

6. Electromagnetic Induction

Module 3 : Mutual Inductance and Self-Inductance

This module is designed to help you to:

- explain the phenomena of self-inductance and mutual inductance using Faraday's law;
- describe growth and decay of current in an inductor;
- describe working of AC generator and DC generator ;
- explain self-inductance as the electromagnetic analogue of mass in mechanics.

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Continuous Professional Development

6.3.1 Introduction

In the previous modules we discussed the concept of electromagnetic induction, induced e.m.f, and induced currents. We also discussed Lenz's which can be very well applied to find the direction of induced current in a loop when the magnetic flux is linked with the loop changes. We further discussed eddy currents and their applications in our daily life. In this module, we shall study the quantification of electromagnetic induction which will be done through the introduction of two terms – mutual and self-inductance. We shall also be dealing with devices like Inductor, AC generator, and DC generator which manifest the practical applications of the phenomenon of EMI.

6.3.2 Mutual inductance and self-inductance

We have seen that electric current can be induced in a coil by flux change produced in another coil placed in its proximity. It is seen that the magnetic flux ϕ_{21} linked with coil 2 due to a flow of current I_1 in Coil 1 is directly proportional to I_1 .

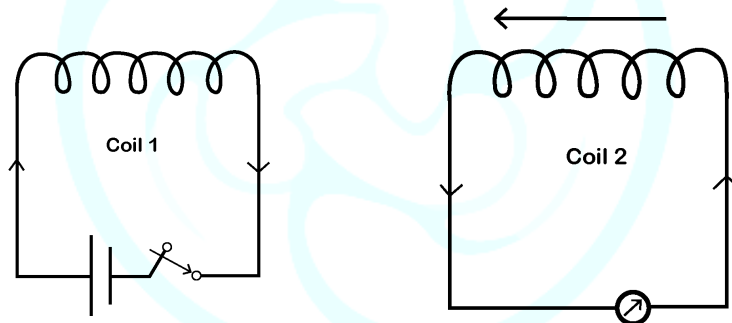


Fig. 6.3.1[(a) and (b)] Showing the phenomenon of mutual inductance between two coils

$$\text{i.e. } \phi_{21} \propto I_1$$

If the geometry of the coil does not vary with time,

then,

$$\frac{d\phi_{21}}{dt} \propto \frac{dI}{dt}$$

For a closely wound coil of N turns, the same magnetic flux gets linked with all the turns. When the flux through the coil changes, each turn contributes to the induced emf. Thus the total flux linkage $= N\phi_{21}$ and in such case,

$$N\phi_{21} \propto I_1$$

$$\text{So, } N\phi_{21} = M_{21}I_1 \quad (6.3.1)$$

where, M_{21} = a constant depending on the geometry of the coils '1' and '2' and it is known as the mutual inductance. From Faraday's law and Lenz's law, we have the emf induced in coil 2, given by

$$\varepsilon = - \frac{d\phi_B}{dt}$$

$$e = \frac{d}{dt}(N\phi_{21}) = - M_{21} \frac{dI_1}{dt} \quad (6.3.2)$$

We see that $|e| = M_{21}$, When $\frac{dI_1}{dt} = 1$

In other words, the mutual inductance between coil 1 and 2 is numerically equal to the emf induced in coil 2, when the rate of change of current in coil 1 is unity. It is also equal to the emf induced in coil 1 when the rate of change of current in coil 2 is unity. In other words,

$$M_{21} = M_{12} \quad (6.3.3)$$

The SI unit of mutual inductance is henry.

