

3.Current Electricity

Module 2 : Wheatstone Bridge

This module is designed to help you to:

- describe the construction/ arrangement of Wheatstone bridge and metre bridge and their working;
- apply Kirchhoff's rules for determining the condition for balanced Wheatstone bridge ;
- analyze the factors affecting the sensitivity of Wheatstone bridge;
- apply the concept of Wheatstone bridge and metre bridge to solve problems in novel situations.

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3.2.1 Introduction

Wheatstone bridge is an arrangement having four resistors and a galvanometer connected in a particular manner. A source (d.c. battery) is connected across the arrangement. Using Kirchhoff's laws the condition for balance of the bridge is obtained. The balance condition of the bridge is used to measure (experimentally) an unknown resistance.

3.2.2 Wheatstone bridge

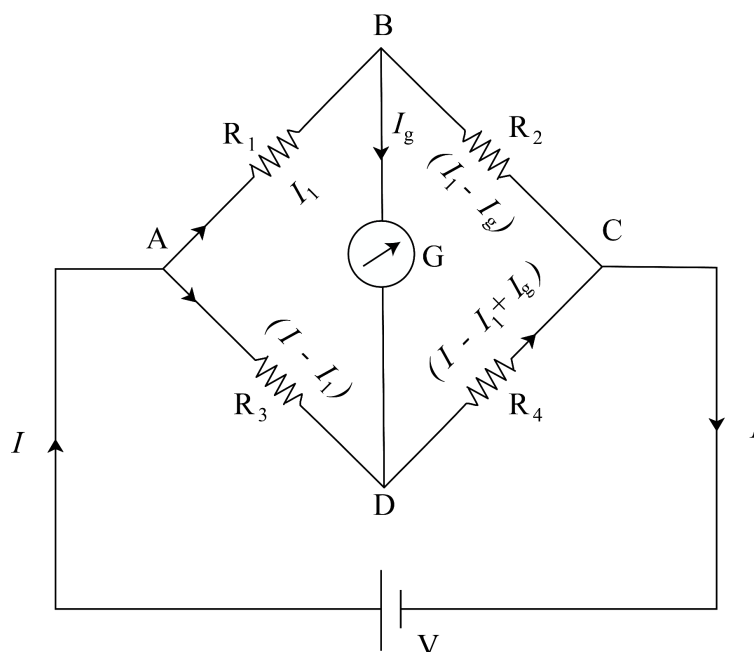


Fig.3.2.1: *Wheatstone bridge*

Fig. 3.2.1 Shows the Wheatstone's bridge. The bridge has four resistors R_1 , R_2 , R_3 and R_4 connected as shown. AB; BC; AD and CD are the four arms of the bridge. The galvanometer of resistance (G) is connected between the two vertices B and D. A battery of potential difference V is connected across diagonally opposite points A and C.

Using Kirchhoff's laws, expression for current in galvanometer (I_g) is calculated. The current distribution, in accordance with Kirchhoff's current law, is shown in Fig 3.2.1. Using Kirchhoff's loop law; for

- (i) loop ABDA

$$\begin{aligned}
 -R_1 I_1 - I_g G + R_3 (I - I_1) &= 0 \\
 \text{or } R_3 I - (R_1 + R_3) I_1 - I_g G &= 0
 \end{aligned}
 \tag{3.2.1}$$

(ii) loop BCDB

$$\begin{aligned}
 -R_2 (I_1 - I_g) + R_4 (I - I_1 + I_g) + G I_g &= 0 \\
 \text{or } R_4 I - (R_2 + R_4) I_1 + (R_2 + R_4 + G) I_g &= 0
 \end{aligned}
 \tag{3.2.2}$$

