

Module 1: AC Voltage and Current

This module is designed to help you to:

- describe different forms of electric current and the graphical representation of their time variation;
- explain the effect of a resistor in a circuit carrying Alternating Current (AC) and hence describe root mean square (rms) value of AC current and power dissipated;
- describe the advantages of using AC over Direct Current (DC) as a source of emf;
- justify the need and utility for high voltage transmission lines in order to reduce transmission losses;
- explain the concept of electron movement as constituting a current in the context of AC;
- discuss the device used to measure AC.

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7.1.1 Introduction

The practical source of electro-motive force (emf) considered so far is a battery. A battery is able to produce a steady emf for an appreciable time due to the nature of the chemicals used. Such steady emf can produce a steady unidirectional direct current (DC).

Although the battery can be a convenient source of emf, it is not suitable for large amounts of electric energy required for domestic and industrial applications.

So, a different method, based on Faraday's discovery of electromagnetic induction, is used for developing emf. Most of the electrical energy that is generated by using this method, converting mechanical energy into electrical energy, develops an alternating emf which in turn produces an alternating current (AC).

7.1.2 Different forms of electric current

Direct current - A current whose magnitude may vary but the direction of flow of current does not change with time is said to be direct current. A current whose magnitude remains fixed with time is called a steady direct current or steady DC. If the magnitude varies but remains always positive (or negative), then it will be simply called Direct Current or DC. A few examples of different types of direct current are shown in Fig.7.1.1 below.

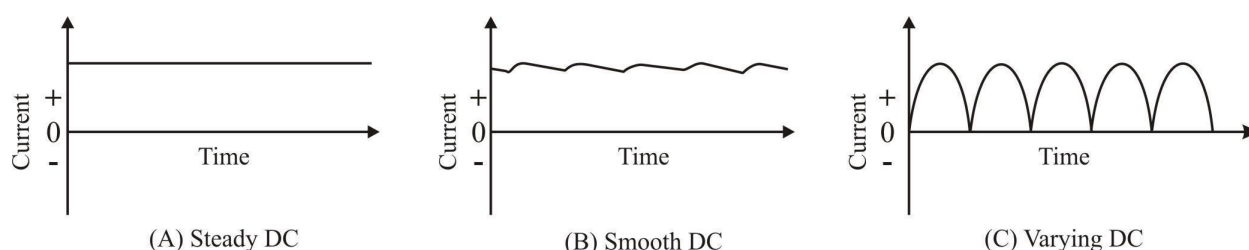


Fig.7.1.1 Time variation of direct current

Periodic Current - A current whose instantaneous value oscillates between two fixed values and repeats itself after a fixed interval of time is called a periodic current. A few examples are shown in Fig.7.1.2.

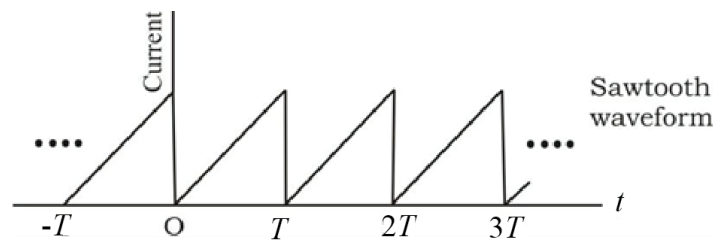


Fig.7.1.2 *Sawtooth waveform*

Pulsating Current - A periodic current that has only positive values (or only negative values) is called pulsating current. in Fig.7.1.3

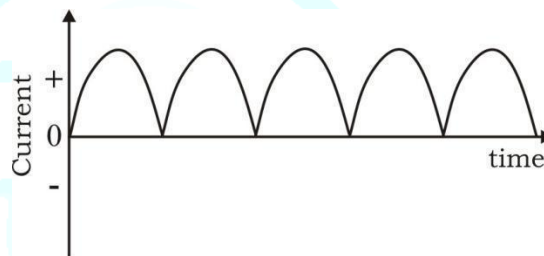


Fig.7.1.3 *Pulsating current*

Alternating current: A periodic current, which reverses direction periodically is called alternating current.

