

13.Nuclei

Module 2: Nuclear Binding Energy

This module is designed to help you to:

- explain the basic concept of binding energy;
- describe the concept of the mass defect and its relation with binding energy;
- discuss the relative stability of nuclei in terms of B.E./nucleon;
- elaborate the relation between B.E./nucleon and mass number of nuclei;
- deduce the characteristics of the binding energy curve.

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13.2.1 Introduction

We have already learnt that all substances are made up of tiny basic building blocks - the atoms. The atoms have a very tiny central core, known as the nucleus. The nucleus contains the entire positive charge and practically the whole mass of the atom. An atomic nucleus has Z protons (positively charged) and N neutrons (electrically neutral) contained in a small space of a sphere of radius of the order of 10^{-14} m. Since we generally expect the atom to be electrically neutral, the number of protons in the nucleus is equal to the number of orbiting electrons. This number is denoted by Z and is called the atomic number of the nucleus.

The size of the nucleus is extremely small. It has been estimated to be about 10^{-14} m. This estimate is based on the Rutherford experiment of scattering of α -particles by gold nuclei and other such experiments.

In an atom, the electron-nucleus system is a bound system because of the attractive forces between the nucleus and the electrons. We express this fact by saying that there is a binding energy associated with each electron in the atom. This indicates that a minimum amount of energy needs to be supplied externally to break the electron-nucleus bound system.

Bohr's theory leads us to the result that the energy of the electron, in the ground state of the hydrogen atom, is nearly -13.6 eV. This implies that an electron, in the ground state of hydrogen, needs to be supplied a minimum of 13.6 eV of energy externally to make it free from the nucleus.

Just as there is an attractive force between the electrons and the nucleus of the atom, there are very strong attractive forces between the nucleons in a nucleus. These forces, known as nuclear forces, cause the nucleons within the nucleus to form a bound system. The binding energies of the nucleons (common name for protons and neutrons) in a nucleus are several orders of magnitude higher than the binding energies of the electrons in an atom. The binding energies of nucleons are of the order of MeVs, i.e. these are 10^4 to 10^5 times the binding energies of electrons to the atom. It is, therefore, much more difficult to bring about a change in the constitution of the nucleus than to bring a change in the constitution of an atom.

Nuclei are generally very stable structures. A large amount of energy is required to break up a nucleus into its constituent particles. This energy is called its binding energy and the binding energy per nucleon is a measure of its stability.

In this module, we discuss the stability of nuclei by plotting the binding energy/nucleon as a function of mass number. It is pointed out that the process of fusion can take place among light nuclei and the process of fission can take place among heavy nuclei. In both cases, the product nuclei are more stable than the original nuclei.

In this module, we will be using the binding energy curve to discuss the characteristics of nuclear forces which keep the nucleons bound in the nucleus.

13.2.2 Binding energy of a nucleus

Let us first try to get a general idea of the term binding energy. Whenever a particle is present in an attractive force field, it would need some (external) energy to free itself from that field. For example, any particle present in earth's gravitational field has to be supplied a certain amount of kinetic energy to enable it to escape this field. We can think of this kinetic energy supplied to it to escape from earth's gravity, as its binding energy in the earth's gravitational field.

A similar consideration would hold for a particle in any other attractive force field. The nucleons, in a nucleus, can be thought of as bound by a force field. The energy needed to separate a proton/neutron from a given nucleus would be the binding energy of the proton/neutron in that nucleus. The total energy needed to separate out all the protons and neutrons of a given nucleus equals the 'binding energy' of that nucleus.

A combination of Z protons and $(A-Z)$ neutrons that forms a nucleus A_ZX , is observed to have a mass $m({}^A_ZX)$ that is smaller than the sum of the masses of the Z protons and $(A-Z)$ neutrons that constitute it. This lost mass or mass defect in units of mass, $\Delta m [= A - m({}^A_ZX)]$ is the mass defect of the given nucleus. The mass defect can also be expressed as equivalent energy $(=\Delta m \cdot c^2)$ according to the mass-energy equivalence relation $(E = m c^2)$.

It is only when this energy is supplied to the nucleus A_ZX , that it can be separated into its constituents [Z protons and $(A-Z)$ neutrons]. This is how we calculate the binding energy of the nucleus.

It is clear, therefore, that a nucleus with a higher binding energy would be more stable than the one with a lower binding energy. It also follows that a nuclear reaction, or change, that results in an (overall) increase in binding energy, would cause energy to be released.

Example 13.2.1

The mass defect of ${}^7_3\text{Li}$ equals 0.042135 u. Calculate its binding energy per nucleon.

Solution :

Recall that u is an atomic mass unit, equal to 1.66×10^{-27} kg. If we convert it to energy, via the Einstein relation $E = mc^2$, then it has the value 931.494 MeV/u. Therefore, the binding energy is $0.042135 \text{ u} \times 931.494 \text{ MeV/u} = 39.25 \text{ MeV}$

Since it has 7 nucleons, the binding energy per nucleon = 5.607 MeV/nucleon. Binding energy of other nuclei can be calculated in a similar manner.

