius r cm and height h cm its curved surface area =
 - (iv) Cone has radius r cm, height h cm slant height l cm. Curved surface area = and total surface area

2. Find the area of the given figure where ABEC is square, CDE is isosceles triangle and EFAE is semicircle.

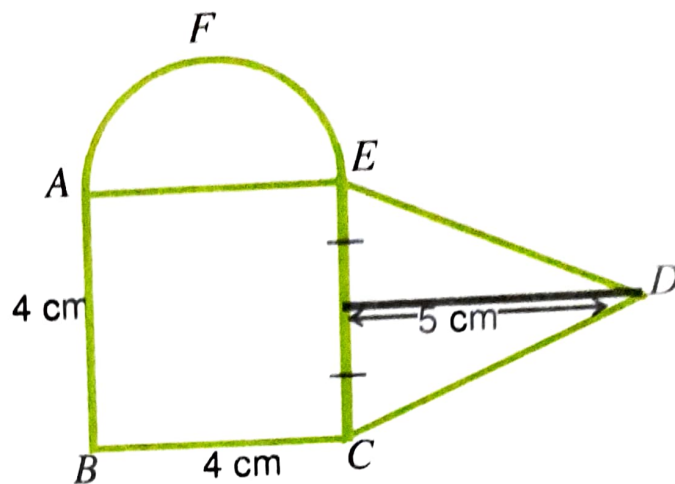


Fig 18.33

3. A cone has height 48 cm and base radius 28 cm. Find its volume, curved surface area and total surface area.
4. A circular park has diameter 48 metre 3.5 metre wide road encircles it. Find the cost of repairing it at the rate of ₹20 per sq. metre.
5. In figure 18.34 the square is of sides 28 cm. It encloses a circle touching all its sides. Find the circular area.

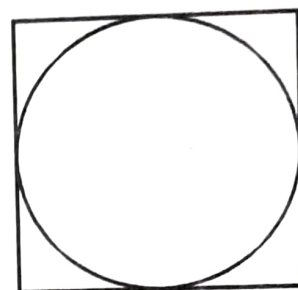


Fig. 18.34

6. A roller has diameter 2.4 m and length 1.68 m. If it has to make 1000 complete rounds to level of the field, then the area of field is-
- 126720 square m
 - 12672 square m
 - 1267.2 square m
 - 12.672 square m
7. A conical tent has base radius 3.5 metre and height 12 metre. Considering thickness of cloth, negligible, find the cost of the spraying the pesticide on the inner and outer walls of the tent, if the rate is ₹ 2.5 per metre square.

(Hint : $l = \sqrt{(3.5)^2 + 12^2} = 12.5 \text{ cm}$)

Required surface = $2 \times \text{slant surface} + \text{box surface}$

We have learnt

1. Area of trapezium = $\frac{1}{2}$ (sum of parallel sides) $\times$ distance between parallel sides
2. Relation between circumference and diameter of circle
$$= \frac{\text{circumference}}{\text{diameter}} = \pi = \frac{22}{7} \text{ (approximate)}$$
3. Circumference of circle $C = \pi D$ (where D is diameter of circle)
$$= 2\pi r$$
 ($D = 2r$, r is radius of circle)
4. Area of circular field $= \pi r^2$ (where r is radius of circle)
5. (i) Curved surface area of right circular cylinder $= 2\pi rh$
(ii) Total surface area of right circular cylinder $= 2\pi r(h + r)$
6. (i) Curved surface area of right circular cone $= \pi rl$ (where l is slant height of cone)
(ii) Total surface area of right circular cone $= \pi r(l + r)$

ANSWERS

Exercise 18(a)

1. 10.5 cm^2 2. 15 cm 3. 26 cm^2 4. 28 square m , 6. 24 cm 12 cm

Exercise 18(b)

1. 5.72 m , 2. 38.50 m 3. 1000 rotation 4. 398 dm 5. 2.04 cm ; 6. 2413714.28 km ;
7. $2 : 3$; 8. 28.70 m , 90.2 m ; 9. 84040 km

Exercise 18(c)

1. 154 dm^2 ; 2. 3.5 dm ; 3. 616 cm^2 , 168 cm^2 ; 4. 6.15 cm ; 5. Area of circle, 33 sq.m .
6. $10 : 1$; 7. $185\frac{1}{7} \text{ cm}^2$; 8. $14\sqrt{2} \text{ cm}$

Exercise 18(d)

1. 1000 cm^3 ; 2. 625 cm^2 ; 3. 285.21 Litre ; 4. 220 sq. km. ; 5. 748 sq. cm ; 6. 924 cm^2 ;
7. 14 cm

Exercise 18(e)

1. 136π sq. cm.; 2. 65π sq. cm; 3. 9.75 m; 4. 30 cm; 5. 452.57 cubic cm;
6. 24π sq. cm; 7. 8 m

Efficiency Practice

1. (i) $\frac{1}{2}$ (sum of parallel sides) $\times$ height (ii) $2\pi r$, πr^2 (iii) $2\pi rh$ (iv) πrl , $\pi r(l + r)$;
2. 32.28 sq cm; 3. 78.57 cm^2 ; 4. (i) 2200 sq cm, 2816 sq cm; 5. ₹ 19250; 6. 616 sq
cm; 7. (b) 12672 sq m.; 8. ₹ 783.75.

