

3.Current Electricity

Module 1 : Kirchhoff's Laws

This module is designed to help you to:

- establish relations between emf, potential difference and currents in various resistive network;
- discuss Kirchhoff's laws governing electric circuits;
- apply Kirchhoff's laws to solve problems involving emfs, potential difference and currents in a resistive network.

Content

3.1.1 Introduction

3.1.2 Potential difference across a resistor

3.1.3 Potential difference across a cell

3.1.4 Potential difference across a combination of resistances and cells

3.1.5 Kirchhoff's laws

3.1.6 Steps involved in solving a problem using Kirchhoff's laws

3.1.7 Summary



3.1.1. Introduction

In general, an electric circuit has a number of resistances and cells interconnected in a complicated way. It is not easy to figure out which resistors are connected in parallel or series. A similar situation arises for different cells used. It is not convenient to reduce the circuit to one where Ohm's law can be applied to obtain (i) current and (ii) voltage; in different branches of the circuit. Kirchhoff's laws are one of the convenient ways to analyze these types of circuits.

3.1.2. Potential difference across a resistor

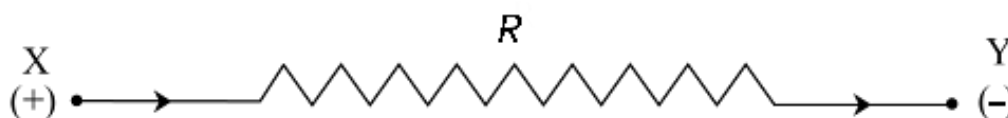


Fig. 3.1.1 Resistance R carrying a current I

Fig (3.1.1) shows a resistance R carrying a current I . From Ohm's law, potential difference (V) across resistance equals IR . The end X of the resistor is at a higher potential than the end Y. Therefore,

$$V_X - V_Y = IR$$

ΔV = The difference of potential across the ends of R as one moves from X to Y

$$= V_Y - V_X = - (IR)$$

The change in potential across the ends of R is, therefore, assigned a negative sign. The negative sign indicates moving from a point (X) at a higher potential to another point (Y) at a lower potential.

This implies that the change in potential, across a resistor, would be assigned a negative sign whenever the path is being traversed along the direction of flow of the (conventional) current through it.

3.1.3. Potential difference across a cell

Case(i)

Consider a cell of e.m.f. (E) and internal resistance r . Fig 3.1.2(a) shows current I being drawn from the cell.

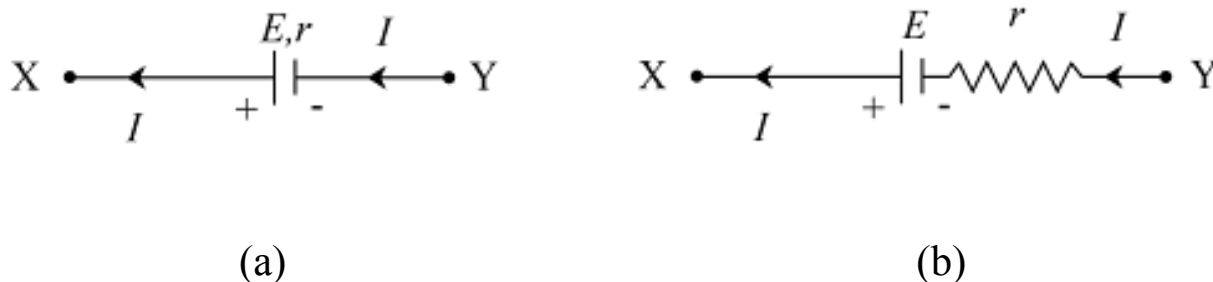


Fig. 3.1.2 Current I being drawn from the cell.

Fig.3.1.2 (b) shows e.m.f. (E) and internal resistance r separately. The p.d. (V) across the ends X and Y of the cell as we move from Y to X is

$$V = V_X - V_Y = E - Ir$$

The end X is at a higher potential than the end Y. The change in potential in moving from end X to end Y of cell is

$$V_Y - V_X = -(E - Ir)$$

The negative sign has the same significance as in a resistor, i.e. in traversing a cell from point X to Y, one moves from a point at a higher potential to a point at lower potential. The cell shown has a p.d. (V) of magnitude less than its e.m.f. E .

Case (ii)

Fig 3.1.3 shows the above considered cell, getting a current (I). This corresponds to a cell being charged.

