

2. Electrostatic Potential and Capacitance

Module 1: Electrostatic Potential; Potential Energy of a System of Charges

This module is designed to help you to:

- explain the meaning and significance of the concept of electric potential;
- justify why do we define a (scalar) potential only for conservative fields;
- derive the relation between electric potential and electric field;
- use the principle of superposition to calculate the electric potential due to
 - (i) a collection of point charges
 - (ii) a continuous charge distribution;
- describe the meaning of the term potential gradient;
- describe the concept of equipotential surfaces and their characteristics/properties;
- explain the meaning of the potential energy associated with a general charge distribution;
- calculate the potential energy of a system of point charges.

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2.1.1 Introduction

In the previous chapter we have learnt about the basic concepts of the force between the two static electric charges thereby we arrive at the concepts of the electric field and potential. We shall extend the concept further in this module particularly by falling back on the conservative nature of electrostatic fields. We shall use the very important relation that the electric field is the negative gradient of electric potential. The above relation will help us to exploit the advantage of electric potential being a scalar field. For solving problems on the electric field, we shall first determine the potential and take its derivative to calculate the electric field.

Equipotential surfaces form a very useful concept in electrostatics, which we shall study in this module and will explore the connection between the geometry of such surfaces and electric field lines. We shall revisit the principle of superposition for obtaining the values of electrostatic potential due to a system of charges.

2.1.2 Potential

The concept of potential provides us with a relatively simple method of calculating electric fields. Potential is a scalar function; and this, by its very nature, makes it more convenient to handle mathematical operations than a vector function (involving vector additions or vector integration).

The concept of potential owes its origin to the conservative nature of the electrostatic field. The electrostatic field, being conservative, the work done, in bringing a given charge, from a standard reference point, to any point (P) in the field, would depend only on the locations of point P and the standard reference point. The work done is path independent because of the conservative nature of the electrostatic field. The location of the standard reference point (usually taken as a very far off point or as the point at infinity) being fixed, we can think of the work done as being a function only of the location of the point P. We can, therefore, think of this scalar function (corresponding to the work done) as having a distinct value at every point of the field. It can, therefore, give us a description of the details of the field in a way similar to the electric field (a vector function) which too has a distinct value at every point of the field.

It is this idea of a scalar function that is used to define the potential at every point of the field. We define the potential, at any point, in a field, as the work done in bringing a unit positive charge, from infinity to that point in the field. Thus

$$\text{Potential} = \text{Work} / \text{Charge}$$

Its SI unit is, therefore, J/C which has been named as the volt (V). For a point charge, Q , the potential, V , at a point distant r from it, turns out to be

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r}$$

In non-conservative force work done is not stored as potential energy.

Example 2.1.1 Two spheres, of radius R and $2R$, carry a charge q each. Will there be any flow of charge if the two spheres are connected by a wire? If yes, find the new values of the charges on the two spheres.

Solution:

The potentials, on the surfaces of the two spheres, are

$$V_1 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q}{r}$$

$$\text{And } V_2 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q}{2R}$$

$$\text{Hence } V_1 > V_2$$

There would, therefore, be a flow of charge, from the smaller sphere, to the larger sphere, till the two spheres acquire the same potential. Let V be the common potential acquired by the two spheres after this flow stops.

The capacitance, of a sphere of radius r , being $4\pi\epsilon_0 r$, we have

Charges on the two spheres (after the flow stops)

$$= (4\pi\epsilon_0 R)V \text{ and } 4\pi\epsilon_0 (2R)V$$

$$\therefore 4\pi\epsilon_0 V[R + 2R] = 2q$$

$$V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{2q}{3R}$$

The new charges on the two spheres, are

$$q_1 = 4\pi\epsilon_0 R \left(\frac{2q}{12\pi\epsilon_0 R} \right) = \frac{2q}{3}$$

$$q_2 = 8\pi\epsilon_0 R \left(\frac{2q}{12\pi\epsilon_0 R} \right) = \frac{4q}{3}$$

Their total charge is again $2q$, as it should be, as per the principle of conservation of charge.

2.1.3 Relation between potential and electric field

How can one use the calculated value of potential (V) at a given field point to calculate the electric field $\mathbf{E}(r)$ at that point? These two functions can be correlated by remembering that the work done in moving a unit charge through a small distance, say dr , in the electric field (due to a positive point charge, say) is $\mathbf{E} \cdot d\mathbf{r}$.