Multiply Eqn (3.2.1) by $(R_2 + R_4)$ and Eqn (3.2.2) by $(R_1 + R_3)$ and subtract. We get

$$I_g [G(R_2 + R_4) + (R_1 + R_3)(R_2 + R_4 + G)] = I [R_3(R_2 + R_4) - R_4(R_1 + R_3)]$$

$$I_g = I \frac{(R_2 R_3 - R_1 R_4)}{G(R_2 + R_4) + (R_1 + R_3)(R_2 + R_4 + G)} \tag{3.2.3}$$

For Eq(3.2.3); it is obvious that I_g can be positive, negative or zero depending on numerator in Eqn (3.2.3). For I_g to be positive; $R_2 R_3 > R_1 R_4$; current in the galvanometer is same direction as in Fig.3.2.1. If $R_2 R_3 < R_1 R_4$. The direction of current in the galvanometer is opposite to the one shown in Fig 3.2.1. The bridge is balanced, When $I_g = 0$. Obviously, the balance condition is

$$\begin{aligned}
 R_2 R_3 - R_1 R_4 &= 0 \\
 \frac{R_1}{R_2} &= \frac{R_3}{R_4}
 \end{aligned}
 \tag{3.2.4}$$

The Wheatstone bridge and its balance condition provide a practical method for the determination of an unknown resistance. Let R_4 be the unknown resistance. Keeping known resistances R_1 and R_2 in the first (AB) and the second (BC) arm of the bridge resistance; R_3 in the third arm (AD) is varied, till there is no deflection in the galvanometer. The bridge is now balanced. From Eq(3.2.4);

$$R_4 = \frac{R_3 R_2}{R_1} \tag{3.2.5}$$

Knowing R_1 , R_2 , and R_3 ; the unknown resistance R_4 is calculated. The condition of balance does not change the position if the battery and the galvanometer are interchanged.

3.2.3. Sensitivity of a Wheatstone bridge

To measure an unknown resistance R_4 ; knowing R_1 and R_2 ; R_3 is varied to obtain the balance of the bridge. The balance of the bridge is observed using a sensitive galvanometer. Let the exact value of R_3 for balance be $(R_3)_T$. Practically when R_3 is slightly more or less than its exact value, there should be an observable deflection in the galvanometer. However, depending on the current sensitivity I_s of the galvanometer; a current less than I_s cannot be observed. The Wheatstone bridge is sensitive if a very small change in R_3 from its true value; $(R_3)_T$; produces an observable deflection in the galvanometer. It can be shown that the bridge is most sensitive for a given galvanometer; if all the four arms of the bridge have resistances of the same order of magnitude.

Example 3.2.1 In the Wheatstone bridge shown in Fig 3.2.2; $R_1 = 5\ \Omega$, $R_2 = 10\ \Omega$, $G = 30\ \Omega$, R_4 is unknown resistance. At balance $R_3 = 10\ \Omega$. What is R_4 ? What is equivalent resistance; at balance; between points A and C?

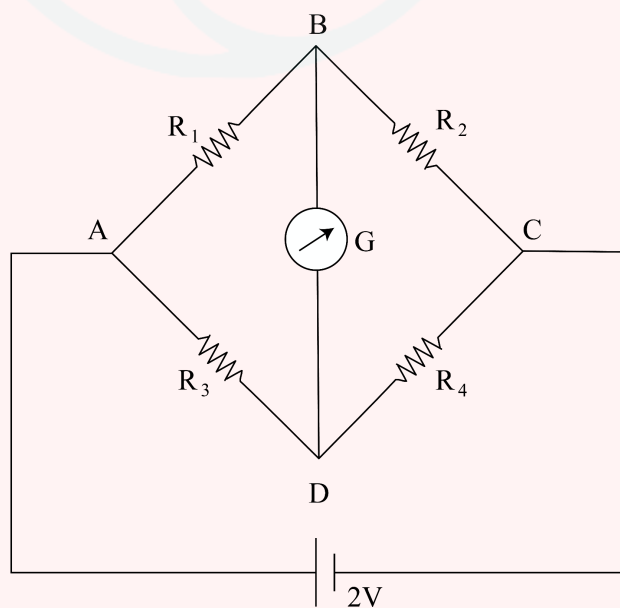


Fig 3.2.2

Solution :