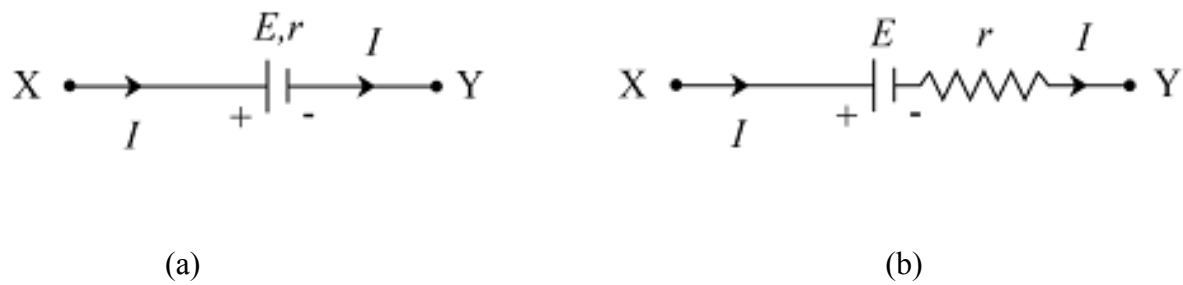


Fig. 3.1.3 A cell getting charged

The p.d. (V) of cell is

$$V = V_X - V_Y = + (E + Ir)$$

Note p.d. (V) of the cell is now more than its e.m.f. (E). Note the change in potential difference in moving from end X to end Y of cell is ($V_Y - V_X$) and is a negative number.

3.1.4. Potential difference across a combination of resistances and cells

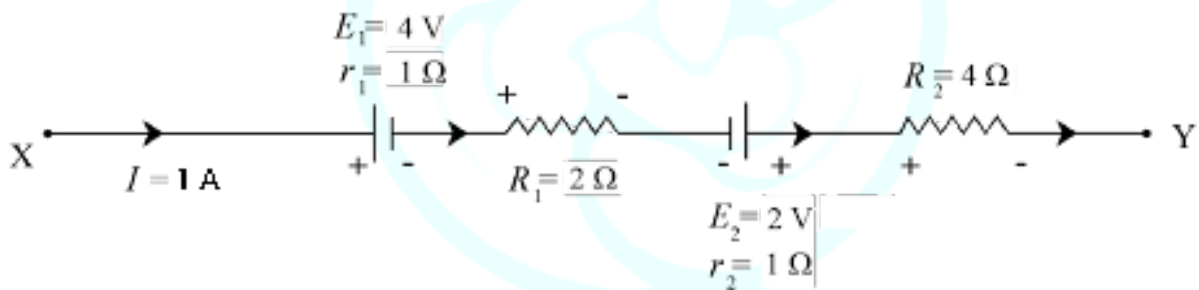


Fig. 3.1.4 Potential difference across a combination of resistances and cells

Fig.3.1.4 shows an arrangement of two cells E_1 and E_2 and external resistances R_1 and R_2 .

To calculate $V_X - V_Y$; we calculate p.d. across each element (cell or resistance) and add them algebraically.

We have

$$(\Delta V)_1 = \text{P. d. across cell } E_1 = E_1 + Ir_1 = 4 + 1 \times 1 = + 5\text{ V}$$

$$(\Delta V)_2 = \text{P. d. across resistance } R_1 = 2 \times 1 = 2\text{ V}$$

$$(\Delta V)_3 = \text{P. d. across cell } E_2 = -E_2 + Ir_2 = -2 + 1 \times 1 = -1 \text{ V}$$

$$(\Delta V)_4 = \text{P. d. across resistance } R_4 = 4 \times 1 = 4 \text{ V}$$

$$\therefore \Delta V = \text{Total P.d. between points X and Y} = V_X - V_Y$$

$$= (\Delta V)_1 + (\Delta V)_2 + (\Delta V)_3 + (\Delta V)_4$$

$$= 5 + (2) + (-1) + (4) = 10 \text{ V}$$

Positive sign of ΔV indicates that point Y is at a lower potential than point X.