The work done equals ΔV , where ΔV represents the difference in the values of potential at these two neighboring points. We can thus put

$$\mathbf{E} \cdot \Delta \mathbf{r} = |\Delta V|$$

$$\text{Now } \Delta V = (V_{\text{final point}} - V_{\text{initial point}})$$

If the unit positive charge were moving in the direction of the electric field, the work done by the electric field would be positive work. However, the potential at the final point (which is a little more distant from the source of the electric field, (assumed to be a positive point charge here) would be a little less than that at the initial point. Hence ΔV , as per its definition, would have a negative sign. We, therefore need to put

in differential notation we have

$$\mathbf{E} \cdot d\mathbf{r} = -dV$$

$$E_x dx + E_y dy + E_z dz = -\left(\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz\right)$$

$$\therefore (E_x + \frac{\partial V}{\partial x})dx + (E_y + \frac{\partial V}{\partial y})dy + (E_z + \frac{\partial V}{\partial z})dz = 0$$

Since dx, dy, dz are independents, each quantity within the brackets is zero.

$$E_x + \frac{\partial V}{\partial x} = E_y + \frac{\partial V}{\partial y} = E_z + \frac{\partial V}{\partial z} = 0$$

$$\therefore E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \mathbf{E} = E_x \hat{\mathbf{i}} + E_y \hat{\mathbf{j}} + E_z \hat{\mathbf{k}}$$

This implies that the electric field equals the negative of the rate of change of potential along its own direction.

In general, if $\mathbf{E}$ and $\Delta \mathbf{r}$ have different directions, the above equality would still be valid if we modify it to say that the: “component of electric field, along any direction, equals the negative of rate of change of

potential along that direction”. This statement provides us with the general relation between electric field and potential. It implies that

$$E_x = -\frac{\Delta V}{\Delta x}, E_y = -\frac{\Delta V}{\Delta y} \text{ and } E_z = -\frac{\Delta V}{\Delta z}$$

We can, therefore, write

$$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\therefore \mathbf{E} = -\text{grad } V = -\nabla V$$

It follows that if we know the potential, due to a given charge configuration, at a general field point (say x, y, z), we can use the above relation to calculate the electric field at that general point. This expression, can in turn, be used to calculate the electric field at any specific point.

Example 2.1.2

A point charge Q , is located at the origin. Find potential due to this charge at a point P (x, y, z) and use the expression, for potential, to find the electric field at the point P.

Solution: We have

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{r}$$

$$\text{Here } r^2 = (x - 0)^2 + (y - 0)^2 + (z - 0)^2 = x^2 + y^2 + z^2$$

$$\therefore V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{\sqrt{x^2 + y^2 + z^2}}$$

it follows that

$$E_x = -\left(\frac{\partial V}{\partial x}\right) = \left[\frac{-Q}{4\pi\epsilon_0} \frac{\partial}{\partial x} (x^2 + y^2 + z^2)^{-\frac{1}{2}}\right]$$

$$= \frac{-Q}{4\pi\epsilon_0} \left(-\frac{1}{2}\right) (x^2 + y^2 + z^2)^{-\frac{3}{2}} \cdot 2x$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} = \frac{Q}{4\pi\epsilon_0} \frac{x}{r^3}$$

$$\text{Similarly } E_y = \left(\frac{Q}{4\pi\epsilon_0} \right) \frac{y}{(x^2+y^2+z^2)^{\frac{3}{2}}} = \left(\frac{Q}{4\pi\epsilon_0} \right) \frac{y}{r^3}$$

$$\text{and } E_z = \left(\frac{Q}{4\pi\epsilon_0} \right) \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}} = \left(\frac{Q}{4\pi\epsilon_0} \right) \frac{z}{r^3}$$

$$\therefore = \left(\frac{Q}{4\pi\epsilon_0 r^3} \right) [x \hat{i} + y \hat{j} + z \hat{k}]$$

$$= \frac{Q}{4\pi\epsilon_0 r^3} \mathbf{r} = \frac{Q}{4\pi r^2} \hat{\mathbf{r}} \left(\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} \right)$$

This is, of course, the standard expression for the electric field due to a point charge.