From, Eq(6.3.2), $\text{henry} = \frac{\text{volt} \cdot \text{sec}}{\text{ampere}} = \text{ohm} \cdot \text{sec}$

An electric current can be induced in a single isolated coil due to flux change produced in the same coil by varying the current through the same coil. This phenomenon is called self - induction. In this case, the flux linkage through a coil having N turn is proportional to the current through the coil and is expressed as,

$$N\phi \propto I$$

$$\text{or } N\phi = LI \quad (6.3.4)$$

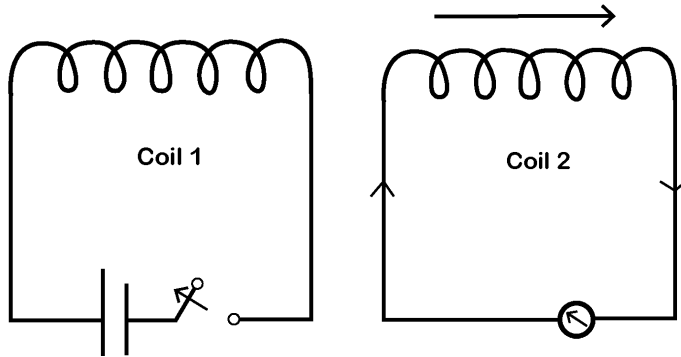


Fig.6.3.2 Showing the phenomenon of mutual inductance between two coils

Where L is a constant depending on the geometry of the coil, and is called the self inductance of the coil.

From Faraday's second law and Eq(6.3.4), the emf induced is given by

$$e = - \frac{d}{dt} (N\phi) = - \frac{d}{dt} (LI)$$

$$\text{or, } e = - L \frac{dI}{dt} \quad (6.3.5)$$

Thus

$$|e| = L, \text{ Where } \frac{dI}{dt} = 1$$

In other words, the self - inductance of a coil is numerically equal to the magnitude of the induced emf in itself when the rate of change of current through itself is unity.

Now, we shall learn to determine the mutual and self - inductances of some system by relating them with their geometry and other factors on which they depend.

6.3.3 Determination of mutual and self - inductance in special cases

First we shall take up the case of mutual induction between two coaxial coils.

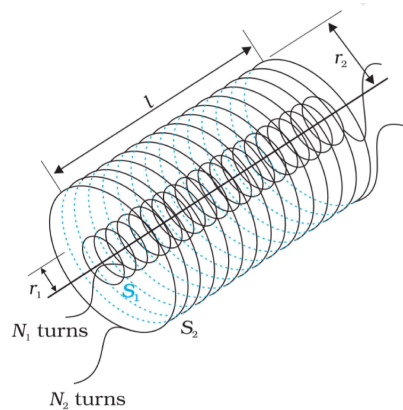


Fig.6.3.1 Two co-axial of same length

Here we have two coils in the form of two solenoids. The primary coil S_1 is surrounded by the secondary coil S_2 . It has been mentioned earlier quite clearly that the mutual and self - inductances depend on the geometry of the coil. Let us present these factors in a Tabular form.

Table :6.3.1 Mutual and self - inductances depend on the geometry of the coil

Factors Coils	Length	Total Number of turns	Number of turns per unit length	Radius	Current	Remarks
S_1	l	N_1	$n_1 = \frac{N_1}{l}$	r_2	I_1	A current I_2 in S_2 sets up a magnetic flux through S_1 . Similarly, I_1 through S_1 sets up flux in S_2
S_2	l	N_2	$n_2 = \frac{N_2}{l}$	$r_2, (r_2 < l)$	I_2	

Following Eq(6.3.1), we get

$$N_1 \phi_1 = M_{12} I_2$$

$$\phi_1 = B_2 A_1 = (\mu_0 n_2 I_2) (\pi r_1^2) \quad , \text{ Where } \mu_0 = \text{magnetic permeability of free space}$$

(6.3.6)

$$\therefore N_1 \phi_1 = (n_1 l) (\pi r_1^2) (\mu_0 n_2 I_2)$$

$$\therefore M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

(6.3.7)

$$\text{Similarly, it can be shown that } M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l \quad (6.3.8)$$

$$\therefore M_{12} = M_{21} = M \text{ (say)}$$

(6.3.9)

We have made two vital assumptions

1. The edge effects have been neglected and the value of the magnetic field in $S_2 (\mu_0 n_2 I_2)$ is considered to be uniform throughout the length and the width of the solenoid S_2 . It is an approximation based on the fact, $l \gg r_2$.