The above calculations give a difference of potential between points X and Y. The value of potential assigned to different points on the network will DEPEND on the choice of point of reference on the network. Fig 3.1.5 shows the arrangement with point b earthed (i.e. assigned potential zero. Obviously $V_b = 0$. To obtain potential V_x at point X and V_y at point Y, we proceed as follows:

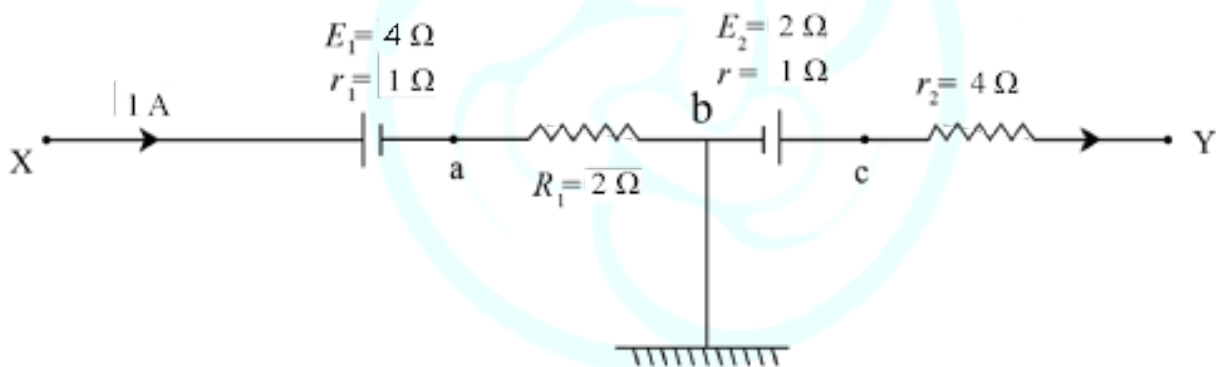


Fig.3.1.5 An electric circuit when one point of it is earthed

$$(\Delta V)_3 = V_c - V_b = 1 \quad \therefore V_c = +1 \text{ V} \quad [\because V_b = 0]$$

$$(\Delta V)_4 = V_y - V_c = -4 \quad \therefore V_y = -4 + V_c = -3 \text{ V}$$

$$(\Delta V)_2 = V_b - V_a = -2$$

$$\therefore V_a = +2 \text{ V}$$

$$\text{and } (\Delta V)_1 = V_a - V_x = -5$$

$$\therefore V_x = 5 + 2 = +7 \text{ V}$$

$$\text{Therefore, } V_x - V_y = 7 - (-3) = 10 \text{ V}$$

Repeat calculations by earthing point a; i.e. $V_a = 0$. The numerical value of the potential at different points will change, but the potential difference across every element remains the same as calculated using Fig.3.1.5. This is because potential is not uniquely defined; however, the difference of potential is uniquely defined; irrespective of the choice and the value of potential assigned to the reference point.

3.1.5. Kirchhoff's laws

Kirchhoff gave two laws (or rules).

(1) Junction rule (or law of current) and (2) Loop rule or (law of voltage)

We now state and explain the rules one by one.

(1) Junction rule (Or law of current)

At any junction (or branch point) in a network, the algebraic sum of currents is zero. This rule is a restatement of the law of conservation of charge. For steady currents, there is no accumulation of charge at

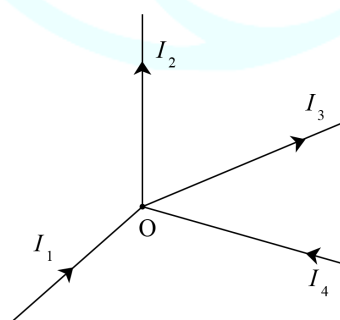


Fig.3.1.6 A junction in a network

any junction (or branch point). The total amount of charge entering a junction equals the total leaving the junction, at any moment of time. Commonly the current entering a branch point is assigned a positive sign (+) and the current leaving the branch point is assigned a negative sign (-).

For currents at junction point O (Fig 3.1.6); I_1 and I_4 are positive whereas I_2 and I_3 are negative. In accordance with the junction rule,

$$I_1 + (-I_2) + (-I_3) + I_4 = 0$$

$$\text{or } I_1 + I_4 = I_2 + I_3$$

We can restate the junction rule as, the sum of currents entering any junction point equals the sum of currents leaving the junction point.

Example 3.1.1

Fig 3.1.7 shows a part of a network. What is I_1 and I_2 ?

