

Module 4: Power in AC Circuit

This module is designed to help you to:

- explain the concept of average power in an LCR series circuit;
- establish the relationship between average power, rms current, rms voltage and power factor;
- explain the relationship between resistor, impedance and power factor;
- describe the construction and working of a transformer;
- describe the use of transformers to step up/down AC voltages and its application in the transmission of electric power.
- explain the common type of losses of the transformer and their remedies.

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7.4.1 Introduction

In the previous module, we have introduced the concept of power in an AC circuit. We have utilised the concept for the understanding of quality factor of an AC circuit. Here, we shall study power in AC circuit in little more detail, and understand the meanings of power factor. We shall apply the concept of power in the study of transformers.

You must have seen huge transformers near electric power stations. There are used to step up the supply voltage to very high values before feeding electrical supply to transmission lines. The transformers near our homes are commonly referred to as sub-stations. These step-down transformers lower the transmitted voltage to appropriate values suitable for supply to our homes. The study of transformers provides an important and interesting insight into the basics of power transmission.

7.4.2 Power in AC circuit: The power factor

Now let us examine the power dissipation in an AC circuit. Power dissipation typically refers to the ability of a component, be it resistive, inductive, or capacitive to deliver power for some activity like heating, lighting, motion, cooling, or other forms of energy. The power that is available for useful activity from a component is called its working power and this is expressed in kW or kilowatt. A resistive load will dissipate power for various applications like heating, lighting, etc which is mentioned as power rating on such appliances. An inductive load, like a motor, compressor, or ballast also requires reactive power to generate and sustain a magnetic field in order to operate and this is referred to as non-working or reactive power and is expressed in kVAR or kilovolt-ampere-reactive. Every establishment like a home, office, or a commercial unit like a factory has a combination of resistive and inductive load. As more and more inductive load is added to the establishment, the ratio of the working to the non-working or reactive power becomes important. This is because more inductive load in the circuit will mean more reactive power in comparison to working power and this will make the circuit inefficient in terms of the utilization of power it is receiving. The total working power and reactive power of the circuit together is called its apparent power and is expressed in kVA or kilovolt-ampere. Thus in order to ensure the efficiency of a circuit in terms of the power utilized we find out the power factor of the circuit defined as the ratio of the working power to apparent power. Thus higher the losses the less power is available for our consumption and usage. In order to understand this further let us look at a series *LCR* circuit whose source voltage is given by

$$V = V_m \sin \omega t \quad (7.4.1)$$

and the current flowing in the circuit is given by

$$i = i_m \sin(\omega t + \phi) \quad (7.4.2)$$

where ϕ is the phase difference between the AC supply voltage and the current flowing in the circuit, given by

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right) \quad (7.4.3)$$

Hence, the instantaneous power supplied by the source is given by

$$p = VI = V_m \sin(\omega t) i_m \sin(\omega t + \phi) \quad (7.4.4)$$

$$p = \frac{v_m i_m}{2} [\cos\phi - \cos(2\omega t + \phi)] \quad (7.4.5)$$

Now in order to calculate the average power dissipated over an entire cycle we take the time average of the terms on the RHS. Since only the second term within the bracket on RHS is time dependent, hence one has to take the time average of this term. Since sine and cosine functions of time when averaged over an entire cycle give zero hence this term goes to zero. Therefore the average power dissipated is given by

$$P = \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos\phi = V_{RMS} I_{RMS} \cos\phi \quad (7.4.6)$$

where V_{RMS} and I_{RMS} are the RMS value of voltage and current. Here $\cos\phi$ is called the power factor of the circuit. We have used Eq (7.4.5) to determine the ‘average’. However, we can also use Eq(7.4.4) to obtain the ‘average’ and it gives a unique insight into the connection between physics and mathematics. From Eq (7.4.4), we get

$$p = V_m i_m (\sin^2 \omega t \cos\phi + \frac{1}{2} \sin 2\omega t \sin\phi).$$

We know that $\sin^2 \omega t = \frac{1}{2}$ and $\sin 2\omega t = 0$. So, $\bar{p} = \frac{1}{2} V_m i_m \cos\phi$. The first term accounts for the dissipation through the resistor, while the second term for the exchange of energy between the reactances and the generator.