2. The flux due to the current I_1 in S_1 is considered to be confined solely inside S_1 since the solenoids are very long.

Eq(6.3.7) has been derived considering air as the medium within the solenoids. If instead, it had been a medium with relative magnetic permeability, then

$$M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l \quad (6.3.10)$$



Let Us Reflect

Q.6.3.1 We have obtained M_{12} for two very long coaxial solenoids. Could we have obtained M_{12} , had the inner solenoid been much shorter and placed well inside the outer solenoid?

Q.6.3.2 What would be the case with the calculation of M_{21} in the above situation?



Answers

A.6.3.1 We should have still obtained the flux linkage because the inner solenoids get effectively immersed in a uniform magnetic field due to the outer solenoid, and thus calculation of M_{12} becomes simpler.

A.6.3.2 Calculation of M_{21} will be difficult as the magnetic field due to the inner solenoid will vary across the length and cross-section of the outer solenoid.

We shall now obtain the value of the self-inductance of a single coil.

For calculation of self-inductance, we have to consider the flux linkage through a coil of say, N turns for a flow of current, I , through itself. We have,

$$N\phi_B \propto I$$

$$\therefore N\phi_B = LI \quad (6.3.11)$$

Where L is a constant depending on the geometry of the coil Eq(6.3.11) is effectively same as Eq(6.3.4)

Now,

$$N\phi_B = (\text{number of turns})(\text{magnetic field})(\text{area})$$

$$= (nl)(\mu_0 nI)(A)$$

$$= \mu_0 n^2 Al I$$

From Eq(6.3.11)

$$L = \frac{N\phi_B}{I}$$

$$\therefore L = \mu_0 n^2 Al \quad (6.3.12)$$

If the medium has a relative magnetic permeability then,

$$L = \mu_r \mu_0 n^2 Al \quad (6.3.13)$$

We shall now study the effect of inductance in an electronic circuit. An electrical accessory (ideally without any resistance) having inductance is called a pure inductor. Let us see how it affects the flow of current in a circuit.

6.3.4 Inductor

Inductors are basically coils or loops of wire that are either wound around a hollow tube (air cored), or wound around some ferromagnetic material (iron cored) to increase their inductance. Inductors can be regarded as a device to store energy in the form of the magnetic field that sets-up when a DC voltage is applied across their terminals.

The growth of the current flowing through an inductor is not instantaneous; the growth rate is determined by the inductor's own self-induced, or back emf. For a given inductor coil, this back emf voltage, V_L , is proportional to the rate of change of the current flowing through it (see Eq(6.3.5)).

The current, through an inductor, 'initially' continues to rise until it reaches its maximum steady state value(Fig.6.3.1). At this point, a steady DC current flows through the coil. Hence, there is no back emf induced, as I become time invariant, which can oppose the current flowing through it. The coil, therefore, now acts just like a resistor (having a resistance equal to the resistance of the wire of the coil) allowing maximum current to flow through it.

Ideally the current takes infinite time to attain the value $I_{\max}$. The I vs. t curve approaches the ordinate through $I_{\max}$ asymptotically.

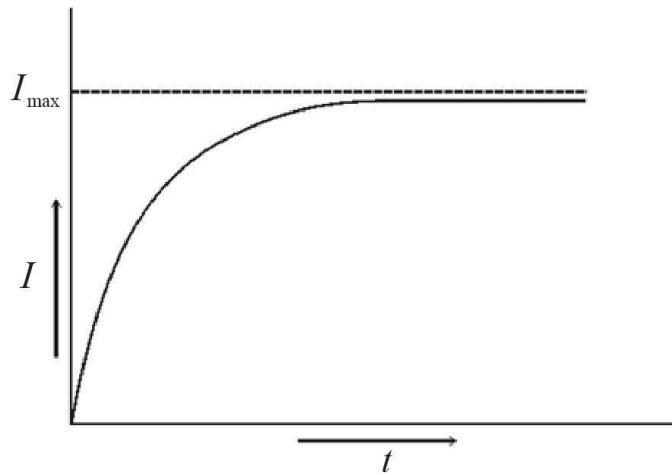


Fig. 6.3.4 Growth of current in a circuit containing inductor (having some resistance)

Now, let us see what happens when a circuit is switched off.