[The idea, illustrated through this simple example, can be carried over to a combination of charges/ a continuous charge distribution. We need to (first) calculate the electric potential at a general field point (x, y, z) and then use this expression to calculate E_x, E_y, E_z . Here it is important to remember that when we are finding $\frac{\partial V}{\partial x}$, we need to consider y and z, in the expression for V , as constants. Similarly, for calculating $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$, we would consider (z and x) and (x and y), respectively, as constants.]

Example 2.1.3

A charge q , is moved, from an internal point A to another internal point B, as shown in Fig.2.1.1 within a charged hollow metallic sphere of radius R .

Find the work done for this displacement of the charge q .

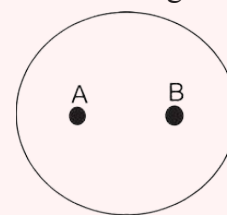


Fig.2.1.1

Solution:

For a charged hollow metallic sphere, the potential, at all internal points, has the same value. There being no difference in potential between points A and B, the work done in moving a charge, q , from A and B, would be zero.

(Work done = charge $\times$ potential difference)

Alternatively

The work done for an infinitesimal displacement dr , in an electric field E , equals

$$dW = E.dr$$

Hence, total work done

$$W = \int E.dr$$

For this hollow metallic sphere, the electric field is zero at all internal points. Hence W , and, therefore, W would be zero.



Let Us Reflect

Q.2.1.1 Calculate the electric potential, at a point A, at a distance z away from the center, on the axis of a ring, due to a charge Q , uniformly distributed along the ring (radius R).



Answers

A.2.1.1 Refer to Fig. 2.1.2

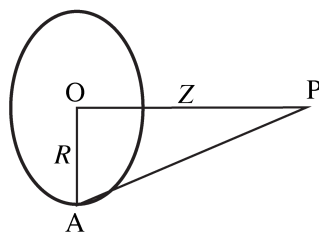


Fig. 2.1.2

let the potential at the point P due to an element of charge, dq on the ring at P be dV .

$$\text{Then, } dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{AP}$$

$$\therefore dV = \frac{1}{4\pi\epsilon_0} \frac{dQ}{\sqrt{R^2 + z^2}}$$

$$\therefore V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dQ}{\sqrt{R^2 + z^2}}$$

where the integration ranges over all the points on the ring, and this is independent of R and Z ,

$$\text{so, } V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}$$

2.1.3.1 The principle of superposition and potential

The principle of superposition (the basis for calculating electric fields due to multiple charges/ continuous charge distributions) is equally valid for electric potential. The total electric potential, at a given field point, due to n point charges, say, is just the sum of the potentials there, due to each of these charges, taken one at a time.

Calculation of the total electric potential, (using the principle of superposition) is obviously much simpler than calculating the electric field. This is because the additions, needed for calculating potential, are just simple scalar additions.

Example 2.1.4 For the four charges arranged as shown in Fig.2.1.3, calculate the total potential at the point N.

$$(q_1 = 5 \mu\text{C}, q_2 = -3 \mu\text{C}, q_3 = +6 \mu\text{C}, q_4 = -7 \mu\text{C})$$

Solution:

The distance, r , of each charge from the point N, is

$$\frac{1}{2}[(4\sqrt{2})\sqrt{2}] = 4 \text{ m}$$

$$\therefore V_n = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 + q_2 + q_3 + q_4}{r} \right] = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{4} \text{ V}$$

$$= 2.25 \times 10^3 \text{ V} = 2.25 \text{ kV}$$

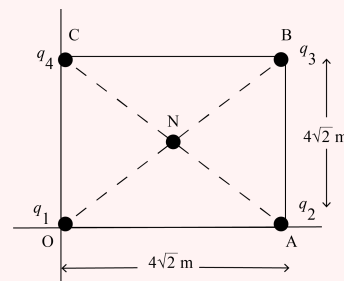


Fig.2.1.3

Example 2.1.5 A circular loop of radius a is uniformly charged to have linear charge density λ . If it is kept in the XY plane, find the electric potential, due to it, at a point on the Z -axis, distant a from the center.