The average power dissipated can also be written in terms of the total impedance Z as

$$P = I_{RMS}^2 Z \cos\phi \quad (7.4.7)$$

Let us look at some specific cases to understand the role of power factor in average power dissipation.

(i) Resistive circuit: In case of a purely resistive circuit the impedance is purely resistive and ϕ is zero therefore $\cos\phi=1$ (Fig. 7.4.1) and $P = I_{RMS}^2 Z \cos\phi = I_{RMS}^2 Z$

This has maximum power dissipation.

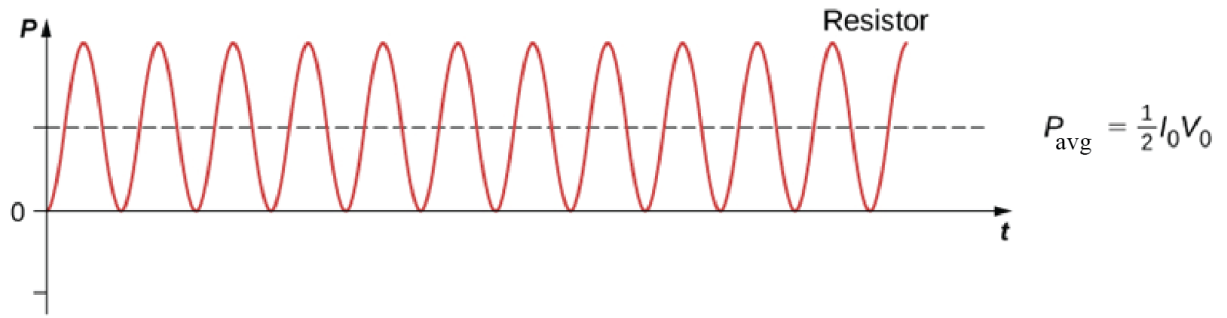


Fig. 7.4.1 Time variation of power in a resistive circuit

(ii) Purely inductive or capacitive circuit: In case of a purely inductive or capacitive circuit $\phi = \pi/2$ with the current lagging or leading the voltage by this phase difference, respectively. Hence $\cos\phi = 0$ (Fig. 7.4.2 and Fig 7.4.2) and the average power dissipation is minimum i.e. zero and such current flowing in the circuit is called wattless current.