Sinusoidal Current -A periodic current whose instantaneous value varies as sine or cosine function of time is called sinusoidal current. refer Fig. 7.1.4

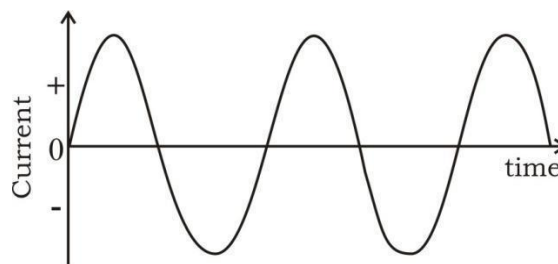


Fig. 7.1.4 *Sinusoidal current*

In a similar manner different types of AC and DC voltages can also be defined.

The most common form of electrical power provided by power companies is in the form of sinusoidal voltages.

It should however be remembered that alternating currents or voltages can be of any shape, not necessarily sinusoidal.

Most of the other types of waveforms are actually composites (superposition) of many individual sinusoidal waves, called harmonics. (For example, a square wave is an example of a waveform that consists of superposition of a number of sine waves. A few of such waveforms are shown below (Fig.7.1.5).

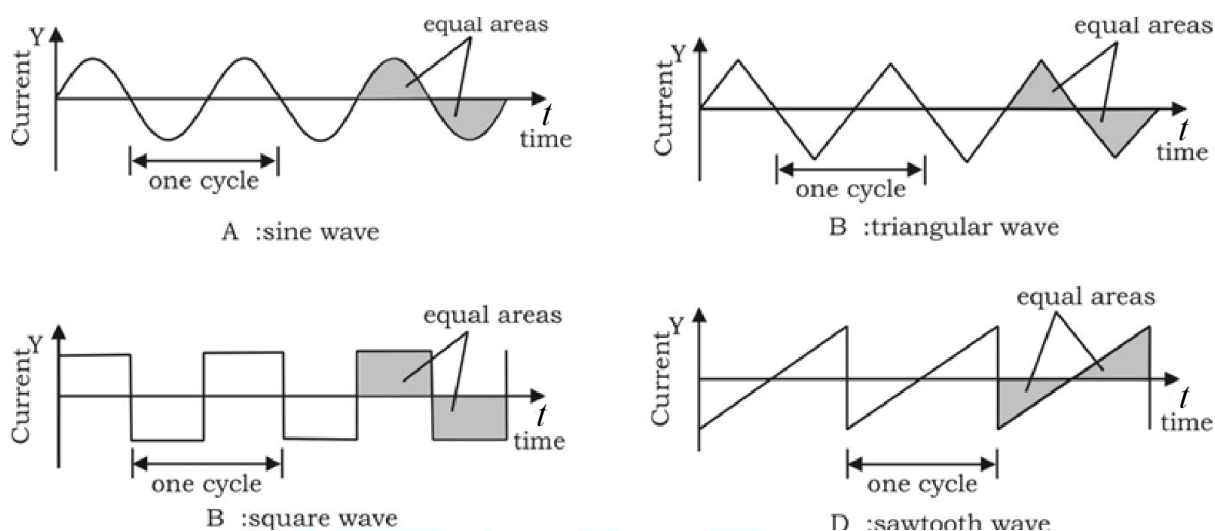


Fig.7.1.5 Examples of waveforms

7.1.3 Alternating current through a resistor: root mean square (rms) value

In order to understand various effects related to AC voltage and current, it is important to consider the simplest circuit consisting of an AC source and a resistor shown in Fig. 7.1.6.



Fig. 7.1.6 Alternating current through a resistor

Here the AC source produces a sinusoidally varying potential difference or voltage given by the following equation.

$$V = V_m \sin \omega t \quad (7.1.1)$$

where V is the instantaneous AC voltage at any time t and V_m is the peak or maximum voltage in one cycle.

Now, the current flowing in the circuit is given by Kirchhoff's loop rule and Ohm's law by the following equation

$$V = IR = V_m \sin \omega t$$

$$I = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \quad (7.1.2)$$

where $I_m = V_m/R$ is the peak or maximum AC current and I is the instantaneous value of AC current in the circuit.

From the equation for the AC current, one can see that it will become zero and maximum at the same time as the AC voltage. Both V and I have the same sinusoidal time variation. This can be seen in Fig.7.1.7.