Solution: The distance of the field point, from all charge elements, on the loop is

$$r = \sqrt{a^2 + a^2} = \sqrt{2a}$$

$\therefore$ The electric potential, due to any one charge element, is

$$dV = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\lambda dl}{r}$$

$$\therefore V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\lambda}{\sqrt{2}a} \int dl = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{\lambda}{\sqrt{2}a} \times 2\pi a = \frac{\lambda}{2\sqrt{2} \epsilon_0} = \frac{\sqrt{2}\lambda}{4\pi\epsilon_0}$$

Example 2.1.6 Three charges $(+q)$, $(+q)$ and $(-q)$ are kept at the three vertices of an equilateral triangle of side a . Find the net electric potential, at the centroid of the triangle.

Solution: The distance of each vertex, from the centroid, is $\left(\frac{a}{\sqrt{3}} \right)$, hence the net potential at the centroid is

$$\begin{aligned} V &= \left(\frac{1}{4\pi\epsilon_0} \right) \left[\frac{q}{a/\sqrt{3}} + \frac{q}{a/\sqrt{3}} + \frac{-q}{a/\sqrt{3}} \right] \\ &= \frac{q}{(4\pi\epsilon_0 a/\sqrt{3})} = \frac{\sqrt{3}q}{12\pi\epsilon_0 a} \end{aligned}$$

Example 2.1.7 Calculate the electric potential, due to a hollow charged sphere, (of radius R , carrying a total charge Q) at a point distance r from the centre, when

- (i) $r > R$ (ii) $r = R$ (iii) $r < R$.

Solution:

Refer to Fig.2.1.4

A simple use of gauss's law tells us that, (for a charged hollow sphere) for external points ($r > R$ to $r \rightarrow R$), the electric field is the same as if the total charge Q , were present at the centre of the sphere. Also the field is zero at all internal ($r < R$) points.

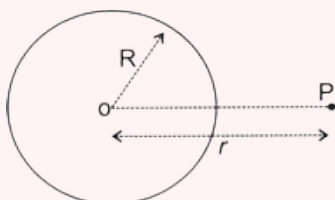


Fig.2.1.4

(i) For $r > R$

$$V_e = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{r}$$

(ii) For $r \rightarrow R$ (going to $r = R$) we can use the above result and say that at $r = R$, we have

$$V_s = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R}$$

(iii) For $r < R$, the field being zero, the rate of change of potential, with distance, in the interior of the hollow sphere, is zero. Hence the potential, V_i , must have the same constant value at all internal points.

This constant value can be arrived at, by considering an internal point for which r is just infinitesimally less than R . The value of potential, at this (just internal) point must be the same as its value at the surface. Hence, for all internal points,

$$V_i = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R}$$

These results lead us to the graph (between V and r) shown at Fig.2.1.5.

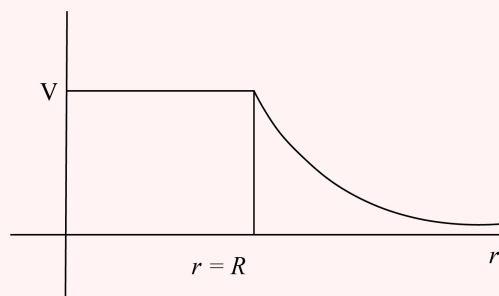


Fig.2.1.5



Let Us Reflect

Q.2.1.2 Three charges, each of magnitude q , are kept at the points $(a, 0, 0)$, $(0, a, 0)$ and $(0, 0, a)$. find the net electric potential, due to them, at the origin.

Q.2.1.3 A circular wire, of radius a , is kept in the XY plane. Its linear charge density, as a function of θ (defined as shown) is given by

$$\lambda(\theta) = \lambda_0 \sin^2 \theta$$

Find the electric potential due to this loop, at the centre of the loop.



Answers

A.2.1.2 The distance, of each charge, from the origin is the same and hence the net electric potential, at the origin, is $V_0 = \frac{1}{4\pi\epsilon_0} \left(3 \times \frac{q}{a} \right) = \frac{3q}{4\pi\epsilon_0 a}$

A.2.1.3 All charge elements are at a distance from the centre. The potential, due to a charge element, around the position $\theta = \theta$, is given by

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\lambda(\theta)(d\theta)}{a} = \frac{\lambda_0 \sin^2 \theta}{4\pi\epsilon_0 a} (d\theta)$$

$$\therefore V = \frac{\lambda_0}{4\pi\epsilon_0 a} \int_0^{2\pi} \sin^2 \theta d\theta = \frac{\lambda_0}{4\pi\epsilon_0 a} \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2} \right) d\theta = \frac{\lambda_0}{4\pi\epsilon_0 a} \cdot \frac{1}{2} \cdot 2\pi + \frac{1}{2} \left| \frac{\sin 2\theta}{2} \right|_0^{2\pi} = \frac{\lambda_0}{4\epsilon_0 a}$$