When current is switched off in a circuit containing an inductor, it again does not reach its zero value instantly; it decreases slowly to its final value (nearly zero), showing an exponential decay (Fig. 6.3.5). The more/ less is the value of the inductance of the inductor, the slower/ faster is this decay, i.e., the more/less the time the current takes to decay to its final value (nearly zero). Here also the approach of the value of I towards zero is asymptotic and ideally it takes infinite time for the current to become zero.

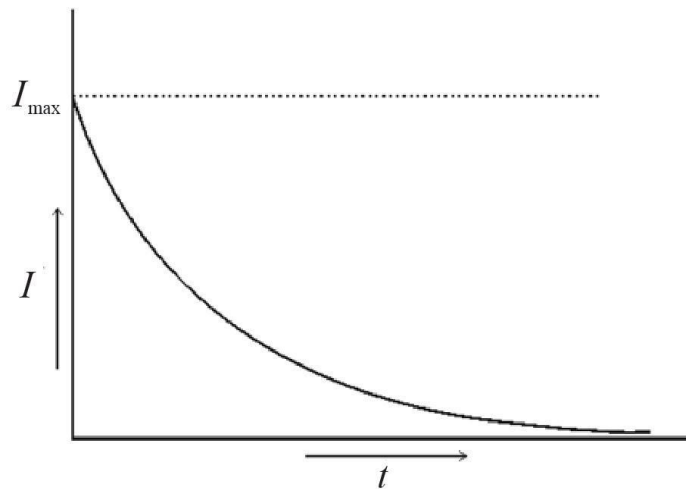


Fig. 6.3.5 Current decreasing exponentially with time



Let Us Reflect

Q.6.3.3 A solenoid is connected to a battery so that a (certain) steady current flows through it. If an iron core is inserted into the solenoid, will this current value increase, decrease or remain unchanged. Explain.



Answers

A.6.3.3 By insertion of iron core, the self inductance increases and thus there will be an instantaneous increase in back emf. So the current decreases at the instant of insertion of the core. We shall now study the application of Faraday's law in explaining the working of AC and DC Generation.

6.3.5 AC and DC generator

Before explaining the principle and working of a generator, let us carry out the following activity.

Activity 6.3.1

Take a simple dc motor. Now connect an LED (in forward bias mode) to the two ends of the terminals of the motor and rotate the motor (mechanically, or otherwise). What do you observe?

The LED glows!

If the motor is kept rotating, the LED keeps on glowing. One can, therefore, say that the rotating motor is converting mechanical energy into electrical energy, i.e. it is acting as a generator.

We shall now see that the basic construction of a motor and a generator are the same. A motor, while rotating, works as a generator also, generating an emf as ‘back emf’.

We shall now study the application of Faraday's law in explaining the working of AC and DC Generator.

6.3.5.1 AC generator

Perhaps the most important application of the phenomenon of electromagnetic induction is the generation of alternating currents with the help of (AC) generators.

Refer to Fig 6.3.6 (a). A coil of many turns (called armature) is rotated mechanically in a uniform magnetic field. Due to rotation of the coil, in the magnetic field, the magnetic flux through it changes and an emf gets induced in it. The variation of emf with time is sinusoidal as shown in Fig 6.3.6(b). The ends of the (armature) coil are connected to an external circuit by means of (suitable) slip rings and brushes.