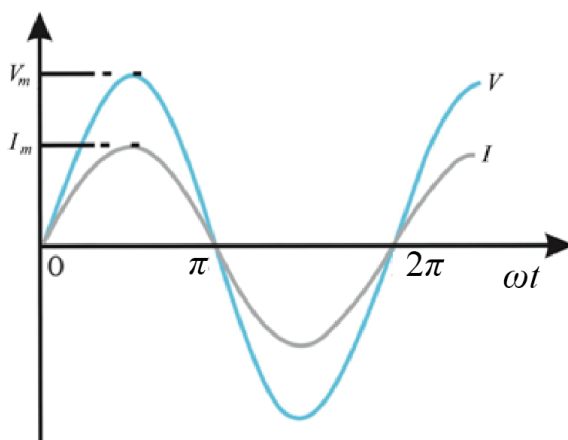


Fig.7.1.7 Sinusoidal AC voltage and current

From Fig.7.1.7 it is also seen that the average value of AC voltage and AC current is zero since each cycle has equal positive and negative half cycles. However AC current also shows Joule heating just like DC current when passed through a resistor.

The power loss due to Joule heating is given by

$$P = I^2 R = I_m^2 R \sin^2 \omega t \quad (7.1.3)$$

where P is the instantaneous power loss due to Joule heating and this instantaneous power loss will always be positive because the square of the AC current will always give a positive value irrespective of whether it is positive or negative (Fig.7.1.8). The average power loss due to Joule heating when an AC current passes through a resistor is

$$\langle P \rangle = \langle I_m^2 R \sin^2 \omega t \rangle = I_m^2 R \langle \sin^2 \omega t \rangle = \frac{1}{2} I_m^2 R \quad (7.1.4)$$

because, $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

This can be understood by the figure given below (Fig.7.1.8)

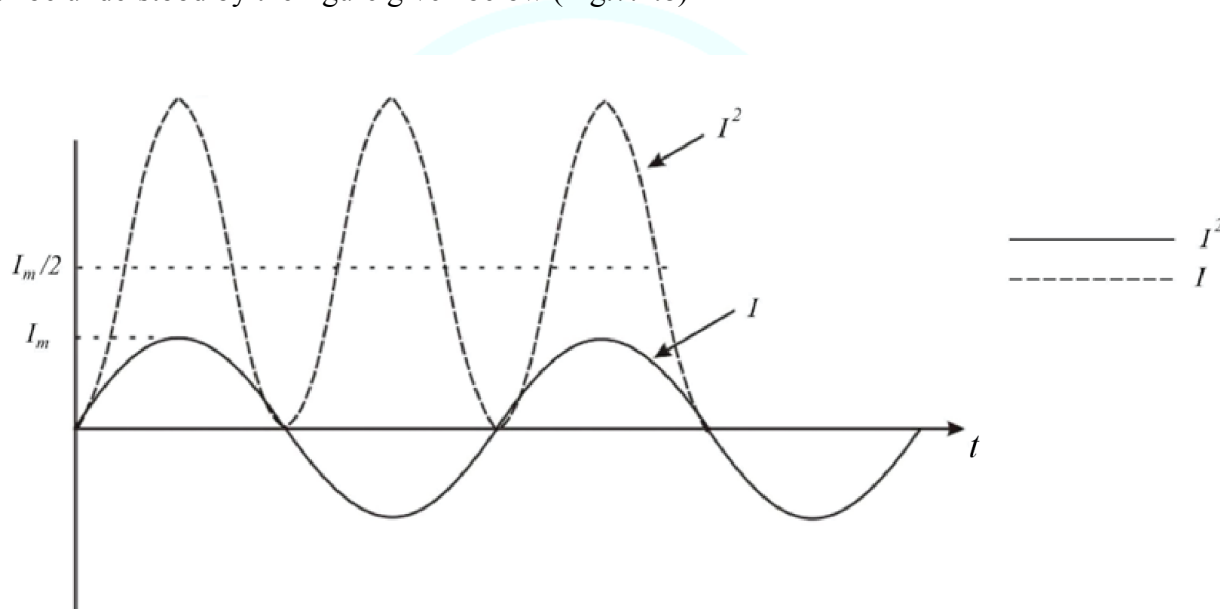


Fig 7.1.8 Graphs of $\sin \theta$ vs θ (continuous line) and $\sin^2 \theta$ vs θ (broken line). The former is symmetrical between the extremes -1 and $+1$, and so the average, $\overline{\sin \theta} = 0$, while latter has same behavior between 0 and 1 , and so the average, $\overline{\sin^2 \theta} = \frac{1}{2}$. This explains the relations $\langle \sin \omega t \rangle = 0$, $\langle \sin^2 \omega t \rangle = \frac{1}{2}$

