

Module 2: Dielectrics and Polarisation

This module is designed to help you to:

- describe the concept of dielectrics and their two main types;
- explain the term polarisation of dielectric and the difference in the process of polarisation of the two types of dielectrics;
- calculate the field due to a polarized dielectric and find the reason for it being opposite to the external electric field that polarized it;
- explain the meaning of the term dielectric constant/relative permittivity;
- obtain the expression for the capacitance of a parallel plate capacitor;
- analyze the two types of combination of capacitors –the series and the parallel;
- derive expressions for the equivalent capacitance of the series and parallel combination of two or more capacitors;
- explain why the presence of dielectrics, between the plates of a capacitor, leads to an increase in the capacitance of a capacitor;
- solve problems where the space, between the plates of a capacitor, is wholly, or partially, filled with dielectric/dielectrics, in different ways.

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2.2.1 Introduction

So far we have learnt the key concepts in electrostatics, now we shall apply them to study the behavior of substances such as dielectrics. We shall restrict our study to polar dielectrics where we shall come across the mechanism of electrostatic polarisation, which basically amounts to the creation of an alignment of the electric dipoles in the medium. Then we shall go over to study about the capacitor and their capacitance. We shall learn about the capacitors of different kinds of geometry such as parallel plate, spherical, etc. We shall learn about the different kinds of combinations (series and parallel) of capacitances and apply them to understand their use in electric circuits.

We shall understand the meaning of the terms dielectric constant and relative permittivity and will find that they are actually equivalent. We shall also study the effect of introduction of a dielectric within a capacitor.

2.2.2 Dielectrics

There are two kinds of dielectrics – polar and non- polar. Normal (neutral) atoms/ molecules, in which the centres of their positive and negative charges coincide, i.e., which have zero dipole moment, are referred to as non-polar dielectrics.

However, the net dipole moment of a macroscopically small volume element of a non-polar dielectric, as well as a polar dielectric, is zero. Why so for polar dielectrics? We can understand this by realizing that a macroscopically small volume element, of a polar dielectric, would still have a very large number of atoms/ molecules and, therefore, a very large number of atomic/ molecular dipoles. These dipoles would, however, have their dipole moments (vectors) distributed in a completely random way. The resultant dipole moment of such a random distribution would be zero.

2.2.2.1 Polarisation of dielectric

A dielectric is said to be polarised when the net dipole moment, of a macroscopically small volume element, acquires a non-zero value. This does not happen (for both nonpolar and polar dielectrics) when the dielectric is left to itself.

Both types of dielectrics, however, get polarized when put in an external electric field. The mechanism of polarisation is, however, different in the two cases.

When a sample of a non-polar dielectric is put in an external field, the (originally) coincident centres, of its positive and negative charges, experience forces in opposite directions- the positive charge in the direction of the field and the negative charge in a direction opposite to that of the field. The field thus makes the two centres

(of charges) non- coincident.

The atom/ molecule thus acquires a (small) dipole moment. This dipole moment, by the very mechanism of its generation, is directed antiparallel to the field direction. The total of all these (parallel to each other) dipole moments, of the atoms/ molecules, in the (macroscopically) small volume elements is the net dipole moment acquired by this dielectric.

The corresponding mechanism for a polar dielectric is different. Here the atoms/ molecules already have a (non- zero) dipole moment. These, however, are randomly oriented for different atoms/molecules. The external electric field exerts a torque on these dipoles and tends to align them parallel to its own direction. The thermal motion (dependent on the temperature) tends to randomize the dipole moment vectors. There is thus a tussle and the ratio of the field strength (E) to the temperature (T) determines the extent of alignment. However, there would always be some alignment and this would make the net dipole moment, of a macroscopically small volume element, as non-zero.

The differences (i) in the basic nature of the two types of dielectrics and (ii) in the mechanism of polarisation, help us to understand why the polarisation of a polar dielectric is

- a. Generally more than that of a non- polar dielectric
- b. Strongly temperature dependent unlike that of a non- polar dielectric (in which the polarisation shows little variation with changes in temperature)

2.2.3 The field of a polarised dielectric

When a dielectric gets polarised, each of its (macroscopically) small volume elements has a net (non-zero) dipole moment. It can, therefore, produce a field of its own- similar to the field of a dipole. The net field, produced by the polarised dielectric, would be the resultant of the fields to its different volume elements.