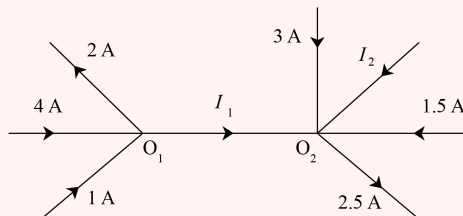


Fig. 3.1.7

Solution: Using Kirchhoff's junction rule, we have

(1) For junction O_1

$$4 + 1 + (-2) + (-I_1) = 0$$

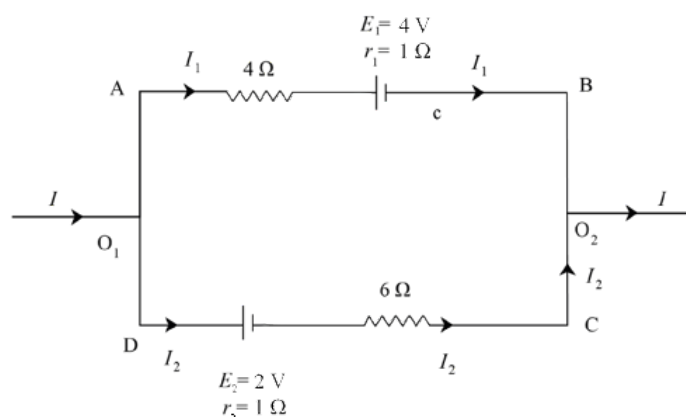
$$\therefore I_1 = 3 \text{ A}$$

(2) For junction O_2

$$3 + I_1 + I_2 + 1.5 = 2.5$$

$$\therefore I_2 = 2.5 - (4.5 + 3) = -5 \text{ A}$$

Negative sign of I_2 indicates that the actual direction of current I_2 is opposite to what is shown in Fig 3.1.7.

(2) The loop rule (or law of voltages)**Fig. 3.1.8** *The loop rule*

The algebraic sum of changes in potential around any closed loop involving resistors and cells (in the loop) is zero.

The rule is obvious; since electric potential depends on the location of the point. Starting from any point and coming back to the same point, the net change in potential must be zero. In a closed loop, we come back to the starting point, and hence the rule. The loop rule is a restatement of the law of conservation of energy.

To illustrate the use of the junction law consider closed loop ABCDA; shown in Fig 3.1.8.

Assigning appropriate + or – sign to the changes in p.d. across the resistance or battery involved; we get

$$-4I_1 + (-4 - I_1) + 6I_2 + (2 + I_2) = 0 \quad \text{(The second and fourth terms are p.d. across the cells)}$$

$$\text{Or } -5I_1 + 7I_2 - 2 = 0$$

$$\therefore -5I_1 + 7I_2 = 2$$

3.1.6 Steps involved in solving a problem using Kirchhoff's laws

The following sequence of steps may help in solving a problem:

- (1) Assign direction to currents in different branches of the given network.
- (2) Current flows in 'one direction' in a part of the loop, having resistors; batteries; etc. connected in series.

- (3) Reduce the number of variables (unknown currents) using the "junction rule"
- (4) On calculating the value of current in a branch, if we get a negative sign, then the direction of current is opposite to what was assigned to it earlier.
- (5) Consider a convenient closed loop. Apply the loop rule, forming an equation involving parameters of the loop.
- (6) The number of loops considered depends on the number of unknown parameters.
- (7) Choose loops giving independent equations. We should have the same number of equations as the number of unknown parameters.
- (8) Solve the equations using knowledge of algebra.

There can be different ways of assigning current directions in different loops. However, the final results obtained are always the same. The parameters of loops do not depend on how one forms or solves the different equations involved.

Example 3.1.2 In the circuit shown in Fig 3.1.9. What is current in $10\ \Omega$ resistance?

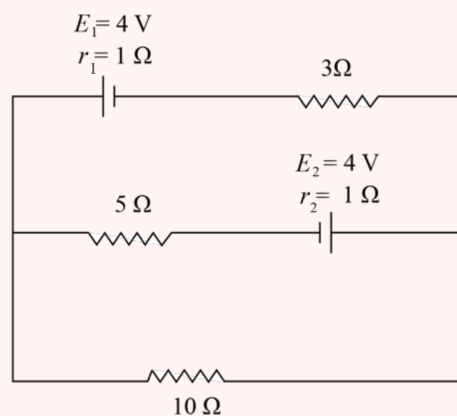


Fig. 3.1.9

Solution:

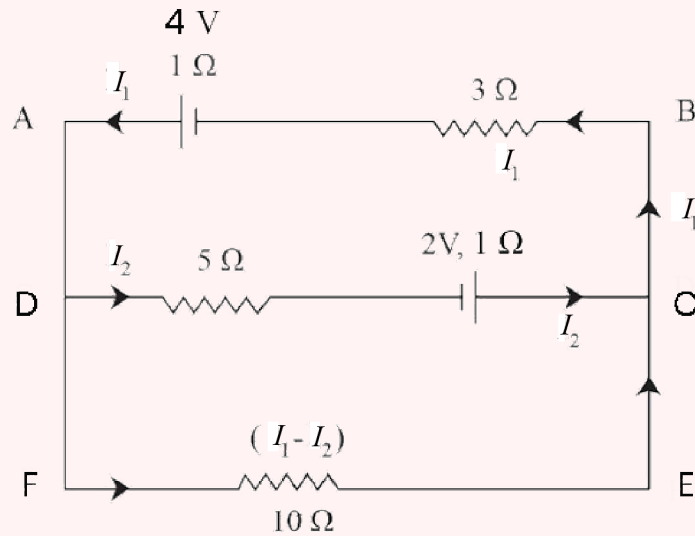


Fig. 3.1.10

The current distribution in circuit, in accordance with Kirchhoff's current law, is shown in Fig 3.1.10.

Applying Kirchhoff's loop law to

(1) loop ABCDA

$$(-4 + I_1) + 3I_1 + (-2 + I_2) + 5I_2 = 0$$

$$\text{or} \quad 2I_1 + 3I_2 = 3 \quad (3.1.1)$$

(2) loop DCEFD

$$-5I_2 + (2 - I_2) + 10(I_1 - I_2) = 0$$

$$\text{or} \quad -10I_1 + 16I_2 = 2 \quad (3.1.2)$$

Solving Eqns (3.1.1) and (3.1.2) for I_1 and I_2 ; we have

$$I_1 = \frac{21}{31} \text{ A}, I_2 = \frac{17}{31} \text{ A},$$

$$\text{Current in } 10 \Omega = I_1 - I_2 = \frac{4}{31} \text{ A}$$



Let Us Reflect

Q.3.1.1 Discuss (i) Kirchhoff's junction law in the light of charge conservation.

(ii) Ohm's law in the light of energy conservation.

Q.3.1.2 What happens if we force the loop in the opposite direction in applying the Kirchhoff loop law?



Answers

A.3.1.1

(i) consider the junction as shown in the Fig.3.1.11. here based on direction of current we can see that three currents i_1 , i_2 and i_4 are arriving at the junction and current i_3 and i_5 are leaving the junction.

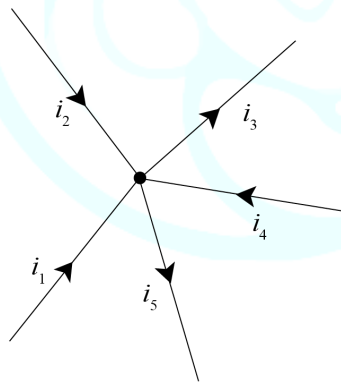


Fig.3.1.11

As no charge accumulates or originates at the junction, the total charge arriving at the junction, must be the total charge leaving the junction in a given time (say 1 s) therefore in the case $i_1 + i_2 + i_4 = i_3 + i_5$

(ii) consider a simple series circuit as shown in the Fig.3.1.12. As per Ohm's law $E = iR$. The charge flowing through this circuit constitutes current

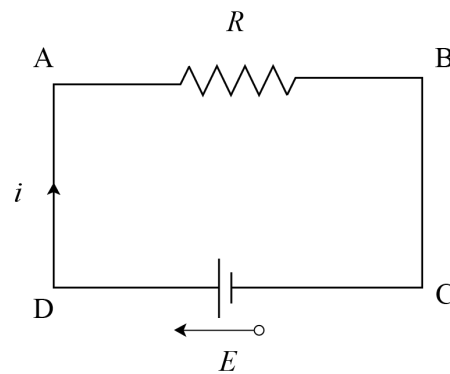


Fig.3.1.12

This charge when flows through the battery from the negative terminal to positive terminal, work is done by battery in raising the charge through potential difference E . if charge q flows through in time t , then work done by battery $= qE$ when $(q = it)$ This works shown up as energy dissipated across resistance $= i^2 R t$ from Joule's law.

According to conservation of energy $qE = (it) \cdot E = i^2 R t$ or $E = iR$, which is Ohm's law.

