

UNIT 19

SQUARE AND CUBE : VEDIC MATHEMATICS



● Square and Cube : Vedic Mathematics

19.1 Introduction:

Vedic mathematics is a method of calculation in which it is possible to calculate the difficult arithmetic calculations in easy way and fast. It consists 16 fundamental formulas and 13 corollaries. These formulas are used to calculate mathematical calculations in different ways. These vedic formulas have gave help to solve the mathematical problems and we felt a satisfaction, also it develop the power of capacity of calculation and it gives a new idea to solve the mathematical problems. In this unit we learn about the calculation of square and cube of given number.

Square of numbers:

We know that if we multiply a number by itself then the product number is called the square of that number.

Study the following table:

Numbers	Square
1	$1^2 = 1 \times 1 = 1$
2	$2^2 = 2 \times 2 = 4$
3	$3^2 = 3 \times 3 = 9$
4	$4^2 = 4 \times 4 = 16$
5	$5^2 = 5 \times 5 = 25$
6	$6^2 = 6 \times 6 = 36$

7	$7^2 = 7 \times 7 = 49$
8	$8^2 = 8 \times 8 = 64$
9	$9^2 = 9 \times 9 = 81$
10	$10^2 = 10 \times 10 = 100$

We can say square of 5 is 25, square of 7 is 49 in the study of the above table. Similarly, the power '2' of number is called the square of that number.

To find the square of numbers by Vedic method :

It is easy to calculate square of small numbers 2,4,8,9 etc. But is it possible to find the square of numbers 35, 48, 99, 999 etc. in short time? It's solution is not so easy and for finding of square of these numbers, we necessarily multiply these numbers by itself.

To find square of the numbers ending with 5:

For finding of square of the numbers ending with 5 we use "Ekadhiken purven" formula of vedic mathematics.

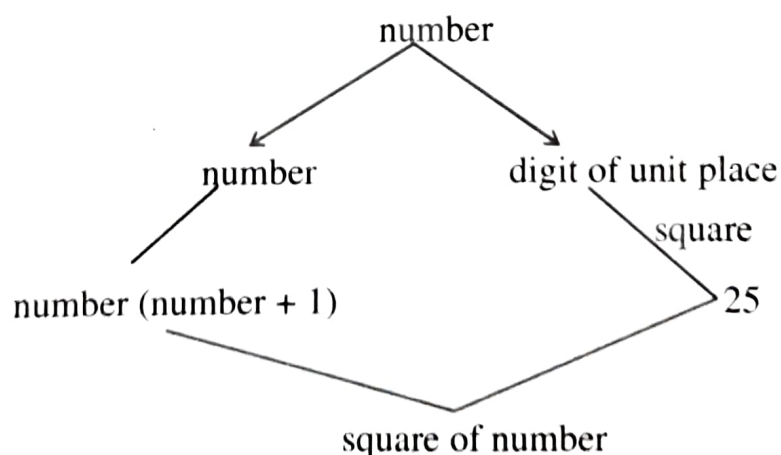
Hindi translation of "Ekadhiken purven" is one "greater than previous" (one more than previous). In mathematical meaning, to find next number we add one in the previous number.

Example 1 : To find 5 add 1 in 4. ($4 + 1 = 5$)