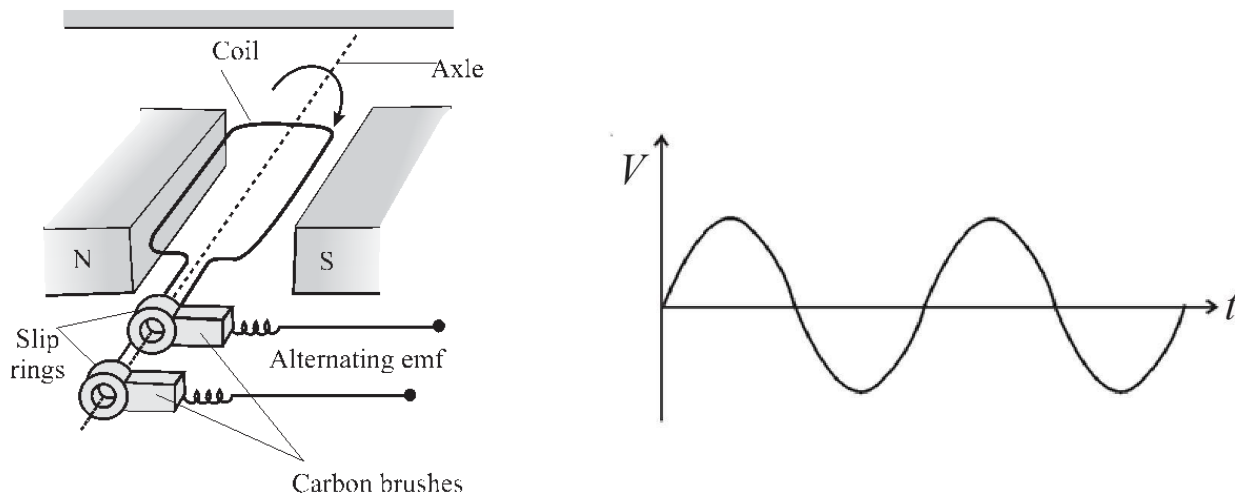


Fig.6.3.3 (a) showing schematic diagram of AC generator **(b)** showing the variation of EMF with time

[**Note:** The basic difference, between an ac generator and a dc generator, is in the way the two ends of the coil are connected to the external circuit. In the ac generator, the ends of the coil are connected to the external circuit by means of slip rings and brushes; in a dc generator, the ends are connected to a commutator or split rings, (made of copper) which serve as its ‘terminals’ for the external circuit.

Why are slip rings and brushes used for connecting the external circuit to the ends of the armature coil? It can be said that this ensures that the connecting wires, to the external circuit, do not keep on rotating and getting twisted continuously due to the continuous rotation of the armature coil.

How does the use of a split ring commutator help in converting the generator’s (alternating emf) output into a unidirectional (loosely referred to as DC) output?

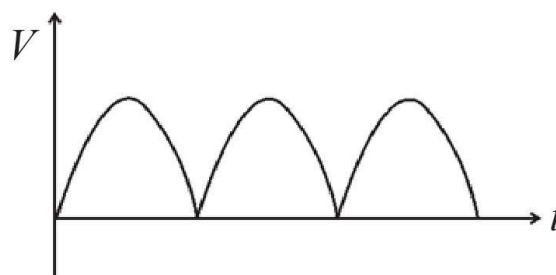


Fig. 6.3.7 A unidirectional voltage

This can be understood in a simple way as follows. The split rings of the commutator (connected to the external circuit) are in touch with alternate brushes after every half rotation. These brushes change their polarity after every half rotation. Hence the polarity of each part of the split ring commutator remains the same throughout the complete rotation of the armature coil. The current, in the external circuit, is, therefore, unidirectional, i.e., it always flows in the same direction.