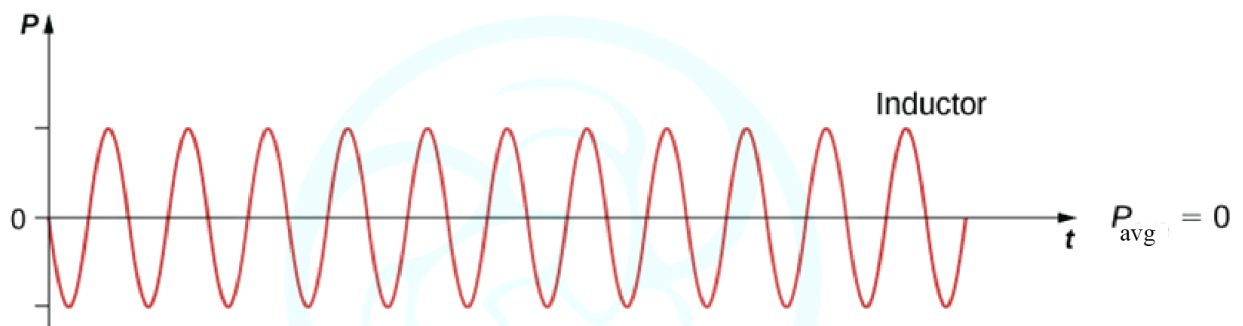


Fig. 7.4.2 Time variation of power in an inductive circuit

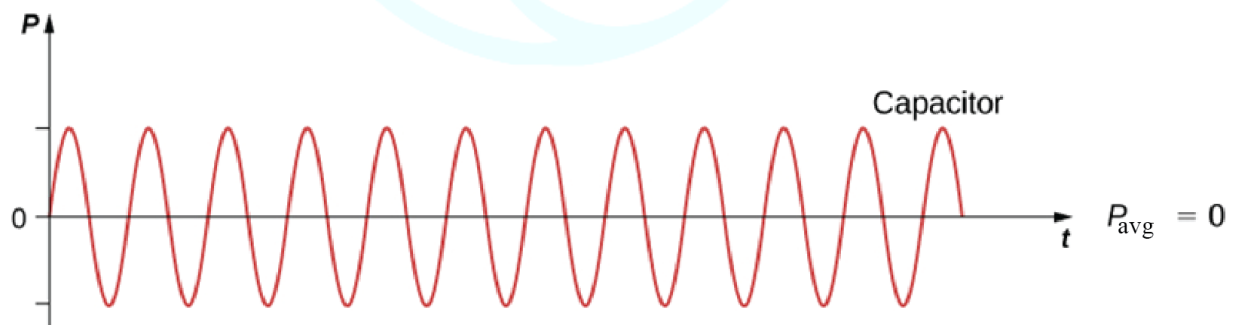


Fig. 7.4.3 Time variation of power in a capacitive circuit

(iii) LCR Series circuit: Here ϕ may be non-zero for a RL , RC , or RLC circuit and the average power dissipation is given by the above equation with the power dissipation occurring only through the resistor (Fig 7.4.4).

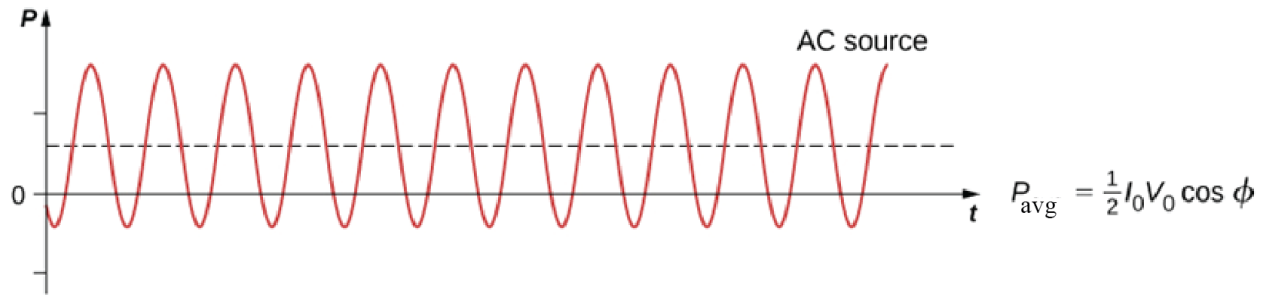


Fig. 7.4.4 Time variation of power in a LCR circuit

(iv) At resonance in *LCR* series circuit: Here $X_c - X_L = 0$ hence ϕ is zero therefore $\cos\phi=1$ and

$$P = I_{RMS}^2 Z \cos\phi = I_{RMS}^2 Z \quad (7.4.8)$$

and there is maximum power dissipation with the power getting dissipated only through the resistor.

Let us look at a specific application of this concept. We have seen above that for a *LR* circuit the power factor shall not be zero and as the inductive reactance load is increased the value of ϕ tends to $\pi/2$ or the power factor tends to zero which means that the circuit will not be very efficient with the efficiency going down as load reactance increases in comparison to the resistive load. In order to compensate for this we can add a capacitive reactance load which will offset the inductance reactance load and for these two reactance loads being equal $\phi=0$ and the power factor is unity. This can be understood from Fig 7.4.5 below . Here we consider the circuit to be predominantly inductive with the voltage leading the current by a phase difference ϕ .