At balance

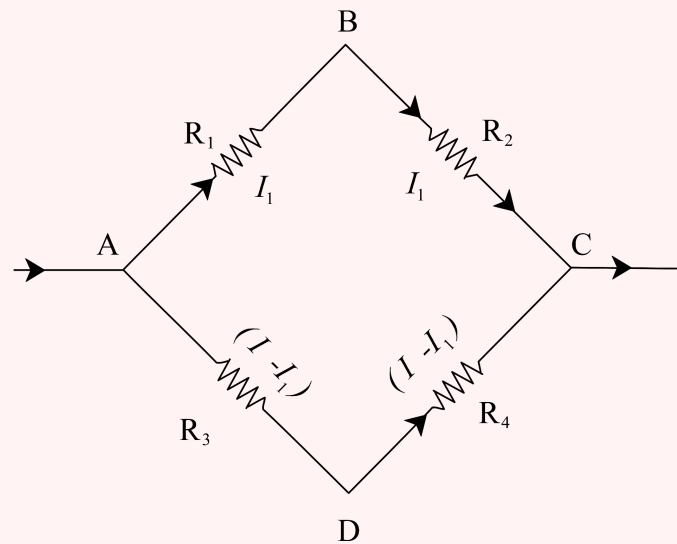


Fig. 3.2.3

$$R_4 = \frac{R_2 R_3}{R_1}$$

Substituting given values

$$R_4 = \frac{10 \times 10}{5} = 20 \, \Omega$$

At balance there is no current in the arm BD. We can redraw the bridge; removing the galvanometer, in arm BD; as shown in Fig 3.2.3.

The current distribution in the balanced Wheatstone bridge is shown in Fig 3.2.3. Obviously (R_1 and R_2) as well as, (R_3 and R_4) are in series with one another. $R' = R_1 + R_2 = 5 + 10 = 15 \, \Omega$ and $R'' = R_3 + R_4 = 10 + 20 = 30 \, \Omega$ are joined in parallel with one another. The equivalent resistance, R_{eq} , between A and C is

$$R_{eq} = \frac{15 \times 30}{15 + 30} = 10 \, \Omega$$

Example 3.2.2

(1) Is the Wheatstone bridge shown in Fig 3.2.4 balanced ? Is point B at higher or lower potential at point D ?

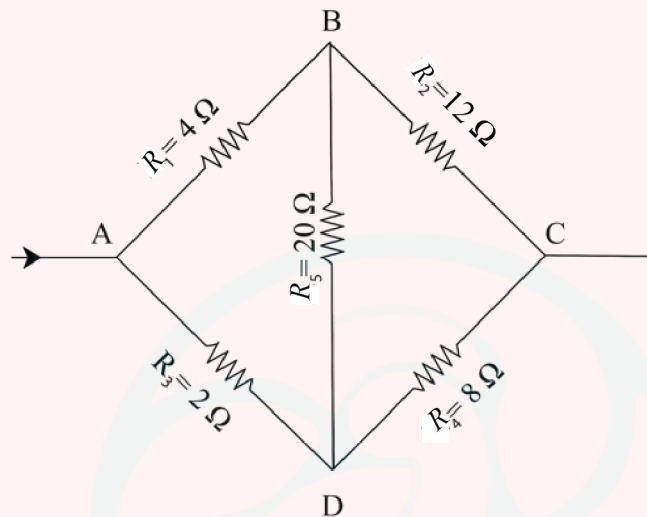


Fig 3.2.4

(b) How can you balance the bridge using one additional resistance in

- (i) arm AD;
- (ii) arm CD

Solution:- Given

$$\frac{R_1}{R_2} = \frac{4}{12} = \frac{1}{3} \text{ and } \frac{R_3}{R_4} = \frac{2}{8} = \frac{1}{4}$$

Since $\frac{R_1}{R_2} \neq \frac{R_3}{R_4}$; the given bridge is unbalanced. Also $R_2 R_3 = 12 \times 2 = 24$ and $R_1 R_4 = 4 \times 8 = 32$. Since $R_2 R_3 < R_1 R_4$; current in arm BD flows from D to B [Eqn (3.2.5)]. Therefore point B is at a lower potential than point D.