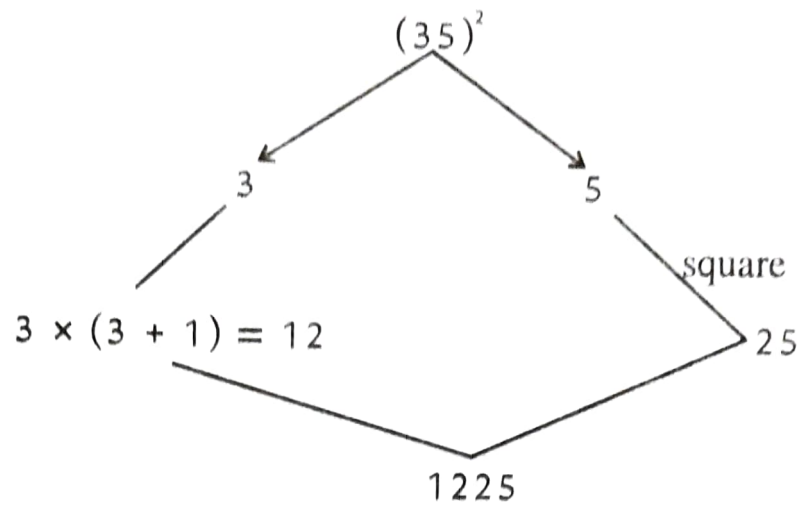
Attempt it : To write next number of the numbers 25, 38, 39, 99, 104 by using "Ekadhiken Purven"

Step 1 - (Unit) remaining all numbers multiply by next number (left of last digit 5).

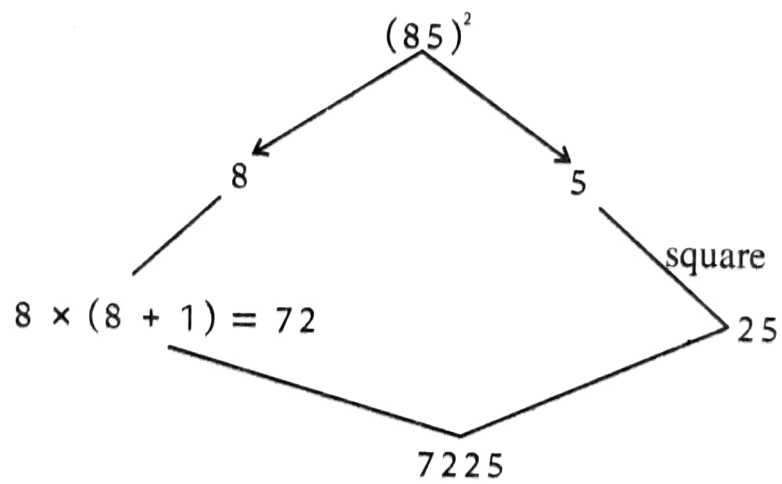
Step 2 - (Unit) Write square of 5 in the last digit.



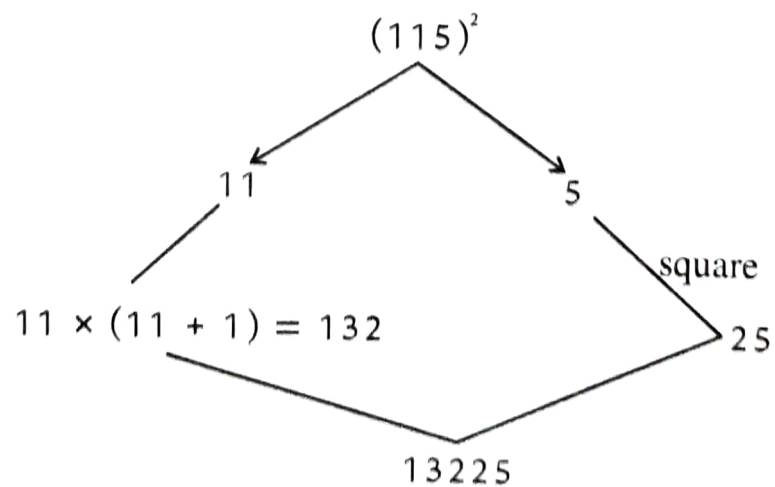
(i) $(35)^2$



(ii) $(85)^2$



(iii) $(115)^2$



$$\begin{aligned}
 \text{(iv)} \quad (205)^2 &= [20 \times (20 + 1)] 25 = [420] 25 = 42025 \\
 \text{(v)} \quad (495)^2 &= [49 \times (49 + 1)] 25 = [2450] 25 = 245025 \\
 \text{(vi)} \quad (945)^2 &= [94 \times (94 + 1)] 25 = [8930] 25 = 893025
 \end{aligned}$$

Proof : Take a number whose unit place number is 5. i.e. (95)

$$\begin{aligned}
 (95)^2 &= (10x + 5)^2 && \text{(We take } a = x) \\
 &= 100x^2 + 100x + 25 && [(a+b)^2 = a^2 + b^2 + 2ab] \\
 &= 100(x + 1)x + 25 \\
 &= x(x + 1) \times 100 + 25
 \end{aligned}$$

Attempt it : To find square of 195, 225, 185.