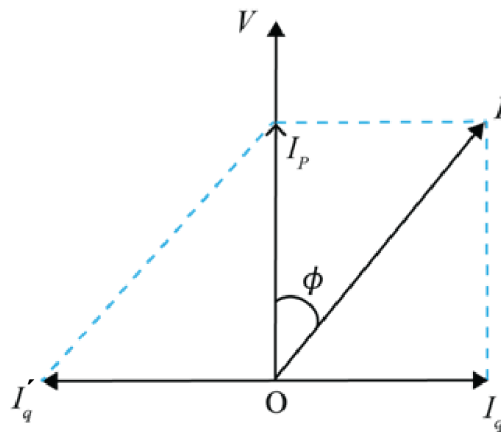


Fig.7.4.5 Phsor diagram: improving Power factor by use of a capacitor. Power component I_p is in phase with V ; I_q and I'_q cancel each other and P is effectively I_p

When phase difference ϕ has values lying between zero and $\pi/2$ then the current flowing through the circuit can be resolved into two components one parallel to the voltage, I_p and other which is perpendicular to the voltage, I_q . For the perpendicular component, the average power dissipated is zero as $\phi = \pi/2$. I_q is referred to as wattless current as there is no power dissipation due to this current in the circuit. While I_p is known as the power component as it is in phase with the source voltage. Now in order to improve the power factor the wattless current I_q must be neutralized and for doing this we add a capacitor of capacitive reactance load in parallel to the circuit, which sets up a leading wattless current I'_q flowing in the opposite direction and equal in magnitude to I_q . Thus these two wattless currents cancel each other and the power factor is unity and in this manner, the power factor is improved by adding power factor correction capacitors.

7.4.3 Transformers

Transformers are devices, which transform supply or source AC voltage to a higher or lower value using the principle of mutual induction. They have two sets of coils insulated from each other wound on a soft iron core in two different arrangements (a) one on top of the other with insulation in between and (b) on separate arms of the transformer (Fig 7.4.6).

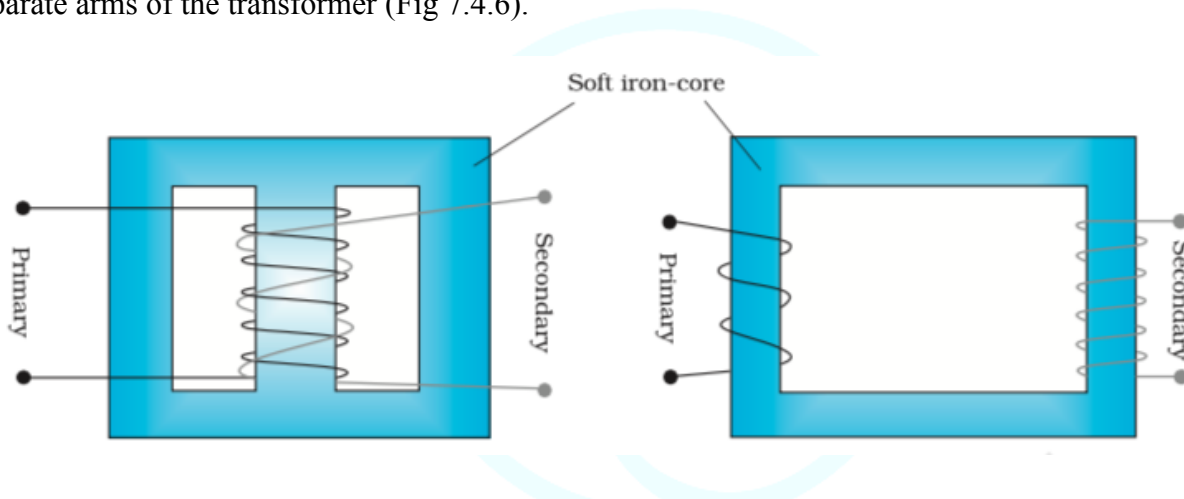


Fig. 7.4.6 Two arrangements for winding of primary and secondary coil in a transformer: (a) two coils on top of each other; (b) two coils on separate limbs of the core

The coil connected to the source or supply voltage is generally referred to as the primary coil while the other coil which is connected to the output is referred to as the secondary coil. The operation of stepping up or down of voltage is dependent on the number of turns on the coil in the primary (N_p) and secondary (N_s), respectively. If $N_s > N_p$ then the transformer is a step-up transformer and if $N_p > N_s$ then it is a step-down transformer. Normally, step-up transformers are used near the power stations for stepping up the voltage for supplying to the transmission lines in order to minimize transmission losses ($I_{RMS}^2 R$). Step-down transformers are used near our homes, offices or business and commercial establishments to step down the voltage of the transmission lines to a value, which can be supplied for commercial or

domestic use. The mechanism of step-up and step-down transformers can be understood from the treatment given below.