6.3.5.2 DC generator

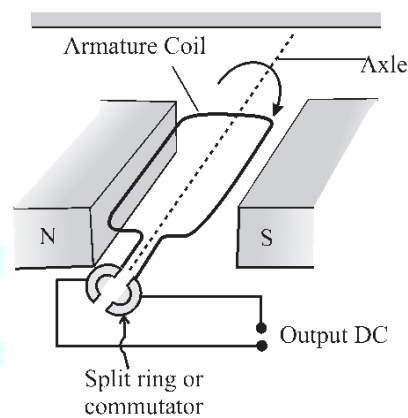


Fig. 6.3.8 Schematic of DC generator

An AC generator can be converted into a DC generator by replacing the slip rings and carbon brushes with split rings. Split rings are metal rings divided into two isolated halves (segments) which are mounted in the axis. Now each terminal of the armature coil is in contact with a split ring. When the armature coil rotates within the magnetic field an AC voltage (EMF) is induced in the armature coil exactly as in an AC generator. However, since the split rings are fixed therefore the terminal of the armature coil in contact with a particular split ring is for only one half cycle. For the other half cycle the armature coil terminal moves to the other split ring. Thus before the sign of the induced voltage changes on the terminal of the armature coil at the end of a half cycle it has left contact with that split ring.

In this manner, each split ring acts as a mechanical rectifier such that it converts the AC input into a DC output of the form shown in Fig 6.3.8



Let Us Reflect

Q.6.3.4 Where does the energy, obtained from an ac generator, come from?

Q.6.3.5 Motional emf is in a way, just an application of Faraday's law to a specific case. Why is it then viewed/ regarded as an important example that needs to be discussed explicitly?

Q.6.3.6 In an ac generator, the magnetic flux and the emf, do not attain their maximum values simultaneously. How can one justify this?



Answers

A.6.3.4 It comes from the mechanical energy of rotation of the armature coil.

A.6.3.5 Faraday's law, as we know, relates the rate of change of magnetic flux with the induced emf. Now Magnetic flux equals $\int_s B \cdot dA$, and emf equals $\oint E \cdot dl$. Faraday's law is, therefore, providing us a link between magnetic and electric fields.

The special case of motional emf helps us to have a simple justification for Faraday's law. In this case, we can calculate the induced emf

(i) by applying Faraday's law, i.e. by calculating the rate of change of magnetic flux

and (ii) by using the basic definition of emf $\left(emf = \oint E \cdot dl \right)$. For this calculation we use the Lorentz force,

acting on the moving charges, and calculate the (induced) emf through its definition given above.

We find that the values of the induced emf, calculated via these two different approaches, are identical.

[Important Note: The electric field (viewed simply as force per unit charge), associated with the Lorentz force on moving charges, is not quite the same as the electric field defined in electrostatics. It does not have,

for example, the conservative nature that is an important characteristic of the electric field due to static charges, i.e. electric fields in electrostatics. The only similarity is that here also, we are referring to the force per unit charge as an electric field.]

A.6.3.6 The induced emf, as we know, equals the rate of change of magnetic flux, i.e.

$$\varepsilon = \frac{-d\phi}{dt}$$

Both ϕ and ε are functions of time, but ε , being the rate of change of ϕ with time, need not have the same type of time dependence as ϕ has. In general, a function and its rate of change, i.e. its derivative, do not have a similar nature. This is easily visualised by looking at the graph of $y = \sin\theta$ as a function of $x = \theta$. We see from the graph that at

$x = \theta$, $y = \sin\theta$

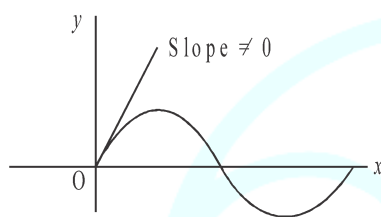


Fig.6.3.9

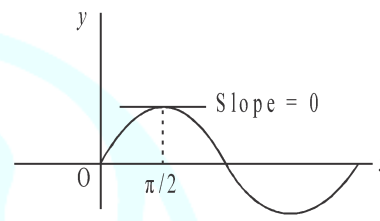


Fig.6.3.10

But $\frac{dy}{dx}$ = slope of the tangent to the graph, at $x = 0$ is not zero. but it is 1.