To find square of a number :

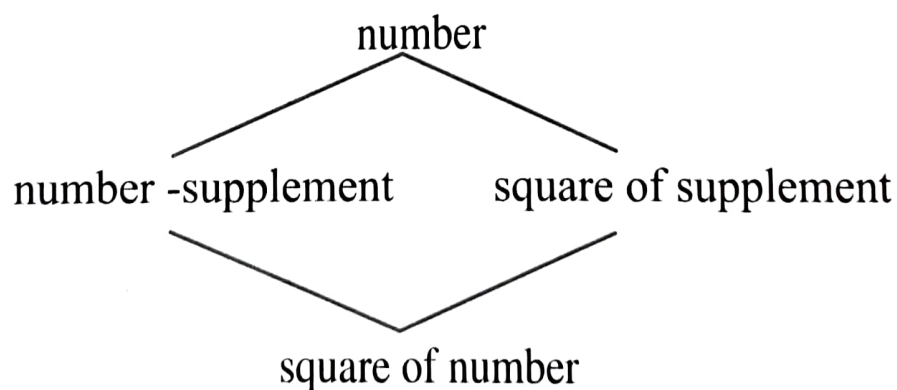
To find the square of a number, it is used a formula of vedic mathematics “Yavdunam Tadvunikritya, vargh yojyet.”

Hindi translation :

Square of supplementary number is found by a number replaced by supplementary.

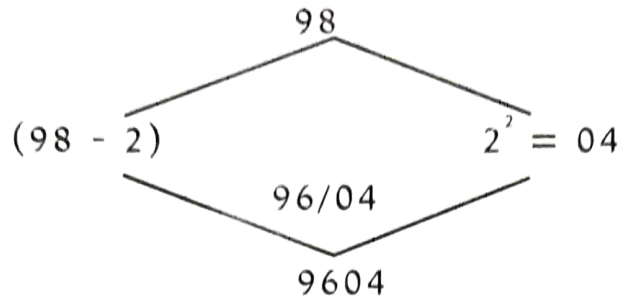
Step 1 - Right hand side: Square of supplement.

Step 2 - Left hand side: To replace the number from the supplement to find square.

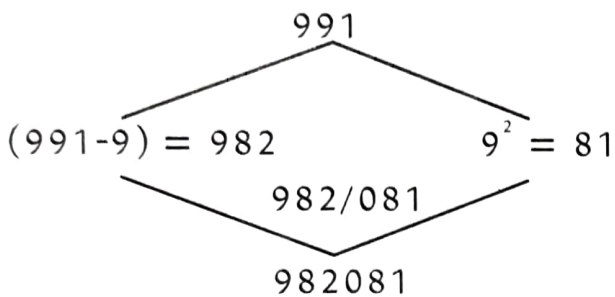


Example -

(i) $(98)^2$

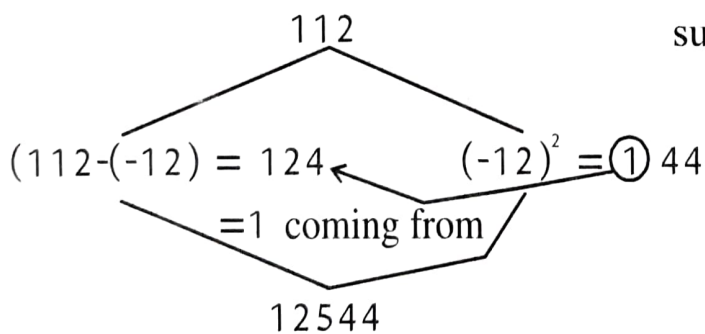


(ii) $(991)^2$



81 has two digits i.e. it has one less number than the base 1000 so, we write 081 in place of 81