Fig 7.4.7 shows a step-up transformer. An AC voltage is applied to the primary coil of the transformer producing a current in it, the current in the primary coil induces an EMF in the secondary coil due to an alternating magnetic flux, which gets linked to the secondary coil. Here we assume that the transformer is an ideal transformer such that resistance in the primary coil is negligible and all the alternating magnetic flux in the core links both primary and secondary coil windings with negligible flux being lost.

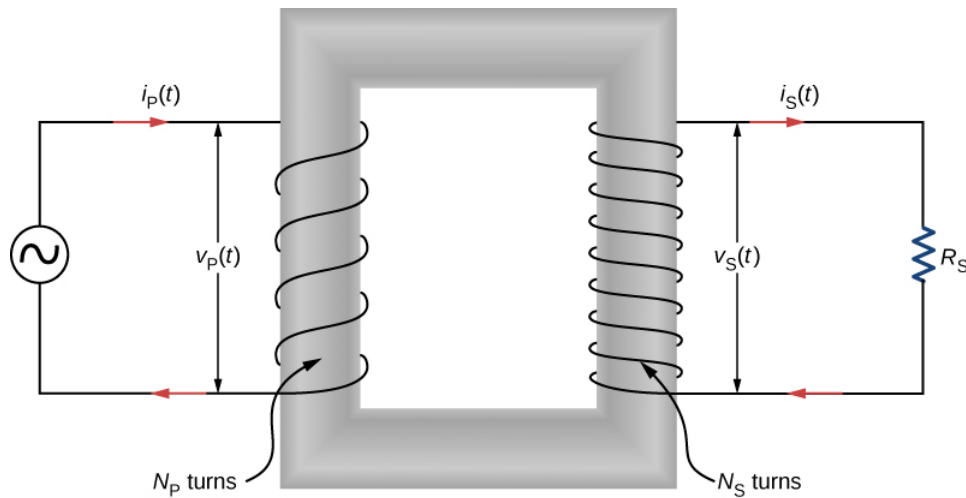


Fig. 7.4.7 A step-up transformer

If ϕ is the instantaneous alternating magnetic flux linked to each turn in the core due to current flowing in the primary and v_p is the AC supply or source voltage then the induced EMF in the secondary coil is given by

$$\varepsilon_s = -N_s \frac{d\phi}{dt} \quad (7.4.9)$$

Further the same flux also induces an EMF in the primary coil of the transformer called the back EMF given by

$$\varepsilon_p = -N_p \frac{d\phi}{dt} \quad (7.4.10)$$

Since we have neglected the resistance in the primary coil hence

$$\varepsilon_p = v_p$$

$$\Rightarrow v_p = -N_p \frac{d\phi}{dt} \quad (7.4.11)$$

else the current in the primary coil will be infinite. Further, if the secondary coil is an open circuit or the current drawn from it is negligible then to a good approximation

$$\begin{aligned}\varepsilon_s &= v_s \\ \Rightarrow v_s &= -N_s \frac{d\phi}{dt}\end{aligned}\quad (7.4.12)$$

where v_s is the voltage appearing across the secondary available as output voltage.