It is interesting to note that the field, due to the polarised dielectric, would oppose the external field that caused it to get polarised. This has to be so because if the field of the polarised dielectric were to support the external field, the net field would go up. This, in turn, would mean greater polarisation, hence a greater net field- an infinitum. Such a happening would go against the principle of energy conservation. The field of a polarised dielectric is thus opposite to the external electric field that polarised it.

2.2.4 Dielectrics and insulators

Do these two terms, often used as two distinct terms, refer to clearly distinct and separate types of materials? Not quite. The term dielectric is a general term which can be used for all materials. Usually, however, the term is used more often for insulators. We, however, use one, or the other, of these terms, depending on the context.

When we are explicitly making use of the polarised behavior of a material in an external electric field, we use the term dielectric for that material. When the material is being used for obstructing/ stopping the flow of electric currents/ charges, we refer to it as an insulator.

There is thus no basic difference between dielectrics and insulators. It is more the context in which the material is being used that usually determines which of these terms we use in that set-up/ situation.



Let Us Reflect

Q.2.2.1 Which of the two words (a) creation (b) alignment can be associated with polarisation (by an external electric field) of a (i) polar (ii) non-polar dielectric? Justify your choice.

Q.2.2.2 The field of a polarized dielectric behaves in a way similar to the emf induced in a conductor present in a changing magnetic field. justify, with reason, your agreement/disagreement with the above statement.



Answers

A.2.2.1 (i) We can associate the word creation with the mechanism of polarisation in a non-polar dielectric.

This is because the atoms/molecules of such dielectrics have zero dipole moment in the absence of an external electric field. The external electric field causes each atom/molecule to acquire a (small) non-zero dipole moment parallel to its own direction. We can, therefore, say that the external electric field creates a dipole moment, (in each atom/molecule of a non-polar dielectric) where there was no dipole moment to start with. This leads to the polarisation of the dielectric.

(ii) We can associate the word alignment with the mechanism of polarisation of a polar dielectric.

This is because the atoms/molecules of such dielectrics have a (non-zero) dipole moment associated with them. However, these dipole moments are randomly oriented and the net dipole moment of macroscopic small volume elements is zero. The external electric field exerts torques on these individual

atomic/molecular dipoles that tends to align them in the field direction. There is, therefore, a partial alignment of the atomic/molecular dipoles. This makes the net dipole moment of a macroscopic small volume element have a finite (non-zero) value, i.e., causes the dipole to get polarized.

A.2.2.2 We agree with this statement. The field of a polarized dielectric opposes the external electric field that caused the dielectric to get polarized. We can, therefore, say that it opposes the cause of its creation. This is similar to the behaviors of the emf induced in a conductor present in a changing magnetic field. There also, the emf induced is always such as to oppose the cause of its creation. In both cases, the observed behavior is in agreement with the law of conservation of energy.

2.2.5 Capacitors

The concept of capacitors, a combination of two (usually identical) conductors, kept in close vicinity of each other arose from the realization that the capacity of a single isolated conductor is usually very small. Here capacity implies the amount of charge that needs to be added to bring a unit change in the potential of the conductor. Thus the capacity, C , of a conductor, equals $\left(\frac{Q}{V}\right)$ where V is the potential acquired by the conductor when a charge Q is added to it.

For a spherical conductor of radius R , the potential, V , corresponding to a charge Q , equals $\frac{1}{4\pi\epsilon_0} \frac{Q}{R}$.

Hence the capacity, C , is,

$$C = \left(\frac{Q}{V}\right) = 4\pi\epsilon_0 R$$

It is thus seen to have a very low value not only for ordinary spherical conductors but even for an (imaginary) spherical conductor that may have a radius as large as that of the earth itself.

2.2.6 Why does a capacitor have a larger capacitance as compared to an isolated conductor?

The capacitor is a combination of (usually) two identical conductors, kept in close vicinity of each other. The two conductors carry equal and opposite charges, $(\pm Q)$. The capacity C , of the capacitor, equals the ratio of the magnitude of charge (on either conductor) to the potential difference, V , between them.

It is easy to realize that, because of the close proximity of the two conductors (that carry equal and opposite charges), the potential difference, between them, would tend to have quite a low value- compared to the potential of either conductor, in isolation, (when it has a charge of magnitude Q on it). Thus the capacity of a capacitor becomes quite large in comparison to that of an isolated conductor.