Let us now consider the two circuits shown below in Fig.7.1.9 having identical light bulbs (i.e. having same resistance). In the first circuit there is a DC source and in the second circuit there is an AC source.

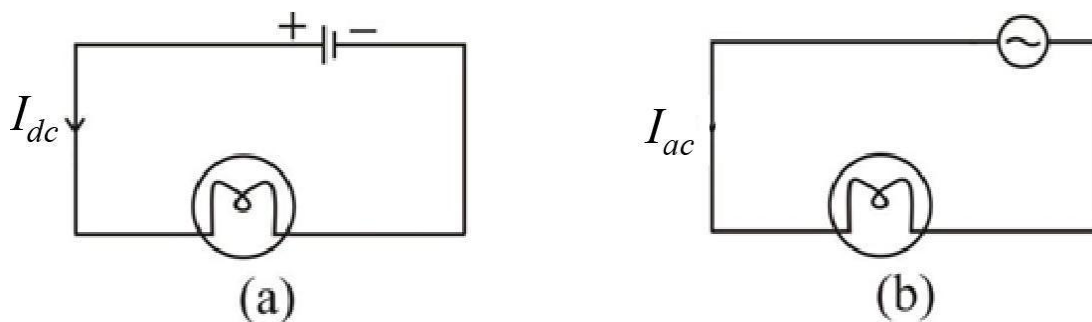


Fig. 7.1.9 (a) circuit with DC source ; (b) circuit with AC source

If the bulbs light with the same brightness at an instance, that is they are working with the same power, then it can be said that the current i_{ac} is equivalent to the current I_{dc} .

The simple mathematical average value of I_{ac} over one complete cycle is equal to zero. Therefore, average power consumed or electrical energy dissipated by alternating current in the above circuit appears to be zero (mathematically). But neither the average power consumed nor true electrical energy dissipated is zero as the electrical bulb glows with AC. This is so because the energy dissipated varies directly as square of the current and not as current.

In order to bring the AC on the same footing as DC for calculating the power loss and for other applications we use the **root mean square (rms)** value. Mathematically, rms value of AC current can be defined as the square root of the mean of squares of the instantaneous values of the alternating current.

Likewise, rms value of AC voltage can also be defined as the square root of the mean of squares of the instantaneous values of the AC voltage.

Alternatively, we can also define the rms AC as under:

The rms value of AC is that value of steady DC which produces the same amount of heat in a given resistance in a given time as is done by the alternating current.

It should be remembered that whenever AC current or voltage is referred to, it means rms value.

$$I = \sqrt{\langle I^2 \rangle} = \sqrt{\langle I_m^2 \sin^2 \omega t \rangle} = \sqrt{I_m^2 \langle \sin^2 \omega t \rangle} = \sqrt{I_m^2 \left(\frac{1}{2}\right)}$$

$$I = \frac{I_m}{\sqrt{2}} = (0.707) I_m \quad (7.1.5)$$

Therefore the average power loss is given by

$$\langle P \rangle = \frac{1}{2} I_m^2 R = I^2 R \quad (7.1.6)$$

Similarly rms value of AC voltage is given by

$$V = \sqrt{\langle V^2 \rangle} = \sqrt{\langle V_m^2 \sin^2 \omega t \rangle} = \sqrt{I_m^2 R^2 \langle \sin^2 \omega t \rangle} = \sqrt{\frac{1}{2} V_m^2}$$

$$V = \frac{V_m}{\sqrt{2}} = 0.707 V_m = IR \quad (7.1.7)$$

In terms of the RMS value of AC voltage the average power loss is given by

$$\langle P \rangle = I^2 R = VI = \frac{V^2}{R} \quad (7.1.8)$$

7.1.4 What does the value of any AC voltage (or current) mean?

As the instantaneous value of an AC voltage (or current) keeps on changing with time, we cannot use any of the instantaneous values to represent the AC voltage (or current) over a cycle. If we try to take a simple average of these ac voltage (or current) over a cycle, it comes out equal to zero. Therefore we use rms value as the effective value of AC current and voltage that is commonly referred to.