2.1.4 Potential gradient

The rate of change of potential, with distance, in a given direction, may be referred to as the gradient of potential (or potential gradient), in that direction. It is easy to realize that, in general, the potential gradient would be a direction dependent quantity, i.e. the change in potential (in the electric field due to a given charge configuration), for the same change in distance, would not have the same value along different directions.

We can combine the values of the rate of change of potential, along the x , y and z axis, to define a vector function, say $\mathbf{S}$, given by

$$\mathbf{S} = \left[\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right]$$

this vector function is known as the gradient of potential, In terms of this function, one can say

$$\mathbf{E} = -\mathbf{S} = -\text{grad } V = -\nabla V$$

[Note: The symbol, used for the potential gradient function, is not S but $-\nabla V$ or $\text{grad } V$]

What is important to realize is that unlike the potential, which is a scalar quantity, the potential gradient is a vector function. In general, a function and its rate of change can have quite different natures.

2.1.5 Equipotential surface

An equipotential surface is a surface with a constant value of potential for all points on its surface.

The shape of the equipotential surface, for a given charge distribution, depends on the nature of that charge distribution. For a point charge, the equipotential surfaces are concentric spherical surfaces centered at the charge. For a charge configuration, producing a uniform electric field like a uniformly charged infinite plane sheet of charge, the equipotential surfaces would be a set of parallel planes, normal to the field lines of that

uniform field.

There being no difference in potential, at different points on an equipotential surface, the work done in moving a charge, from one point to another, on an equipotential, would be zero. This, in turn, also implies that the rate of change of potential (and therefore, the component of electric field), along an equipotential, has to be zero. The electric field therefore cannot have any component along an equipotential. The angle between the direction of the electric field, and the equipotential, has to be

$$\frac{\pi}{2} \left(\because \cos \frac{\pi}{2} = 0 \right)$$

Hence the electric field has to be normal to the equipotential at every point.



Let Us Reflect

Q.2.1.4 Two point charges of magnitude $+q$ and $-q$ are placed at $\left(\frac{-d}{2}, 0, 0\right)$ and $\left(\frac{d}{2}, 0, 0\right)$, respectively. Find the equation of the equipotential surface where the potential is zero.



Answers

A.2.1.4

The equation is that of the (Y-Z) plane defined by $x = 0$.

2.1.6 Potential energy of a system of charges

We have seen that potential, at a point P, is defined as the work done in bringing a unit positive charge from infinity to that point. The work done, in changing the position (configuration) of an object, is viewed as the change in potential energy of that object. It follows that we can think of any charge configuration as having a potential energy associated with it. This equals the work done in assembling that charge configuration from an

initial set-up in which all the charges, (making up that charge configuration) were located infinitely apart from one another.

Using the definition of potential and its expression for a point charge, it is easy to see that the potential energy, associated with a system of n point charges, $q_1, q_2, \dots, q_n$ is

$$U = \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \left[q_1 \left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}} + \dots + \frac{q_n}{r_{1n}} \right) + q_2 \left(\frac{q_2}{r_{21}} + \frac{q_3}{r_{23}} + \dots + \frac{q_n}{r_{2n}} \right) + \dots + q_n \left(\frac{q_2}{r_{n1}} + \frac{q_3}{r_{n2}} + \dots + \frac{q_{n-1}}{r_{n(n-1)}} \right) \right]$$

The factor $\frac{1}{2}$ has to be put because each pair appears twice in this way of writing the potential energy.

This expression, for the potential energy, is to be used when the n charges are assembled (via. their mutual interactions only) in a field free region. If the charges are brought in a region where there is already an electric field (due to some other external (to the system) charge configuration), we would then need to add n terms, of the form $q_i V_i$ ($i = 1, 2, \dots, n$) where V_i is the electric potential (due to the (external) electric field) at the location of the charge q_i . Thus in the region of a field, the total potential energy, associated with the above assembly of n point charges, would be

$$U_{total} = U(\text{as given above}) + \sum_{i=1}^n q_i V_i$$



Q.2.1.5 Choose the correct option

A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge

- (a) remains a constant because the electric field is uniform.
- (b) increases because the charge moves along the electric field.
- (c) decreases because the charge moves along the electric field.
- (d) decreases because the charge moves opposite to the electric field.