(b) (i).

$$\frac{R_1}{R_2} = \frac{R_3 + x}{R_4} ; R_3 + x = \left(\frac{8}{3}\right) \Omega ; x = \left(\frac{2}{3}\right) \Omega$$

Resistance in arm AD is increased to $\left(\frac{8}{3}\right) \Omega$. This is so if resistance $\left(\frac{2}{3}\right) \Omega$ is connected in series in arm AD.

now, $\frac{R_1}{R_2} = \frac{1}{3}$ and $\frac{R_3 + x}{R_4} = \frac{8}{3 \times 8} = \frac{1}{3}$

Hence, the bridge is balanced if an additional resistance $\left(\frac{2}{3}\right) \Omega$ is connected in series in arm AD.

(ii)
$$\frac{R_1}{R_2} = \frac{R_3}{\frac{1}{R_4} + \frac{1}{x}} = \frac{R_3 (R_4 + x)}{R_4 x} ; x = 24 \Omega$$

Resistance in arm CD is reduced to 6Ω . This is so if an external resistance of 24Ω is connected in parallel across R_4 .

now,

$$\frac{R_1}{R_2} = \frac{1}{3} \text{ and } \frac{R_3}{\frac{1}{R_4} + \frac{1}{x}} = \frac{1}{3}$$

Hence, the bridge is balanced if an additional resistance 24Ω is connected in parallel in arm CD.

Example 3.2.3

Fig 3.2.5 shows an unbalanced Wheatstone bridge. What is

- current drawn from the battery?
- potential difference between points B and D?

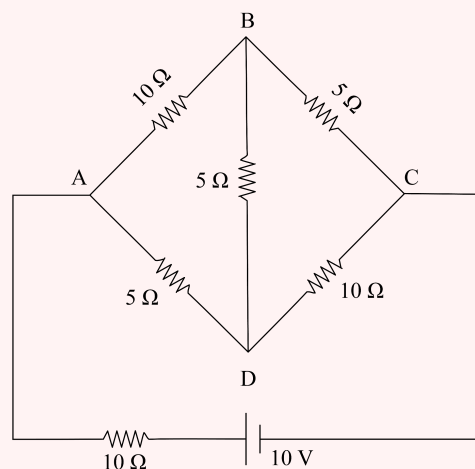
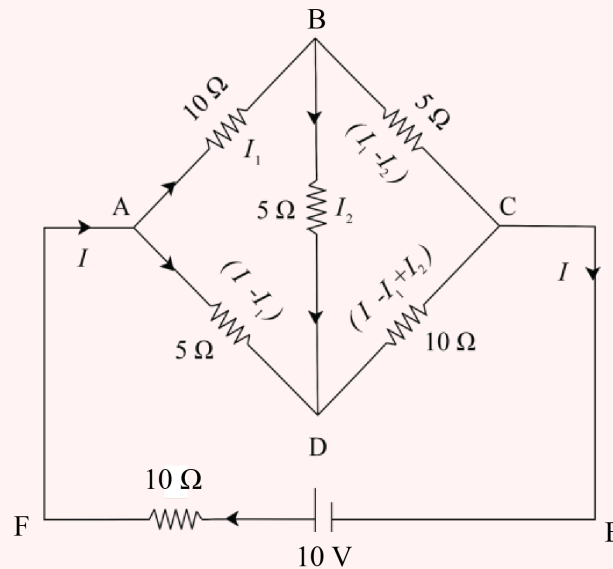