A household supply of 220 V refers to the rms value of AC voltage. Likewise, 15 A refers to the rms value of AC.

7.1.5 Advantages of AC over DC

Most of the electrical power is obtained from alternating sources, as there are several advantages of AC over DC. Some of them are given below:

- It is easier to design an AC generator than a DC generator.

- Alternating voltage can easily be stepped up or stepped down with the help of transformers according to need and requirement.
- Transmission of electrical power at higher AC voltages can reduce transmission loss.
- AC motors are mechanically rugged requiring lesser attention in operation and maintenance.
- Whenever required, AC can be easily converted to DC using rectifiers and filters.

An Illustration, which shows the importance of transporting electrical power at higher voltages

To minimize the I^2R (power loss) in transmission lines, it is economical to use high voltage and low current. The safety and other considerations, however, require that power be delivered to consumers at lower voltages and therefore with higher currents.

Let us take an example of a city with a population of 1,00,000 and average electrical power consumption per person in this city is 1.2 kW.

At 240 V, the current required for each person would be

$$I = P/V = 1200/240 = 5 \text{ A}$$

The total current for 1,00,000 people would then be 5,00,000 A. The transmission of such a huge current from a power plant to a city many kilometers away would require very thick conductors and the I^2R power loss would be huge. Therefore rather than transporting the power at 240 V, step up transformers can be used at the power plant to step up the voltage to some very large value, such as 2,75,000 V.

To transmit a fixed power through a certain resistance, the current needed is calculated as under:

$$P = V_1 I_1 = V_2 I_2 ; \quad I_2 = (V_1/V_2) I_1$$

$$I_2 = (240/275000) \times 500000 = 436.36 \sim 440 \text{ A}$$

Now you can see that the magnitude of this current (440 A) is much smaller than that in the previous case (5,00,000 A). This in turn reduces the line losses.

But this voltage (275000 V) is very dangerous for domestic/industrial use. Therefore it should be reduced to a safer value for transmission within the city. For this purpose, a power substation is made just outside the city, where a step-down transformer converts this very high voltage to a comparatively lower voltage.

Now, let us solve some problems to get an idea of power loss in transmitting electrical power over long distances.



Let Us Reflect

Q.7.1.1 A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of two-wire line carrying power is $0.5 \, \Omega$ per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.

- Estimate the line power loss in the form of heat.
- How much power must the plant supply, assuming there is negligible power loss due to leakage?
- Characterize the step up transformer at the plant.

Q.7.1.2 Repeat the above exercise replacing the earlier transformer by a 40,000-220 V step-down transformer. Neglect leakage loss. Hence, explain why high voltage transmission is preferred?

Q.7.1.3 Compute the average value of the sawtooth wave (Fig. 7.1.10)

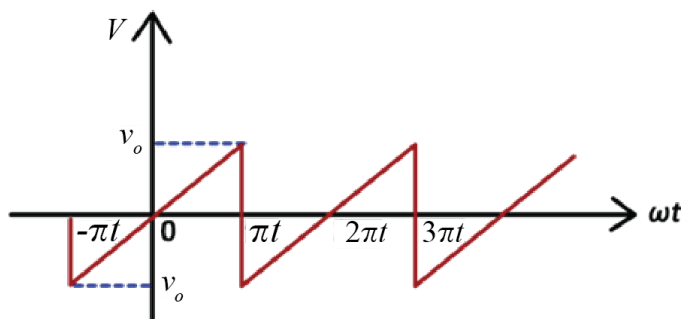


Fig. 7.1.10

Q.7.1.4 An alternating voltage has the equation $V = 141.4 \sin 377 t$ volt.

Compute the value of (i) rms voltage (ii) frequency (iii) instantaneous voltage at $t = 2$ ms (iv) time period.

Q.7.1.5 Give the expression for the instantaneous current for the Fig.7.1.11 shown below.