2.2.7 Parallel plate capacitor

The capacitance C of a parallel plate capacitor, made of plates of area A each, separated by a (vacuum/air) distance d , is given by

$$C = \frac{\epsilon_0 A}{d}$$

We can easily derive this result as follows. The electric field, in the region between the plates of the capacitor, due to the

positively charged plate is $\left(\frac{\sigma}{2\epsilon_0}\right)$ directed (normally) away from this plate. The electric field, due to the negatively charged plate, also has a magnitude, $\left(\frac{\sigma}{2\epsilon_0}\right)$ and this is directed (normally) toward the negatively charged plate. The two fields add up to produce a net field E between the plates where

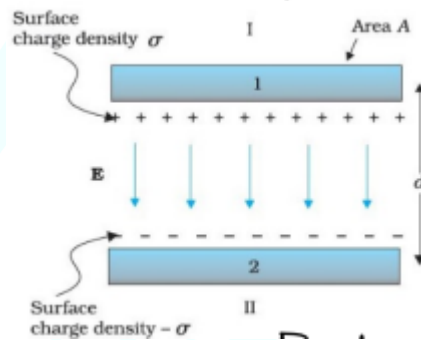


Fig.2.2.1 A parallel plate capacitor

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

The potential difference, between the plates, is, therefore,

$$V = Ed = \frac{Q}{\epsilon_0} \frac{d}{A}$$

The capacitance, C , of the parallel plate capacitor, is, therefore,

$$C = \frac{Q}{V} = \frac{Q \cdot \epsilon_0 A}{Q \cdot d} = \frac{\epsilon_0 A}{d}$$

Thus, for a parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

2.2.8 Capacitor combinations

A number of capacitors can be combined to have a series combination or a parallel combination or a mixed combination. We know the formulae for the equivalent capacitance (C) for the series as well as the parallel combinations. Their formulae are

Series combination

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \sum_{c=1}^n \frac{1}{C_n}$$

Parallel combination

$$C = C_1 + C_2 + \dots + C_n = \sum_{c=1}^n C_n$$

The charges, on the plates of a number of capacitors, put in series, have the same value. The potential differences across the different capacitors of the combination are, however, in inverse proportion to their individual capacitances. For a parallel combination, the potential difference across the plates of all the capacitors have identical values. However, the charges, on different capacitors, are in direct proportion to their individual capacitances.

From the formulae for the two combinations, it is also easy to realize that the equivalent capacitance of a series combination has a value less than the least among the values of the individual capacitors.

Parallel combination has a value greater than the greatest among the values of the individual capacitors.

Equivalent capacitance :What does the term equivalent capacitance imply?

The equivalent capacitance, of any given combination of capacitors, is the capacitance of a single capacitor, which, when connected across a given voltage source, acquires a charge that equals the total charge, acquired (from the same voltage source) by all the capacitors making up that combination.

Example 2.2.1

Two spherical conductors, carrying charges q and $2q$, have radii R and $R/2$, respectively. Find the ratio of

- (i) their capacitances
- (ii) potential values at their respective surfaces.

Solution:

(i) The capacitance of a given conductor/capacitor, is a characteristic constant of that conductor/capacitor. It does not depend on the charge present on it.

For a spherical conductor, of radius R ,

$$\text{Capacitance } C = 4\pi\epsilon_0 R$$

Ratio of capacitances for the two given conductors

$$= \frac{R}{(R/2)} = 2:1$$

(ii) Potential, V_1 , on the surface of the first conductor

$$V_1 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{q}{R}$$

and potential, V_2 , on the surface of the second conductor,

$$V_2 = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{2q}{(R/2)} = \left(\frac{1}{4\pi\epsilon_0} \right) \frac{4q}{R}$$

$$V_1 : V_2 = 1 : 4$$

2.2.9 Effect of dielectric on the capacitance of a capacitor

We can understand this in terms of the polarisation of a dielectric. The dielectric, when present between the plates of a charged capacitor, is effectively in an external electric field. This electric field was (before the dielectric was put) equal to V/d where V = potential difference between the capacitor plates and d = distance between the two plates. (the capacitor is being taken as a parallel plate capacitor). This (external) electric field polarises the dielectric. As we have already seen, the polarised dielectric would produce a field of its own that opposes this external electric field. The net electric field, in the presence of the dielectric, thus becomes lower than what it was in the absence of the dielectric. The net result is —the potential difference between the capacitor plates (which still have the same charges $+Q$ and $-Q$) on them, gets reduced. The capacity (= charge/potential difference), therefore, increases. The extent of this increase would depend on the nature of the dielectric. An interesting fact about this phenomenon is: For a given dielectric, the factor by which the capacitance increases, remains the same irrespective of their shape, size or the capacitance value of the original (without dielectric) capacitor.