13.2.3 Binding energy per nucleon

The total binding energy of a nucleus A_ZX , when divided by its mass number A (i.e. by the total number of nucleons in the nucleus), gives the binding energy per nucleon. This may be thought of as the average energy/nucleon required to separate a nucleus into its constituent nucleons.

A study of the dependence of the binding energy per nucleon on the mass number A would give us a better understanding of the mutual interaction between a pair of nucleons. In other words, this study would give us a better and clearer picture of the nature and characteristics of forces that exist between nucleons.

The curve of binding energy per nucleon, as a function of the mass number, has the form shown in Fig 13.2.1

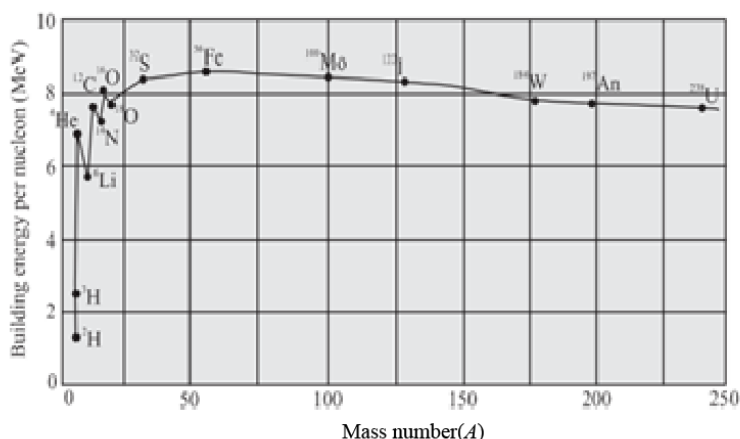


Fig. 13.2.1: The binding energy per nucleon as a function of mass number

13.2.4 Binding energy curve and its characteristics

- The B.E./nucleon has values of the order of MeV/nucleon for all the nuclei.
- For low mass number nuclei, the B.E./nucleon has a lower value that generally increases with an increase in mass number. There are, however, some mass number ranges [mass number (5 to 7), (12 to 14), (16 to 18)] in which the B.E./nucleon shows a decrease with the increase in mass number.
- In the lower mass number range, the B.E./nucleon, the binding energy graph (Fig.13.2.2) shows peaks at $A = 4$, $A = 8$, $A = 12$ and $A = 16$.

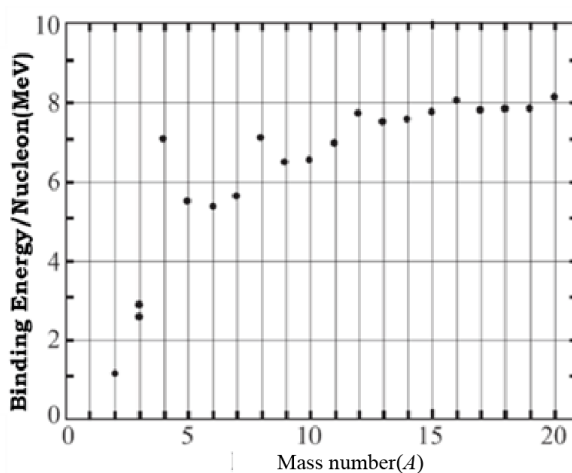


Fig. 13.2.2 : Details of the binding energy curve for nuclei of mass number in the range 1 – 20. Notice the peaks at $A = 4$, $A = 8$, $A = 12$ and $A = 16$.

- (iv) From $A=30$ to $A=170$, the curve gives a nearly constant value (~ 8 MeV/nucleon). The curve peaks in the range $A=50$ to $A=60$ with $A=56$ showing the highest value (~ 8.75 MeV/nucleon) of B.E./nucleon. ${}^{56}\text{Fe}$ is, thus, the most stable nucleus in the universe.
- (v) Beyond $A=170$, the B.E./nucleon decreases to a value of nearly 7.6 MeV/nucleon for $A=238$.

[Note: Nuclei having $A > 238$, are not found in noticeable amounts in the earth's crust. The heaviest known stable nucleus is ${}^{209}_{83}\text{Bi}$. Nuclei with $Z > 83$ and $A > 209$ tend to decay spontaneously.]

We now try to understand how the above features of this curve enable us to get some ideas about the characteristics of the nuclear force.

a) The quite large values ($\sim$ MeV/nucleon) of the binding energy per nucleon indicate that the nuclear force must be a very strong attractive force. Here it is worth noticing that the binding energies of the electron in an atom have values only in eV. This force (within the nucleus) has to be very much stronger than the electrical force as it is able to hold together, or bind, the protons in spite of the (large) repulsive electrical forces ($>200\text{N}$) they exert on one another.

b) Let us, for a moment, imagine that each nucleon interacts with all the other nucleons in the nucleus. For a nucleus of mass number A , the net interaction is then likely to increase as $\frac{A(A-1)}{2}$. We can understand this by realizing that two nucleons, out of a total of A nucleons, can be paired in ${}^A C_2$ ways. This would imply that the B.E./nucleon should keep on increasing with increasing A , rather than maintaining a near constant value over a large range of values of A . On the other hand, if we assume the nuclear interaction to be saturated and short range, that is, a nucleon interacts only with its immediate neighbors, the B.E./nucleon would be (nearly) independent of A .