Again, at $x = \frac{\pi}{2}$, $y (= \sin\theta) = 1$

However, $\frac{dy}{dx}$ = slope of the tangent to the graph, at $x = \frac{\pi}{2}$ is zero.

In the case of ac generator, $\phi = NBA \cos\omega t$

and $\varepsilon = \frac{-d\phi}{dt} = NBA\omega \sin\omega t = NBA\omega \cos\left(\omega t - \frac{\pi}{2}\right)$

The induced emf, ε , is thus seen to be $\frac{\pi}{2}$ lagging in phase w.r.t. ϕ . Thus when ϕ is maximum, ε has its minimum value and vice-versa.

6.3.6 Self inductance : the electromagnetic analogue of mass in mechanics

In mechanics, the tendency of a body to maintain its status quo (i.e., to maintain its state of rest or of uniform motion or to resist/oppose any change in the existing condition) increases with an increase in its mass. The more the mass of a body, the more is the external force needed to bring a given change in its state of motion or rest.

In electromagnetism, self-inductance plays a similar role. The more the self-inductance of a coil, the more is the (self) induced emf in it. This induced emf, as per Lenz's law, opposes the cause to which it is produced. Thus when the external emf is trying to increase the current in a circuit, the (self) induced emf tends to oppose that increase i.e., it tends to maintain the status quo. The self-induced emf again does a similar, status quo-maintaining, action when the current in the circuit tends to decrease. This implies that the more 'self-inductance' of a circuit, the more would be the energy that an external source would have to spend, to bring about a change in the current following in the circuit.

The last statement follows from the fact that the energy needed to establish a current I in a circuit, equals $\frac{1}{2} LI^2$ where L is the self - inductance of that circuit.

We thus see a clear similarity between the role of self - inductance in electromagnetism and that of mass in mechanics. We are thus quite justified in regarding self-inductance as the electromagnetic analogue of mass in mechanics.

6.3.7 Summary

- When current flowing through a coil varies with time, a current is induced in a neighbouring coil to oppose this change. This phenomenon is called mutual induction.
- Mutual induction between two coils is quantified by mutual inductance M which is numerically equal to the magnetic flux linked with the second coil when the current flowing through the first coil is unity.

- When current flowing through a coil varies, a back emf and hence a current is induced in the same coil to oppose this change. This phenomenon is called self - induction. It is also called as inertia of electricity.
- Self - induction of a coil is quantified by self - inductance or inductance L which is numerically equal to the magnetic flux linked with the same coil when the current flowing through it is unity.
- An inductor is a device that exhibits self-inductance. These are basically coils or loops of wire that are either wound around a hollow tube or around some ferromagnetic material (iron, steel, nickel, cobalt) to increase their inductance. They can be regarded as a device to store energy in the form of the magnetic field that sets-up when a DC voltage is applied across their terminals.
- An AC generator is a device that converts mechanical energy into electrical energy by way of the mechanical motion of a coil in a magnetic field.
- An electric motor working in reverse orientation works as an AC generator.
- In a DC generator current is made unidirectional by replacing slip rings used in AC generators with split rings or commutators.
- Inductance may be viewed as electrical inertia. We find a clear similarity between the role of self-inductance in electromagnetism and that of mass in mechanics.

Development Team

C.K. Ghosh, Former Director, NCIDE, IGNOU, New Delhi

Review Team

Atul K. Mody, Former Associate Professor, VES College, Mumbai

Nidhi Kalucha, PGT, Sanskriti School, New Delhi

Suruchi Agrawal, PGT, Amity International School Sector-1, Vasundhara Ghaziabad

G.D.Sharma, Vice Principal, KV, Sector-8, Rohini, New Delhi

Member- Coordinator

Shashi Prabha, Professor, CIET, NCERT, New Delhi

Technical support

Riya Rani, JPF, CIET, NCERT, New Delhi

Bhawana Bhatt, Technical Content Developer, CIET, NCERT, New Delhi

Rajat Kumar, Graphic Designer, CIET, NCERT, New Delhi