Thus comparing the two equations we have

$$\frac{v_s}{v_p} = \frac{N_s}{N_p} \quad (7.4.13)$$

Assuming the transformer to be 100% efficient i.e. no energy losses, we have that input power, p_p should be equal to the output power, p_s

$$\begin{aligned}p_p &= p_s \\ \Rightarrow v_p i_p &= v_s i_s \\ \Rightarrow \frac{i_p}{i_s} &= \frac{v_s}{v_p} = \frac{N_s}{N_p}\end{aligned}\quad (7.4.14)$$

If I_{RMS} and V_{RMS} are the RMS values of the voltage and current respectively and the fact that both the voltage and current oscillate with the same frequency as the AC source for both primary and secondary then

$$V_{S\,RMS} = \left(\frac{N_s}{N_p}\right) V_{P\,RMS}$$

$$I_{S\,RMS} = \left(\frac{N_s}{N_p}\right) I_{P\,RMS}$$

These equations show that if the number of turns on the secondary coil is greater than that of the primary coil, i.e. $N_s > N_p$, then the voltage is stepped up and if the number of turns on primary coil is greater than secondary coil i.e. $N_p > N_s$, then the voltage is stepped down. In case of current, this gets reversed with current in the secondary coil decreasing for a step-up transformer and increasing for a step-down transformer.

Real world transformers may show some losses as assumptions we have made above in case of an ideal transformer may not be fully satisfied. In such cases, the common losses that are encountered and their remedy are listed below.

- (i) Flux leakage: Since the core may have air gaps or other design flaws hence not all of the flux due to primary passes through the secondary resulting in flux leakage. Winding the primary and secondary coils one over the other can help in reducing flux leakage.
- (ii) Resistance of the windings: The assumption that the wires used for the winding have no resistance does not hold true even for the best of conductors and there will be some energy loss due to heat produced in the wire (I^2R). One can minimize these losses in high current, low voltage windings by using thick wire.
- (iii) Eddy currents: The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by using a laminated core.
- (iv) Hysteresis: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material, which has a low hysteresis loss.



Let Us Reflect

Q.7.4.1 An AC circuit consists of the series combination of resistance of $1\text{ k}\Omega$, an inductance of 1 H and a capacitance of $10\text{ }\mu\text{F}$ connected to an AC source of frequency 50 Hz . Calculate power factor of this circuit. What would be the power factor if the resistance is removed (i.e made zero)?

Q.7.4.2 A transmission line carries 11000 volts which has to be brought down to 440 volts . If primary coil has 1000 turns , how many coils should there be in secondary?



Answers

A.7.4.1 Power factor = $\cos \phi$ where $\phi = \tan^{-1} \left(\frac{\omega L - \frac{1}{\omega C}}{R} \right)$

$$= \tan^{-1} \left(\frac{-4.15}{10^3} \right)$$

$$\phi \approx -4.15 \times 10^{-3}$$

$$\begin{aligned}\therefore \text{Power factor} &= \cos \phi = \cos [-0.00415] \\ &= 0.9999 \\ &\approx 1\end{aligned}$$

If $R = 0$, $\phi = \tan^{-1}(\infty) = \frac{\pi}{2}$

$$\therefore \text{Power factor} = \cos \frac{\pi}{2} = 0$$

This means this circuit does not consume any power. Actually power is consumed only when energy dissipates across resistance in the form of heat. Inductor and capacitor ideally do not consume any power.

A.7.4.2 $\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{440}{11000}$

$$\therefore N_s = \frac{N_p}{2S} = \frac{1000}{2S} = 40$$

7.4.4 Summary

- If V_{RMS} and I_{RMS} are rms value of voltage current in an AC circuit , then average power dissipated is $P = V_{RMS} \times I_{RMS} \cdot \cos\phi$
where $\cos\phi$ is a power factor
(i) in a purely resistive circuit $\phi = 0$ and $\cos\phi = 1$
(ii) purely inductive or capacitive circuit : $R = 0$, $\phi = 90^\circ$ and $\cos\phi = 0$
Thus means no power dissipated in such a circuit.
(iii) in general $\cos\phi$ decides what fraction of power available is actually dissipating in a circuit.

- In case of transformers , relation between voltage across secondary coil and primary coil is given by
$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Where N_s and N_p are the number of turns of secondary and primary coil respectively.

- Transformers may show some losses as assumptions we have made in case of an ideal transformer may not be fully satisfied. These are due to (i) Flux leakage (ii) Resistance of the windings (iii) Eddy currents (iv) Hysteresis.
- If the transformer is 100 % efficient, that is there are no energy losses, the input power would be equal to the output power, and we have,

$$\frac{i_p}{i_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

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