(iii) $(112)^2$



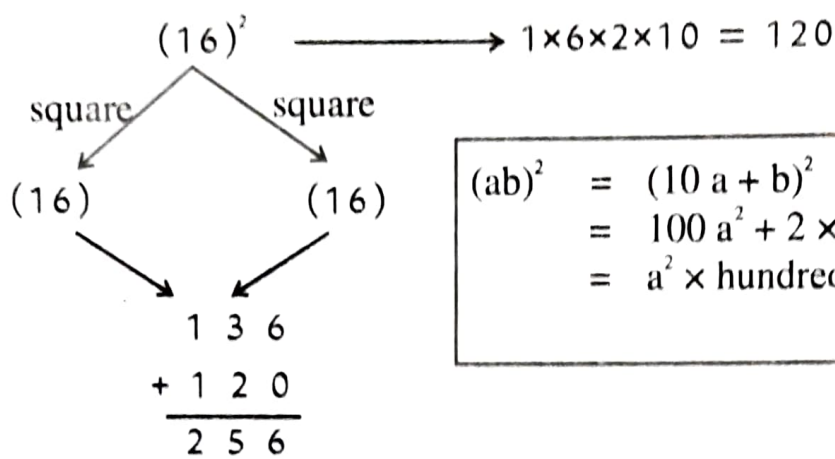
Base = 100
supplement = - 12

144 has three digits, base is 100; which one more than the zero is 100 so, 1 goes left side

Attempt it : To find square of 99, 188, 908

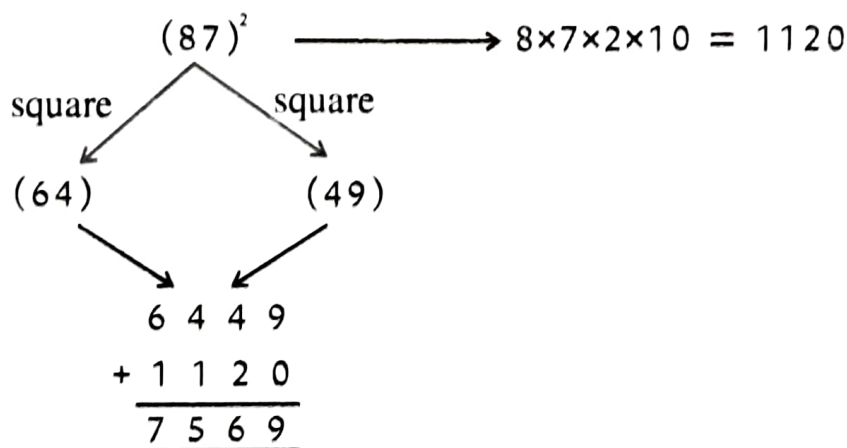
Square of two digit numbers:

(i) $(16)^2$

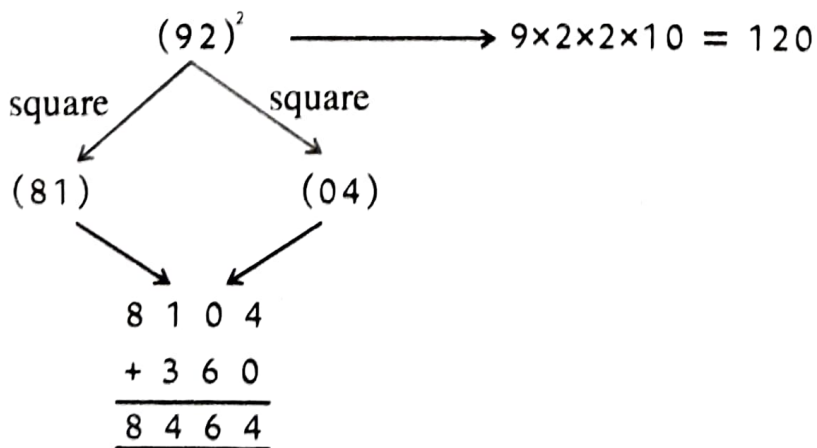


$$\begin{aligned} (ab)^2 &= (10a + b)^2 \\ &= 100a^2 + 2 \times 10 \times b + b^2 \\ &= a^2 \times \text{hundred} + b^2 + 2 \times a \times b \times 10 \end{aligned}$$