A.3.1.2 consider a simple potential divider circuit as shown in adjacent Fig.3.1.13(a) and (b)

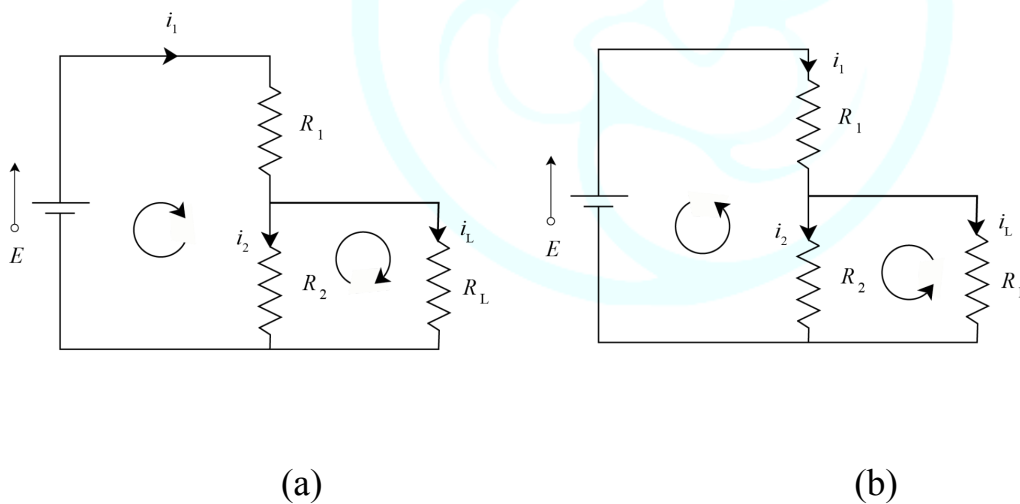


Fig. 3.1.13

In Fig. 3.1.3 (a) we are tracing both the loops clockwise and in Fig. 3.1.13(b) we are tracing them anti-clockwise

In case (a) we get eqn 1st loop $E = i_1 R_1 - i_2 R_2 = 0$

$$\text{or } E = i_1 R_1 + i_2 R_2 = 0 \quad (3.1.3)$$

$$\text{and 2nd loop } E = i_2 R_2 - i_L R_L = 0$$

$$\text{or } i_2 R_2 = i_L R_L \quad (3.1.4)$$

In case of (b) we get Eqn 1st loop $-E + i_2 R_2 + i_1 R_1 = 0$

$$\text{or } E = i_1 R_1 + i_2 R_2 \text{ same as (3.1.3)}$$

$$\text{2nd loop } -i_2 R_2 + i_L R_L = 0$$

$$\text{or } i_2 R_2 = i_L R_L \text{ same as (3.1.4)}$$

Thus, irrespective of the sense of traveling the loop, we get the same equation.

Note that it is important to trace the entire loop in the same sense.

Example 3.1.3 In the circuit shown in Fig 3.1.14 current in the galvanometer of resistance G is same when key K is either open or closed. What is the value of G in terms of P, Q, R ?

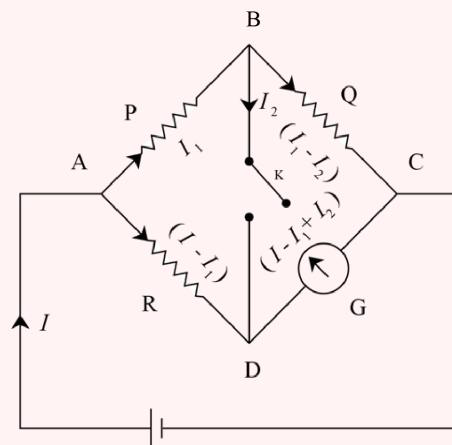


Fig. 3.1.14

Solution: Fig 3.1.14 shows the current distribution in the circuit in accordance with Kirchhoff's current law. Applying Kirchhoff's loop law to

(i) Loop ABDA

$$-PI_1 + R(I - I_1) = 0$$

$$(P+R)I_1 - RI = 0 \quad (3.1.5)$$

(ii) Loop BDCB

$$-(I - I_1 + I_2)G + (I_1 - I_2)Q = 0$$

$$GI - (G+Q)I_1 + (G+Q)I_2 = 0 \quad (3.1.6)$$

Deflection in galvanometer G remains same whether key K is open or closed if $I_2 = 0$

From Eq.(3.1.6) we have

$$GI - (G+Q)I_1 = 0 \quad (3.1.7)$$

Multiply Eqn(3.1.5) by $(G+Q)$ and Eqn (3.1.7) by $(P+R)$ we have

$$(P+R)(G+Q)I_1 = (G+Q)RI \text{ and}$$

$$(P+R)(G+Q)I_1 = G(P+R)I,$$

Therefore,

$$(G+Q)R = G(P+R)$$

$$\text{or } QR = GP$$

$$\therefore G = \left(\frac{R}{P}\right)Q$$

Example 3.1.4 In the given circuit AB is a uniform resistance wire of total length L ; resistance per unit length λ . Calculate current in galvanometer of resistance G .