2.2.10 The dielectric constant, relative permittivity

The multiplicative factor, for the capacitance, is thus seen to be a characteristic of a given dielectric. We call this factor the dielectric constant (k) of that dielectric.

We also use the term relative permittivity for this characteristic constant of a given dielectric. The symbol used for relative permittivity is ϵ_r . The value for the relative permittivity (ϵ_r) of a given dielectric is the same as that of its dielectric constant (k). The reason for this equality can be understood as follows. The relative permittivity is the factor by which the force between two given charges in a vacuum gets reduced when the (originally vacuum) separation between these charges gets filled with the given dielectric. It is the ratio of the

permittivity of a substance to the permittivity of vacuum.

The capacitance of a capacitor increases by a factor k (the dielectric constant) because the dielectric reduces the electric field (for the same charges) between its plates by factor k .

$$k = \frac{\epsilon}{\epsilon_0}$$

Electric field is viewed as the force per unit charge. A reduction in the electric field by a factor k , thus, implies a reduction in force between charges by a factor k . However, as discussed above, this factor is relative permittivity ϵ_r . We thus have $k = \frac{\epsilon}{\epsilon_0} = \epsilon_r$

2.2.11 Calculating the capacitance of a capacitor partially filled with a dielectric

The capacitance of a (vacuum) parallel plate capacitor is

$$C = \frac{\epsilon_0 A}{d}$$

With the dielectric, the capacitance becomes where

$$C' = k \frac{\epsilon_0 A}{d}$$

We now rewrite as

$$C' = \frac{\epsilon_0 A}{(d/k)} = \frac{\epsilon_0 A}{d'}$$

We can now view C' as the capacitance of a vacuum capacitor in which the plate separation is d' i.e. smaller than the given separation by a factor k . We can use this result to say that:

A thickness d , of a dielectric, is equivalent to a vacuum thickness $d' = \left(\frac{d}{k}\right)$

We illustrate the use of this result by an example.

Example 2.2.2

A dielectric, of dielectric constant k , is used to half fill the separation, d , between the plates of a parallel plate capacitor having plates of area A each. Find the capacitance of this capacitor now.

Solution:

In the new capacitor, the vacuum (or air) thickness is $d/2$. The thickness, $d/2$, of the dielectric, is equivalent to a vacuum thickness $\frac{d}{2k}$. Hence the total effective vacuum thickness is

$$d' = \frac{d}{2} + \frac{d}{2k} = \frac{d(k+1)}{2k}$$

$\therefore$ The new value of capacitance is

$$C' = \frac{\epsilon_0 A}{d} = \frac{2k\epsilon_0 A}{(k+1)d} = \left(\frac{2k}{k+1}\right)C$$



Let Us Reflect

Q.2.2.3 It is known that the capacitance of a spherical conductor of radius R equals $4\pi\epsilon_0 R$. Given that the breakdown (or sparking) electric field, near a conductor equals E_C , Obtain an expression for the maximum potential to which a spherical conductor, of radius R , can be raised.

Q.2.2.4 A circuit is set up as shown in Fig. 2.2.2. The key K_1 is closed, keeping key K_2 open. Let the energy stored in the capacitors in this system, in this set up be E_1 . The key K_1 is now opened and the capacitor (1) is shorted. The space between capacitors (1) and (2) is now filled with a dielectric of dielectric constant K . Again the key K_1 is closed keeping key K_2 open. Next the key K_1 is opened and key K_2 closed. Let the energy stored be E_2 . Obtain an expression for the ratio (E_1/E_2) .