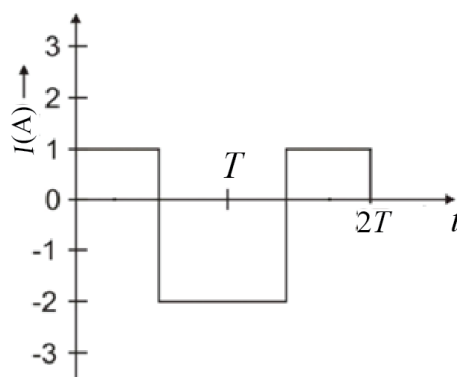


Fig.7.1.11

Q.7.1.6 Give the expression for the instantaneous current in a $22\ \Omega$ resistor connected to 220 V, 50 Hz.



Answers

A.7.1.1 Line resistance $= 2 \times 15 \times 0.5 = 15\ \Omega$.

rms current in the line $= \frac{800 \times 1000\ \text{W}}{4000\ \text{V}} = 200\ \text{A}$

(a) Line power loss $= (200\ \text{A})^2 \times 15\ \Omega = 600\ \text{kW}$

(b) Power to be supplied by the plant $= 800\ \text{kW} + 600\ \text{kW} = 1400\ \text{kW}$

(c) Voltage drop on the line $= 200\ \text{A} \times 15\ \Omega = 3000\ \text{V}$

The step-up transformer at the plant is 440 V – 7000 V

A.7.1.2

rms current in the line $= \frac{800 \times 1000\ \text{W}}{40000\ \text{V}} = 20\ \text{A}$

(a) Line power loss $= (20\ \text{A})^2 \times (15\ \Omega) = 6\ \text{kW}$.

(b) Power to be supplied by the plant $= 800\ \text{kW} + 6\ \text{kW} = 806\ \text{kW}$.

(c) Voltage drop on the line $= 20\ \text{A} \times 15\ \Omega = 300\ \text{V}$.

The step-up transformer required at the plant is 440 V – 40,300 V. It is clear that percentage power loss is greatly reduced by high voltage transmission. In Q.7.1.1 this power loss is $(600/1400) \times 100 = 43\%$.

In this case, it is only $(6/806) \times 100 = 0.74 \%$.

$$\text{A.7.1.3 } V_{av} = \int_{-\pi}^{\pi} \left(\frac{V}{\pi} \right) t dt = \frac{V}{2\pi} t^2 \Big|_{-\pi}^{+\pi} = 0$$

$$\text{A.7.1.4 } V = 141.4 \sin 377 t = V_0 \sin \omega t$$

$$\text{(i) rms voltage } V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100 \text{ V}$$

$$\text{(ii) Frequency } f = \frac{\omega}{2\pi} = \frac{377}{2\pi} = 60 \text{ Hz.}$$

$$\text{(iii) at } t = 2 \text{ ms} = 2 \times 10^{-3} \text{ s}$$

$$V = (141.4) \sin(377 \times 2 \times 10^{-3}) = (141.4) \sin(0.754)$$

$$= (141.4) \sin(43.2^\circ) = (141.4) \times 0.6845$$

$$= 96.6 \text{ V}$$

$$\text{(iv) time period } T = \frac{1}{f} = \frac{1}{60} = 16.66 \text{ ms}$$

$$\text{A.7.1.5 } I = 1 \text{ A for } \frac{-T}{2} \leq t < \frac{T}{2}$$

$$= -2 \text{ A for } \frac{T}{2} \leq t < \frac{3T}{2} \text{ with time period } 2T.$$

$$\text{A.7.1.6 } i = \frac{220\sqrt{2}}{22} \sin 100\pi t$$

$$\therefore i = 100\sqrt{2} \sin 100\pi t \text{ A}$$

7.1.6 How do electrons move in case of alternating current

Generally it is a misconception among students that during the flow of AC, electrons continuously oscillate at their respective places. In fact, the electrons drift towards the positive terminal.

Current is defined as the rate of flow of charges crossing any section, through the conductor. During any half cycle of an AC, the net flow of charges (electrons) across a given cross-section is in a particular direction. During the next half cycle of AC, the polarity gets reversed and this flow is in the opposite direction. For example, for AC of frequency 50 Hz, the time period of one cycle is 0.02 sec. For 0.01 second current flows in one direction and for the next 0.01 second, it flows in the opposite direction.