Solution:

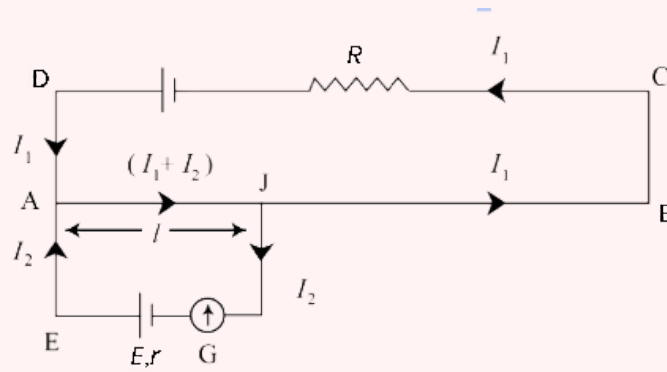


Fig. 3.1.15

Fig 3.1.5 shows current distribution in the circuit. Using Kirchhoff's loop law for

1) loop ABCDA (length of the segment AJ is l)

$$- \lambda l (I_1 + I_2) - \lambda(L - l)I_1 - I_1 R + V = 0$$

$$(\lambda L + R)I_1 + \lambda l I_2 = V \quad (3.1.8)$$

(ii) loop AJGEA

$$- (\lambda l) (I_1 + I_2) - I_2 G - I_2 r + E = 0 \quad c \quad (3.1.9)$$

Multiply Eq(3.1.8) by λl and Eq(3.1.) by $(\lambda L + R)$

$$(\lambda L + R) \lambda l I_1 + (\lambda L) (\lambda l) I_2 = (\lambda l) V \text{ and}$$

$$I_2 [\lambda^2 l^2 - (\lambda L + r)(\lambda l + G + r)] = \lambda l V - E (\lambda L + R)$$

$$I_2 = \frac{\lambda l V - E (\lambda L + R)}{\lambda^2 l^2 - (\lambda L + r)(\lambda l + G + r)}$$

if $I_2 = 0$ we have

$$E = \frac{\lambda l V}{\lambda L + R}, I_1 = \frac{V}{\lambda L + R}.$$

Example 3.1.5 In circuit shown in Fig.3.1.16, two cells of emfs E_1 , E_2 and internal resistances r_1 and r_2 , respectively are connected in parallel. What is (i) the e.m.f. and (ii) internal resistance of the equivalent cell of E_1 and E_2 ?

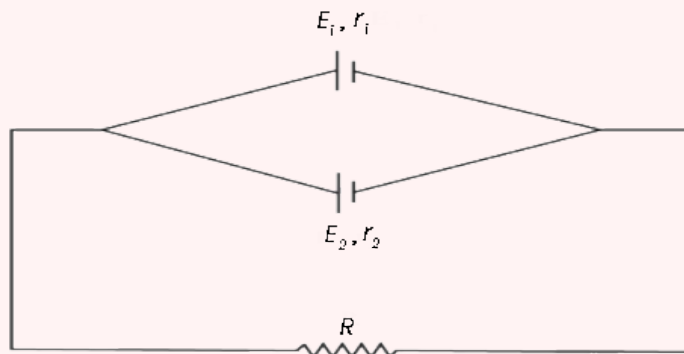


Fig.3.1.16

Solution:

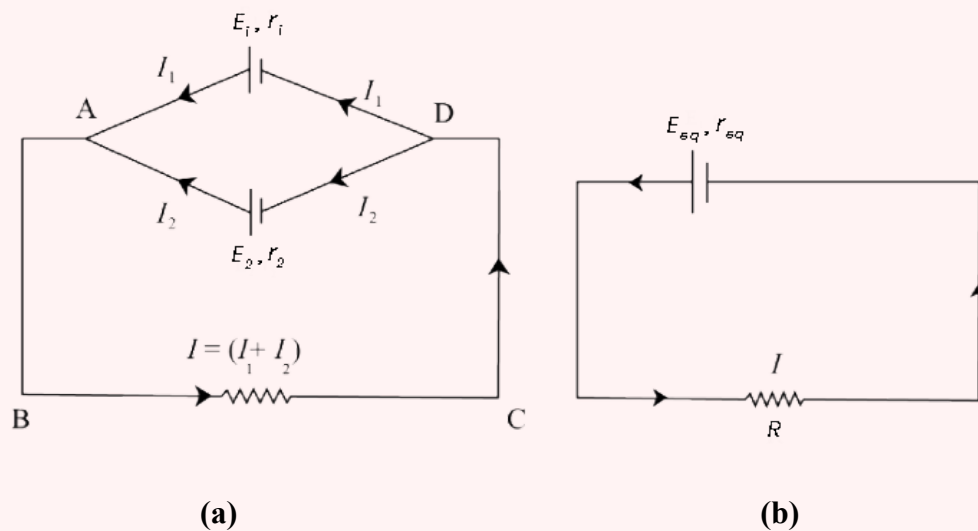


Fig. 3.1.17 Shows current distribution in the given arrangement

(a) Applying Kurchhoff's loop law to

(i) loop ABCDE₁A

$$-(I_1 + I_2)R - I_1 r_1 + E_1 = 0$$

$$I_1(R + r_1) + I_2 R = E_1 \quad (3.1.10)$$

(ii) loop ABCDE₂A;

$$-(I_1 + I_2)R - I_2 r_2 + E_2 = 0$$

$$\text{or, } I_1 R + I_2(R + r_2) = E_2 \quad (3.1.11)$$

Multiply Eq(3.1.10) by R and Eq(3.1.11) by $(R+r_1)$ and subtract. We get

$$I_2 = \frac{(E_2 - E_1)R + E_2 r_1}{R(r_1 + r_2) + r_1 r_2} \quad (3.1.12)$$

Similarly,

$$I_1 = \frac{(E_1 - E_2)R + E_1 r_2}{R(r_1 + r_2) + r_1 r_2} \quad (3.1.13)$$

Therefore current (I) in external resistance (R) is

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{(E_1 r_2 + E_2 r_1)}{R(r_1 + r_2) + r_1 r_2} \\ &= \frac{(E_1 r_2 + E_2 r_1) / (r_1 + r_2)}{R + \left(\frac{r_1 r_2}{r_1 + r_2} \right)} \end{aligned} \quad (3.1.14)$$

Using the relation for equivalent resistance of resistance in parallel; the equivalent internal resistance (r_{eq}) of the two cells; is

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Let E_{eq} be the equivalent e.m.f of the two cells. From Fig (3.1.14)

$$I = \frac{E_{eq}}{R + r_{eq}} \quad (3.1.15)$$

Comparing Eqn(3.1.14) and Eq(3.1.15); we have

$$E_{eq} = \frac{E_2 r_1 + E_1 r_2}{r_1 + r_2} \quad (3.1.16)$$

Eq(3.1.16) can be generalized for, n cells connected in parallel as

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \dots + \frac{E_n}{r_n} \quad \text{and} \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_n}.$$

3.1.7 Summary

- When a source of emf ε is connected to an external resistance R , the voltage V_{ext} across R is given by

$$V_{ext} = IR = \frac{E}{R + r} R$$

where r is the internal resistance of source.

- An electric circuit, in general, has a number of resistances and cells interconnected in a complicated way. It is not convenient to reduce the circuit to one where Ohm's law can be applied to obtain (i) current and (ii) voltage; in different branches of the circuit. Kirchhoff's laws are one of the convenient ways to analyze these types of circuits.
- Kirchhoff's junction rule: Algebraic sum of current at a junction is zero. i.e. Total current coming towards the junction is equal to total current going away from the junction.
- The sign convention we adopt is : current coming towards the junction taken with (+ve sign) and current leaving the junction taken with (-ve sign).

- Kirchhoff's loop rule: Algebraic sum of potential difference in a closed loop is zero.
- The sign convention we adopt is: a change in potential across a resistor would be assigned a negative sign, whenever the path is being traversed along the direction of flow of conventional current through it, and if we traverse the path from a negative terminal to positive terminal through the battery, then emf should be taken with a positive sign.



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