(ii) $(87)^2$



(iii) $(92)^2$



To find cube of the given numbers by vedic mathematics:

It is easy to find cube of the numbers 1 to 10 as:

Number	Cube
1	$1 \times 1 \times 1 = 1$
2	$2 \times 2 \times 2 = 8$
3	$3 \times 3 \times 3 = 27$
4	$4 \times 4 \times 4 = 64$
5	$5 \times 5 \times 5 = 125$
6	$6 \times 6 \times 6 = 216$
7	$7 \times 7 \times 7 = 343$
8	$8 \times 8 \times 8 = 512$
9	$9 \times 9 \times 9 = 729$
10	$10 \times 10 \times 10 = 1000$

But when we take a number greater than 10 then it is difficult to find the cube of the numbers by the traditional method and it take more times. It is more easier than to find the cube of a number by vedic mathematics than the traditional methods.

We use the following formulae of vedic mathematics to find the cube of a number.

Vedic formula for finding the cube:

Yavdunam

Nikhilam

Anurupyen

Yavdunam :

It's fraction will be Yavt+unam, where meaning of 'Yavt' is how (Jitna) and meaning of unam is less. So, meaning of Yavdunam is "how many less". This method is used for finding of

cube of those numbers which are near to its base 10^n , where n is a natural number (as: 10, 100, 1000, 10000.....). Therefore, this formula has limited applications.

Working rule:

- (i) To find the cube of a number, firstly we check that the given number is near to which base number (10, 100, 1000.....) As: base number for the number 12 is 10, base number for 96 is 100, base number for 997 is 1000.
- (ii) We find deviation for the given number after the finding of base number.

$$\text{deviation (d)} = \text{original number (given number)} - \text{base number}$$
- (iii) The following three sides after the finding of base and deviation are:

$$\begin{aligned} \text{First term (I)} &= (a + 2d) \\ \text{Second term (II)} &= [(a+2d) - \text{base (b)}] \times \text{deviation (d)} \\ \text{Third term (III)} &= \text{deviation (d)}^3 \end{aligned}$$

where, a = given number,
 b = base number,
 d = deviation
- (iv) We use the following formula $(a + 2d) / [(a + 2d) - b] \times d / d^3$ for finding of the cube of a number after the calculation of the above three steps, which is a actual cube of the number:

Example 1: Find cube of 12 by using of formula Yavdunam.

Solution :

Step 1 - For finding cube of 12, which is near to 10 so, base will be 10 for the number 12.

Step 2 - Deviation (d) for number 12 is = given number - base number (b)

$$= 12 - 10$$

$$= 2$$

Step 3 - I $= (a + 2d)$

$$= 12 + 2 \times 2$$

$$= 12 + 4$$

$$= 16$$

$$\begin{aligned}
 \text{II} &= [(a + 2d) - b] \times d \\
 &= [16 - 10] \times 2 \\
 &= 62 \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 \text{III} &= d^3 \\
 &= (2)^3 \\
 &= 2 \times 2 \times 2 \\
 &= 8
 \end{aligned}$$

So, cube of 12 is $(12)^3 = \text{I} / \text{II} / \text{III}$

The cube of the given number 12 will be-

$$\begin{aligned}
 (12)^3 &= \text{I} / \text{II} / \text{III} \\
 &= 16 / 12 / 8 \\
 &= (16 + 1) / 2 / 8 \quad (\text{add 1 to I as carry over}) \\
 &= 17 / 2 / 8 \\
 &= 1728
 \end{aligned}$$

So, cube of 12 is 1728.