7.1.7 How to measure AC

We measure DC, generally by its magnetic effect. A needle attached to the coil of an ammeter gets deflected due to magnetic effect of current (i.e. torque experienced by the current carrying coil placed in a magnetic field.)

But in case of AC, direction of current reverses at a very rapid rate. So, the magnetic effect cannot be utilized for measuring AC (since the galvanometer coil will not be able to respond to this rapid change due to its inertia). So, for measuring AC, we use the heating effect of electric current, which is independent of direction of its flow. This heating is termed as Joule's heating and is proportional to the square of the current.

7.1.7.1 Hot wire ammeter or thermal ammeter

A hot wire ammeter consists of a stretched wire whose length depends on the current passing through it. This is because the current produces a heating effect bringing about a change in length of the wire. The instrument is suitably calibrated so that these changes in length can indicate the values of currents causing these changes. The scale would not be a linear one as the heat produced by a current, (in a given resistor) is proportional to the square of the current.

The heating effect, being similar for both AC and DC, a hot wire ammeter can be used to measure both the AC as well as DC.

This instrument is designed to read the root mean square (rms) value of AC directly. The following diagram (Fig.7.1.12) of the instrument explains its working.

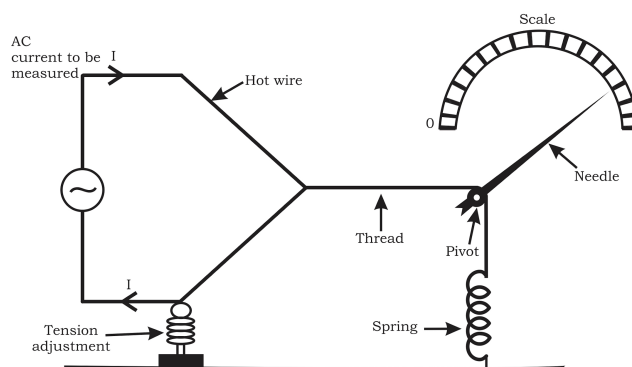


Fig.7.1.12 Instrument to read the root mean square (rms) value of (AC) current directly.

7.1.8 Summary

- An alternating voltage $V = V_m \sin \omega t$ applied to a resistor R drives a current $I = I_m \sin \omega t$ in the resistor, $I_m = V_m/R$. The current is in phase with the applied voltage.
- For an AC current $I = I_m \sin \omega t$ passing through a resistor R , the average power loss P (averaged over a cycle) due to Joule heating is $\langle P \rangle = \frac{1}{2} I_m^2 R$. To express it in the same form as the DC power ($P = I^2 R$), a special value of current is used. It is called Root Mean Square (RMS) current and is denoted by I :

$$I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, the RMS voltage is defined by

$$V = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

We have $P = IV = I^2 R$

- A given value for AC voltage or current is ordinarily the rms value. The voltage across the terminals of an outlet in your room is normally 240V. This refers to the rms value of the voltage. The amplitude of this voltage is $V_m = \sqrt{2}V = \sqrt{2} \times 240 = 340 \text{ V}$
- There are several advantages of AC over DC. Some of them are (i) It is easier to design an AC generator than a DC generator. (ii) Alternating voltage can easily be stepped up or stepped down with

the help of transformers according to need and requirement. (iii)Transmission of electrical power at higher AC voltages can reduce transmission loss. (iv)AC motors are mechanically rugged requiring lesser attention in operation and maintenance. (v)Whenever required, AC can be easily converted to DC using rectifiers and filters.

- Both AC and DC are measured in ampere. AC ampere cannot be defined using the mutual attraction of two parallel wires carrying AC, as the DC ampere is defined, as AC changes direction with the source frequency and the attractive force would average to zero. Thus, the AC ampere must be defined in terms of some property that is independent of the direction of the current. Joule heating is such a property, and there is one ampere of rms value of AC in a circuit if the current produces the same average heating effect as one ampere of DC current would produce under the same conditions.
- The heating effect, being similar for both AC and DC , a hot wire ammeter can be used to measure both the AC as well as DC.

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