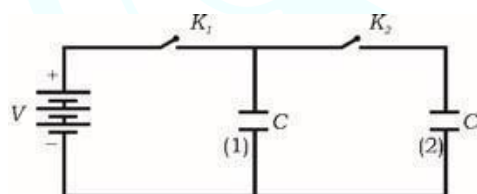


Fig. 2.2.2

Q.2.2.5 Find the charge on the capacitors of capacitance $5\ \mu\text{F}$ and $3\ \mu\text{F}$ when the key K in the circuit (in Fig. 2.2.3) is closed.

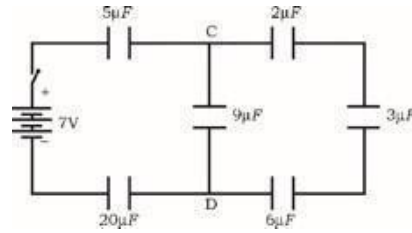


Fig.2.2.3

The space, between the plates of two parallel plate capacitors, each of capacitance C , is filled with equal amounts of three dielectrics of dielectric constants k_1 , k_2 , and k_3 , in two different ways, as shown in Fig. 2.2.3(a) and 2.2.3(b). Find the ratio of the equivalent capacitance of system (1) and system (2).

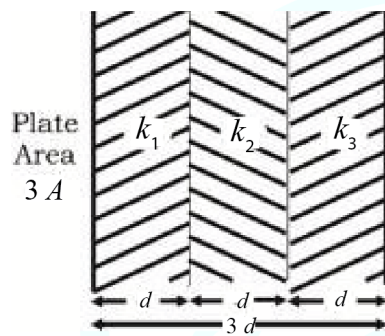


Fig.2.2.3(a)

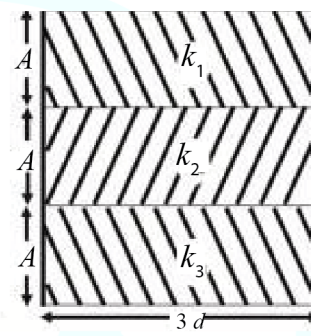


Fig.2.2.3(b)



Answers

A.2.2.3 Let V be the required maximum potential. The total charge, on the spherical conductor, of radius R , when it is raised to potential V_0 , is

$$Q = CV_0 = 4\pi\epsilon_0 RV_0$$

The surface density of charge would then be

$$\sigma = \frac{Q}{4\pi R^2} = \frac{4\pi\epsilon_0 V_0}{R}$$

The electric field, near the surface of a conductor, when it has a surface charge density σ , is

$$E = \frac{\sigma}{\epsilon_0} = \frac{V}{R}$$

We, therefore, have

$$\frac{V}{R} = E_c$$

$$V_0 = R E_c$$

A.2.2.4 We have

$$E_1 = \frac{1}{2} CV^2$$

In the second case, when key K_1 is closed, keeping K_2 open, let the charge on capacitor (1) be Q . When key K_2 is closed, and K_1 is opened, the equivalent capacitance of the system is $2kC$. Hence the common potential, acquired by it, would be

$$V' = \frac{Q'}{2kC} = \frac{kC.V}{2kC} = \frac{V}{2}$$

The energy

$$E_2 = \frac{1}{2}(2kC) \cdot \left(\frac{V}{2}\right)^2 = \frac{kCV^2}{4}$$

$$\therefore \frac{E_1}{E_2} = \frac{\frac{1}{2} CV^2}{\frac{kCV^2}{4}} = \frac{2}{K}$$

A.2.2.5 The equivalent capacitance (C') of capacitors $2 \mu\text{F}$, $3 \mu\text{F}$, $6 \mu\text{F}$, connected in series, is given by

$$\frac{1}{C'} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6}$$

$$\therefore C' = 1 \mu\text{F}$$

We then have $5 \mu\text{F}$, $(9 + 1) \mu\text{F}$ and $20 \mu\text{F}$ in series. As shown in Fig.2.2.4(a) the equivalent capacitance C of the circuit is given by

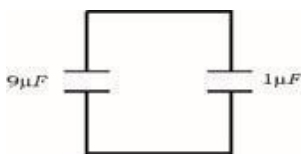


Fig.2.2.4(a)

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{4+2+1}{20}$$

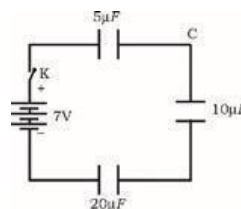


Fig.2.2.4(b)

$$\therefore C' = \frac{20}{7} \mu\text{F}$$

Total charge on the circuit shown in Fig.2.2.4(b), as a whole, is

$$Q = CV = \frac{20}{7} \times 7 \mu\text{C} = 20 \mu\text{C}$$

The circuit is equivalent to the setup shown in Fig.2.2.5.