A Answers

A.2.1.5 (c)

$$E = - \left(\frac{dV}{dx} \right)$$

$$\therefore dV = -(E dx)$$

Example 2.1.8

Calculate the potential energy associated with a uniformly charged solid sphere, of radius R and carrying a total charge Q .

Solution:

For this continuous charge distribution, we can think of the charged sphere as being assembled by adding a hollow shell of radius $(r + dr)$ to an already present charged part of the sphere of radius r ; here r varies continuously from $r = 0$ to $r = R$

The charge present in the sphere of radius r

$$= q = \left(\frac{4\pi}{3} r^3 \right) \rho = \left(\frac{4\pi}{3} r^3 \right) \frac{Q}{\left(\frac{4\pi}{3} R^3 \right)} = Q \cdot \frac{r^3}{R^3}$$

The charge added, when a ring of radius $r + dr$, is just put over this (already assembled) sphere of radius r , is

$$\begin{aligned} dq &= (4\pi r^2 dr) \rho = 4\pi r^2 dr \left(\frac{3Q}{4\pi R^3} \right) \\ &= \left(\frac{3Q}{R^3} \right) r^2 dr \end{aligned}$$

This charge is being added, at the potential, V that is the potential, on the sphere of radius r due to the charge q

$$\text{Hence } V = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q}{r} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{Q}{R^3} r^2$$

$\therefore$ Work done in adding the charge on the ring (lying between radius r and radius $r+dr$), is

$$\begin{aligned} dW &= Vdq = \left(\frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r^2 \right) \left(\frac{3Q}{R^3} r^2 dr \right) \\ &= \frac{3Q^2}{4\pi\epsilon_0 R^6} r^4 dr \end{aligned}$$

$\therefore$ Total work done in assembling the whole sphere

$$W = \left(\frac{3Q}{4\pi\epsilon_0 R^6} \right) \int_0^R r^4 dr = \left(\frac{3Q^2}{4\pi\epsilon_0 R^6} \right) \left(\frac{R^5}{5} \right)$$

Thus the PE, associated with the charged sphere, is

$$P.E = \left(\frac{1}{4\pi\epsilon_0} \right) \cdot \left[\frac{3}{5} \frac{Q^2}{R} \right]$$



Let Us Reflect

Q.2.1.6 Do free electrons move towards a region of higher potential or lower potential?



Answers

A.2.1.6

Higher potential.

2.1.7 Summary

- The potential at any point, in the field of charge is the work done in bringing a unit positive charge from infinity to a point in the field.
- It is scalar quantity and its SI unit is J/C which is named as volt(V).
- The electrostatic potential due to a point charge Q at a distance r from it is $V = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{r}$
- The rate of change of potential, with distance, in a given direction, may be referred to as the gradient of potential (or potential gradient), in that direction. The component of electric field, along any direction, equals to negative of rate of change of potential along that direction. For example in x-direction $E = -\frac{dV}{dx}$
- By the principle of superposition, the total potential due to the collection of point charges at a given field point, is the sum of potential due to each of the point charges

$$V = \left(\frac{1}{4\pi\epsilon_0}\right)\left[\frac{q_1+q_2+\dots+q_n}{r}\right]$$

An equipotential surface is a surface with a constant value of potential for all the points on its surface. The work done in moving a test charge between two points on an equipotential surface is zero.

- The potential energy of charge q at a distance r in an external field is $qv(r)$ and the potential energy due to system of point charge is

$$U = \frac{1}{2} \cdot \frac{1}{4\pi\epsilon_0} \left[q_1 \left(\frac{q_2}{r_{12}} + \frac{q_3}{r_{13}} + \dots + \frac{q_n}{r_{1n}} \right) + q_2 \left(\frac{q_1}{r_{21}} + \frac{q_3}{r_{23}} + \dots + \frac{q_n}{r_{2n}} \right) + \dots + q_n \left(\frac{q_1}{r_{n1}} + \frac{q_2}{r_{n2}} + \dots + \frac{q_{n-1}}{r_{n(n-1)}} \right) \right]$$

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