Fig. 3.2.5

Solution:

Fig. 3.2.5 shows the current distribution in accordance with Kirchhoff's current law. Using Kirchhoff's loop law for


Fig.3.2.6

- (1) loop ABDA
 $-10I_1 - 5I_2 + 5(I - I_1) = 0$
or $5I - 15I_1 - 5I_2 = 0$ (3.2.6)
- (2) loop BCDB
 $-5(I_1 - I_2) + 10(I - I_1 + I_2) + 5I_2 = 0$
or $10I - 15I_1 + 20I_2 = 0$ (3.2.7)
- (3) loop ABCEFA
 $-10I_1 - 5(I_1 - I_2) + 10 - 10I = 0$
or $10I + 15I_1 - 5I_2 = 10$ (3.2.8)

Subtract Eq(3.2.6) from Eq(3.2.7). We have

$$5I + 25I_2 = 0$$

$$\text{or } I = -5I_2$$
(3.2.9)

Adding Eq(3.2.6) and Eq(3.2.8); We get,

$$15I - 10I_2 = 10$$

$$\text{or } 15I + 2I = 10$$

$$\therefore I = \left(\frac{10}{17}\right) \text{ A} \quad (3.2.10)$$

$$\text{and } I_2 = -\left(\frac{2}{17}\right) \text{ A} \quad (3.2.11)$$

Current drawn from battery $I = \left(\frac{10}{17}\right) \text{ A}$

Potential difference between point B and D = $-\left(\frac{2}{17}\right) \times 5 = -\left(\frac{10}{17}\right) \text{ V}$

The negative sign shows that B is at a lower potential than D.

3.2.4 Metre bridge

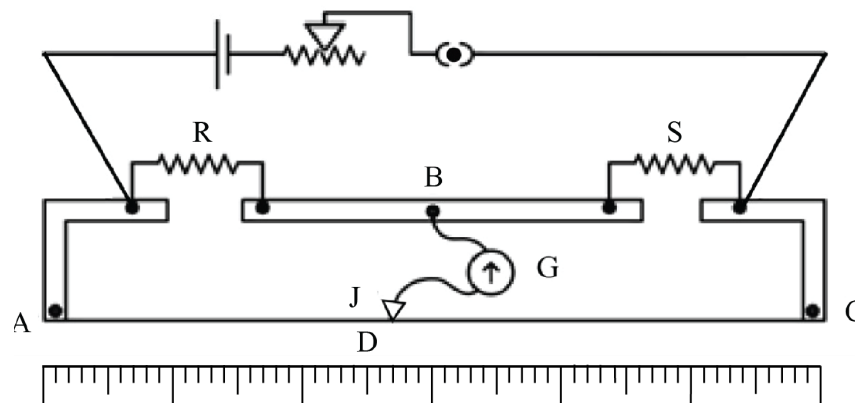


Fig 3.2.7 Metre bridge

An important practical use of Wheatstone bridge is the metre bridge. An unknown resistance can be conveniently measured using the metre bridge. The bridge consists of a uniform, resistance wire AC; 1m long; stretched taut and clamped at its ends on a wooden board. Thick metallic strips bent at right angles are fixed on the wooden board as shown in Fig 3.2.7. There are two gaps across which resistors can be connected. One end of the galvanometer G is connected to the metallic strip, mid-way between the two gaps as shown in Fig 3.2.7. The other end of G is connected

to a jockey (J). A jockey is essentially a metallic rod having a sharp knife edge at one end which can slide over the resistance wire (AC).

R is an unknown resistance connected across one gap. The other gap has a standard, known value resistance S connected across it. Making contact with jockey (J) at point D; we have Wheatstone bridge connection. The four arms AB, BC, AD, and CD are marked in Fig. 3.2.7. Changing the slider contact point D, changes the resistance of the two arms of the bridge. The jockey (J) is moved on wire AC, to balance the bridge. Let $AJ=l$ be the balancing length measured from end A of the bridge.