Hence the charge on each of these three capacitors, in series, is Q . Charge on the $5 \mu\text{F}$ capacitor is also $20 \mu\text{C}$.

The $10 \mu\text{F}$ capacitor (carrying a charge of $20 \mu\text{C}$) is equivalent to a parallel combination of the $9 \mu\text{F}$ capacitor and his (equivalent) capacitor $1 \mu\text{F}$. The total charge (of $20 \mu\text{C}$) get distributed, between these two capacitors, in such a way as to make their potentials equal. Hence charge, on the $1 \mu\text{F}$ capacitor, is

$$\left(\frac{20}{9+1}\right) \times 9 \mu\text{C} = 18 \mu\text{C}$$

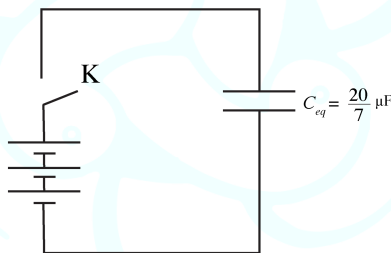


Fig. 2.2.5

This is also the charge on each of the three (in series) capacitors whose equivalent capacitance is $1 \mu\text{F}$. Hence charge on the $3 \mu\text{F}$ capacitor equals $18 \mu\text{F}$

In Fig.2.2.3(a), we have three capacitors connected in series. Their capacitances are $\frac{3\epsilon_0 k_1 A}{d}$,

$\frac{3\epsilon_0 k_2 A}{d}$ and $\frac{3\epsilon_0 k_3 A}{d}$. Hence the equivalent capacitance, C_1 , is given by

$$\frac{1}{C_1} = \frac{d}{\epsilon_0 (3A)} \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \right) = \frac{d(k_2 k_3 + k_3 k_1 + k_1 k_2)}{3\epsilon_0 A k_1 k_2 k_3}$$

$$\therefore C_1 = \left(\frac{3\epsilon_0 A}{d} \right) \frac{k_1 k_2 k_3}{(k_2 k_3 + k_3 k_1 + k_1 k_2)}$$

In Fig.2.2.3(b), we have three capacitors, connected in parallel. Their equivalent capacitances are C_2 , is given by

$$C_2 = \frac{\epsilon_0 A}{3d} (k_1 + k_2 + k_3)$$

$$\therefore \frac{C_1}{C_2} = 9 \left[\frac{k_1 k_2 k_3}{(k_1 + k_2 + k_3)(k_2 k_3 + k_3 k_1 + k_1 k_2)} \right]$$



2.2.12 Summary

- Dielectrics are the substances with no charge carriers and show non-zero net dipole moment when polarised by an external electric field. Dielectrics are of two types polar dielectrics and non polar dielectrics.
- Polarisation is the process of inducing dipole moments in a dielectric material by the effect of an external field.
- The capacitor is a combination of two conductors, kept in close vicinity of each other and separated by an insulator.
- The capacitance of a conductor can be defined as $C = \frac{Q}{V}$, where V is the potential acquired by the conductor when charge Q is added to it.
- Capacitance of parallel plate capacitor, made of plates of area A each and separated by distance d (vacuum/air) is given by $C = \frac{A\epsilon_0}{d}$.
- For the combination of n capacitor in parallel $C = C_1 + C_2 + \dots + C_n$.
- For the combination of n capacitor in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$.
- The relative permittivity is the factor by which the force between two given charges in a vacuum get reduced when the (originally vacuum) separation between these charges gets filled with the given dielectric.
- A dielectric when kept between the plates of a charged capacitor gets polarized and its own electric field gets created within the capacitor which in turn reduces the external electric field of the capacitor and consequently the potential difference between the plates, hence the capacity of capacitor gets increased.
- The capacitance of a capacitor filled with a dielectric material of dielectric constant k can be given as

$$C = \frac{\epsilon_0 A}{d/k}$$

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