2. Find cube of the number 96.

Solution: Here, $a=96$, $b=100$ deviation $d=a-b=96-100= -4$

So, we find the value of all three steps I, II and III with the help of a, b and d.

$$\begin{aligned}
 \text{(I)} &= (a+2d) \\
 &= 96 + 2 \times (-4) \\
 &= 96 - 8 \\
 &= 88 \\
 \text{II} &= [(a + 2d) - b] \times d \\
 &= [88 - 100](-4) \\
 &= (-12)(-4) \\
 &= 48
 \end{aligned}$$

$$\begin{aligned}
\text{III} &= d^3 \\
&= (-4)^3 \\
&= (-4) \times (-4) \times (-4) \\
&= -64
\end{aligned}$$

Now, we put all three step together as

$$\begin{aligned}
(96)^3 &= 8848/(-64) \\
&= 88 / (47 + 1) / (-64) && \text{(In III term is negative, to make it positive we add it with base number)} \\
&= 88 / 47 / (100 - 64) \\
&= 88 / 47 / 36 \\
(96)^3 &= 884736
\end{aligned}$$

3. Find cube of number 105.

Solution :

$$\begin{aligned}
\text{Here, } a &= 105 \\
b &= 100
\end{aligned}$$

$$\begin{aligned}
\text{so, deviation (d)} &= a - b \\
&= 105 - 100 \\
&= 5
\end{aligned}$$

So, we find the value of all three steps I, II and III with the help of values of a, b and d.

$$\begin{aligned}
\text{(I)} &= (a + 2d) \\
&= 105 + 2 \times 5 \\
&= 115 \\
\text{II} &= [(a + 2d) - b] \times d \\
&= [115 - 100]5 \\
&= 155 \\
&= 75
\end{aligned}$$

$$\begin{aligned}
 \text{III} &= d^3 \\
 &= (5)^3 \\
 &= 5 \times 5 \times 5 \\
 &= 125
 \end{aligned}$$

Now, we put all three steps together as,

$$\begin{aligned}
 &\text{I / II / III} \\
 &= 115 / 75 / 125 \\
 &= 115 / (75 + 1) / 25 \\
 &= 115 / 76 / 25 \\
 &= 1157625
 \end{aligned}$$

Note : In each steps of number of digits depends on zeros of the base number. Here base number is 100, in which zeros are 2. So in each steps number of digits be 2.

Nikhilam:

The meaning of this formula is 'Total'. We can find the cube of a number in a easy way to use this method.

Working rule :

- (i) Firstly we find the the base of the given number.
- (ii) If base is 10 then sub base is 1, base is 40 then sub base is 4
- (iii) We calculate the values of the following three steps after the finding of base and sub -base of the number.

$$\text{Step I} = (\text{number} + 2 \times \text{deviation}) \times (\text{sub-base})^2$$

$$\text{Step II} = \{3 \times (\text{deviation})^2\} \times \text{sub-base}$$

$$\text{Step III} = (\text{deviation})^3$$

Example :

1. Find cube of 25 by using of Nikhilam formula.

Solution : Here, number = 25,
 base = 20,
 subbase = 2,

$$\text{deviation} = (\text{number} - \text{base})$$

$$= (25 - 20) = 5$$

$$\text{I} = (25 + 2 \times 5) \times (2)^2$$

$$= (25 + 10) \times 4$$

$$= 35 \times 4$$

$$= 140$$

$$\text{II} = \{3 \times (5)^2\} \times 2$$

$$= 3 \times 25 \times 2$$

$$= 75 \times 2$$

$$= 150$$

$$\text{III} = (5)^3$$

$$= 5 \times 5 \times 5$$

$$= 125$$

Now, we write all three steps I, II and III with together

$$(25)^3 = \text{Step I} / \text{Step II} / \text{Step III}$$

$$= 140 / 150 / 125$$

$$= 140 / (150 + 12) / 5$$

$$= 140 / 162 / 5 \quad (\text{number zero in base 20 is 1})$$

$$= (140 + 16) / 2 / 5$$

$$= 156 / 2 / 5$$

$$(25)^3 = 15625$$

2. Find cube of 58 by Nikhilam method.

Solution:

Here, **number** = 58,

base = 5

subbase = 5

So **deviation (d)** = 58 - 50

$$= 8$$

Step I = $(58 + 2 \times 8) \times (5)^2$

$$= (58 + 16) \times 25$$

$$= 74 \times 25$$

$$= 1850$$

$$\text{Step II} = \{3 \times (8)^2\} \times 5$$

$$= 64 \times 15$$

$$= 960$$

$$\text{Step III} = (8)^3$$

$$= 8 \times 8 \times 8$$

$$= 512$$

So, cube of 58 be

$$(58)^3 = \text{Step I} / \text{Step II} / \text{Step III}$$

$$= 1850 / 960 / 512$$

$$= 1850 / (960 + 51) / 2$$

$$= 1850 / 1011 / 2 \quad (\text{No. of zero in base is 1})$$

$$= (1850 + 101) / 1 / 2$$

$$= 1951 / 1 / 2$$

$$= 195112$$

Anurupyen :

This vedic formula depends on the concept of geometric progression which is not in the syllabus of class-8. So here we cannot study this method.

Exercise

1. Find the square of the following numbers by Nikhilam formula:

(i) 15

(ii) 225

(iii) 97

(iv) 49

(v) 108

(vi) 9999

(vii) 10003

(viii) 2003

2. Find cube of the following numbers by Nikhilam Formula:

(i) 25

(ii) 87

(iii) 117

(iv) 99

(v) 81

(vi) 203

(vii) 999

(viii) 403

INDIAN MATHMETICIANS AND THEIR MASTER PIECE

S.N.	Name	Time	Master Piece
1.	Boudhayan	800 B.C.	Boudhayan Shulka Sutra
2.	Aryabhatta First	476 A.D.	Aryabhatiya (499 AD)
3.	Varah Mihir	505-587 AD	Vrihjjatak, Vrihat Sanhita, Panch Sindhantika
4.	Brahma Gupta	598 AD	Brahmsfut-Sindhatna (628 AD) Khand Khadyak
5.	Bhaskaracharya First	629 AD	Mahabhaskariya, Laghu Bhaskariya.
6.	Mahaviracharya	850 AD	Ganitsar Sangrah
7.	Aryabhatt Second	950 AD	Mahaaryabhatiya; Mahaaryasidhant
8.	Shridharacharya	991 AD	Gonitsar (Trishatika), Jatak Tilak
9.	Shripati Mishra	1039 AD	PatiGanit, BeejGanit, Sidhant Shekhar.
10.	Nenichandar siddhant	11 th Century	Gommatsar
11.	Bhaskaracharya Second	1114 AD, 1185 AD	Sidhant Shiromani, Lilavati Karan-Kutuhel
12.	Naraya Pandit	1356 AD	Ganit Koumudi
13.	Neelkanth	1356 AD	Tajikneelkanthi
14.	Kamalakar	1608 AD	Sidhant Tal Vivek
15.	Samrat Jaganath	1731 AD	Samrat Sidhant, Rekhaganit
16.	Nrisingh Bapudev Shastri	1831 AD	Rekha Ganit, Trikonamiti, Ankaganit
17.	Shriniwas Ramanujan	1887 AD	Ramanujan Ki Diary
18.	Swami Bharti Krishna Tirthji	1884 AD	Vaidik Ganit
19.	Sudhakar Dwivedi	1860 AD- 1922 AD	Goliya Rekhaganit
20.	Dr. Ganesh Prasad	1876 AD- 1935 AD	Mathematical physics and differential equations at the beginning of the twentyth century
21.	Shakuntala Devi (Human Computar)	Date of birth 4.11.1929	Shakultala Devi's the book of Numbers, Shakuntala Devi more puzzles.