Let λ be the uniform resistance per unit length of the meter bridge wire. At balance

$R_1 = \text{resistance of length } AJ \text{ of meter bridge wire} = \lambda l$

$R_2 = \text{resistance of length } CJ \text{ (} 100 - l \text{) of meter bridge wire} = \lambda(100 - l)$

when the bridge is balanced,

$$\frac{R_1}{R_2} = \frac{R}{S}$$

$$\text{or, } R = S \left[\frac{l}{100-l} \right] \quad (3.2.12)$$

Knowing all the parameters involved, R is calculated using Eq(3.2.12). Changing the value of S , different balancing lengths can be observed and the value of R can be calculated for each observation. Rearranging Eq(3.2.12) as

$$S = R \left[\frac{100}{l} - 1 \right] \quad (3.2.13)$$

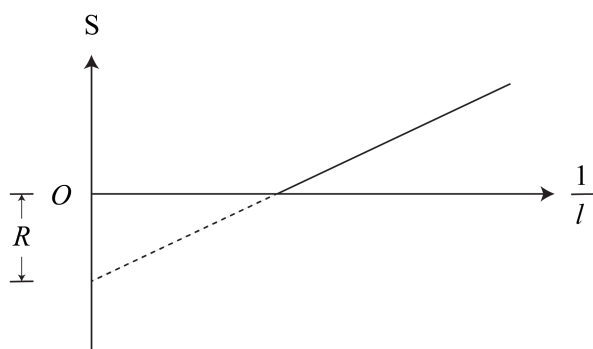


Fig 3.2.8 Graph of S vs $\left(\frac{1}{l}\right)$

Graph of S vs $\left(\frac{1}{l}\right)$ is a straight line. The intercept on the negative S axis equals unknown

resistance. The error in R is minimized by suitably choosing the value of S ; so as to have a balancing point near the middle of the wire.

There is some error in R due to an end error in the metre bridge. The end error is a small, unknown resistance at ends A and C of the bridge wire where the wire is soldered to a copper strip. We have not taken into account the end error at the two ends of the bridge.

Example 3.2.4

In a metre bridge set-up an unknown resistance R is connected across the left-hand gap and a known resistance $S=3\ \Omega$ is connected across the right-hand gap. At balance jockey divides meter bridge wire into two parts having lengths in a ratio of 2:3. What is R ? R and S are interchanged in their respective gaps. What is the % change in balancing length?

Solution:

Let l and $(100 - l)$ be the length of the two parts of the bridge wire at balance. Given

$$\frac{l}{100-l} = \frac{2}{3}$$

$$\therefore l = 40\text{ cm}$$

Using the balance condition; we have

$$\frac{R}{S} = \frac{l}{100-l}$$

or

$$\frac{R}{3} = \frac{40}{60}$$

$$R = 2\ \Omega$$

Let l' be the balancing length when R and S are interchanged. Then

$$\frac{S}{R} = \frac{l'}{100-l'}$$

$$\text{or } \frac{3}{2} = \frac{l'}{100-l'}$$

$$\text{or } l' = 60 \text{ cm}$$

$$\text{Percentage change in balancing length} = \left(\frac{l' - l}{l} \right)$$

$$= \left(\frac{60 - 40}{40} \right) \times 100 = + 50\%$$

Example 3.2.5

Refer to Fig 3.2.7; the metre bridge is balanced when jockey is at mid-point of the bridge wire. An additional resistance of 1Ω is connected in series with R . The balancing length is 20% more than its earlier value.

- (a) What are the values of R and S ? (b) If 1Ω resistance is connected in parallel with R , what will be the balancing length (in cm)?

Solution :

$$l_1 = \text{The balancing length} = 50 \text{ cm}$$

Using the balance condition;

$$\frac{R}{S} = \frac{50}{(100-50)} = 1 \quad (3.2.16)$$

When the resistance of 1Ω is connected in series with R their equivalent resistance $R_1 = R + 1$.

Let l_1 be the balancing length now. Given,

$$l_1 = 50 + \frac{20(50)}{100} = 60 \text{ cm.}$$

Using the balance condition

$$\frac{R+1}{S} = \frac{60}{(100-60)} = \frac{3}{2} \quad (3.2.17)$$

From Eqs (3.2.16) and (3.2.17); we have

$$\frac{R+1}{R} = \frac{3}{2}$$

$$\text{or } R = S = 2 \, \Omega$$

(b) Connecting 1 Ω resistance in parallel with R ; the equivalent resistance R_2 is

$$R_2 = \frac{R \times 1}{R+1} = \frac{2 \times 1}{2+1} = \frac{2}{3}$$

Let l_2 be the balancing length now. At balance

$$\frac{R_2}{S} = \frac{l_2}{(100-l_2)}$$

$$\text{or } \frac{2/3}{2} = \frac{l_2}{(100-l_2)}$$

$$\therefore l_2 = 25 \text{ cm}$$



Let Us Reflect

Q.3.2.1 While working with a balanced Wheatstone Bridge for calculation of current or effective resistance, why do we remove galvanometer or diagonal branch resistance?

Q.3.2.2 The standard resistance in a meter bridge is 40 ohms corresponding two resistance, R_1 and R_2 the balance points are obtained at length 25 cm and 60 cm respectively. What is equivalent resistance when R_1 and R_2 are connected in parallel.



Answers

A.3.2.1 When the bridge is balanced, no current flows through the diagonal branch. This is as if the diagonal branch is open. Therefore, we can remove the diagonal branch without compromising the accuracy of any calculation.

$$\text{A.3.2.2} \quad \frac{R_1}{40} = \frac{25}{75}, \quad R_1 = \frac{40}{3}$$

$$\frac{R_2}{40} = \frac{60}{40}, \quad R_2 = 60$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{\left(\frac{40}{3}\right)(60)}{\frac{40}{3} + 60} = \frac{40}{\frac{2}{3} + 3} = \frac{120}{11}$$

$$\therefore R_{eq} = 10\frac{10}{11} \text{ ohm}$$

3.2.5 Summary

- Wheatstone Bridge in an arrangement of four resistors, a galvanometer and a battery, R_1 , R_2 , R_3 , R_4 and G denote the four resistors and resistance of galvanometer respectively. The current in the galvanometer (I_g) is directly proportional to $(R_2 R_3 - R_1 R_4)$.
- I_g is positive if $R_2 R_3 > R_1 R_4$. Referring to Fig. 3.2.1 point B is at higher potential than point D.
- I_g is negative if $R_2 R_3 < R_1 R_4$. Referring to Fig 3.2.1; Point B is at a lower potential than point D.
- The bridge is balanced when $I_g = 0$. At balance

$$R_2 R_3 = R_1 R_4 \quad \text{or} \quad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

This is the balance condition of the Wheatstone Bridge.

- Interchanging the battery and galvanometer does not affect the condition of balance.
- Wheatstone bridge is most sensitive when the resistance of all the four arms is of the same order of magnitude.
- The metre bridge, based on Wheatstone Bridge, is a convenient arrangement to measure an unknown resistance.
- The metre bridge has a uniform resistance wire fixed on a wooden board. The ends of the wire are attached to thick copper strips bent at right angle. There are two gaps across which resistance can be connected.
- R and S denote resistance connected across the left-hand and right gap on the meter bridge respectively. l denotes the balancing length on meter bridge wire. From balance condition;
$$\frac{R}{S} = \frac{l}{(100-l)}$$
- An unknown small resistance gets introduced at the two ends of the bridge; where the resistance wire is joined to copper strips. This is not accounted for and is referred to as end error.

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