The observed (near) constant of B.E./nucleon from $A=30$ to $A=170$, thus, helps us to think of the nuclear force as a short-range (saturated) force, implying that a particular nucleon interacts effectively only with its immediate neighbors.

- a) The nuclear size being nearly proportional to $A^{1/3}$, the ratio of its surface area to its volume would change as $\frac{A^{2/3}}{A}$; that is as $A^{-1/3}$. Nuclei with a lower value of A would then have a higher value for surface area to volume ratio. We can reasonably expect the 'surface nucleons' to be less tightly bound than the interior ones. This implies that the B.E./nucleon is likely to have a lower value for low mass number nuclei. This is in accord with the experimental observations.
- b) The helium nucleus ($Z=2$, $A=4$), having two neutrons and two protons, is a relatively stable nucleus as it has a larger value of B.E./nucleon as compared to its neighbors. We can understand this stability in terms of the very tight binding of its pairs of two neutrons and two protons. This stable nature of the helium nucleus also helps us to understand the more stable nature of nuclei with $A=8$, $A=12$ and $A=16$, that is, nuclei whose mass numbers are integral multiples of 4, the mass number of helium.

- c) The decrease in B.E./nucleon, for $A > 170$, can be understood in terms of the increasing repulsive effect of the larger number of protons present in them. The combined effect of the repulsive Coulomb forces and the attractive nuclear forces, results in a decrease in B.E./nucleon for $A > 170$.
- d) Beyond $Z > 83$ and $A > 209$, the effects of the repulsive Coulomb forces become large enough to make such nuclei undergo spontaneous decay on their own.
- e) The half-life for such spontaneous decay becomes very small for nuclei with $Z > 92$ and $A > 238$. Hence elements with atomic number higher than that of uranium (called trans-uranic elements) are not found in nature.
- f) The B.E./nucleon, both for $A < 30$, and $A > 170$, is (generally) smaller than that for nuclei in the range $A=30$ to $A=170$. The fusion of two light nuclei, or the fission of a heavy nucleus in two or more nuclei, is, therefore, likely to result in the increase of the B.E./nucleon of the product nuclei. Such a change, as we have already noticed, would result in a release of energy. We, therefore, get nuclear energy, both through the process of nuclear fusion and through the process of nuclear fission(Fig.13.2.3).

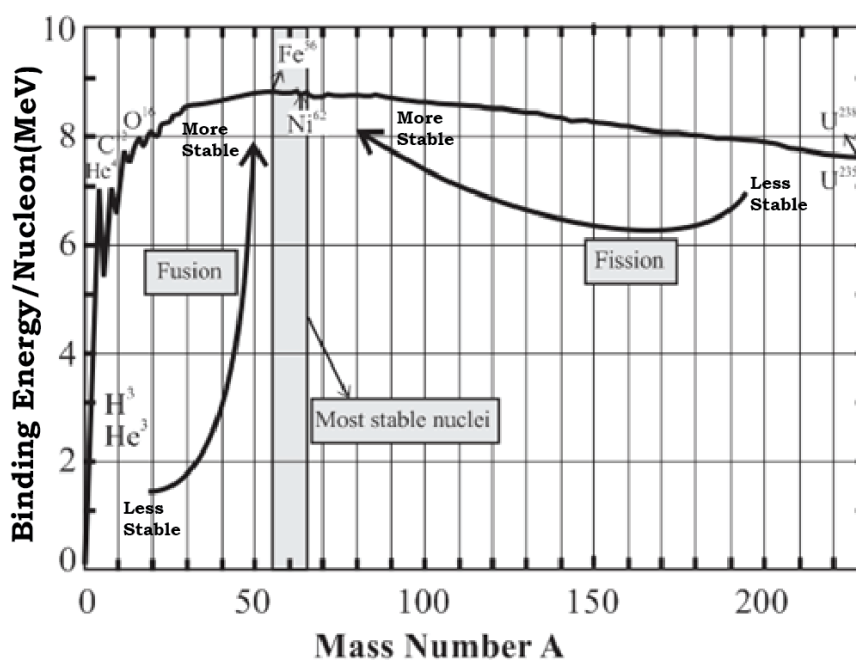


Fig. 13.2.3: The nuclear binding energy curve shows that nuclei with A in the range 55 – 65 are the most stable nuclei. Nuclei on the right of this band can undergo fission to produce nuclei that are closer to this stable band. Nuclei to the left undergo fusion to produce nuclei that are closer to this band.

13.2.5 Stability of nuclei

It is important to notice that an increase in B.E./nucleon occurs during fusion of hydrogen nuclei to form a helium nucleus. Nuclear fusion is, therefore, a source of energy.

We now have strong reasons to believe that the nuclear fusion reaction is the basic source of the enormous energy radiated by the sun and the stars.

It is observed that there are certain peaks in the B.E./nucleon versus mass number curve. A close scrutiny reveals that these peaks occur for nuclei whose proton or neutron number or both have the values 2, 8, 20, 28, 50, 82 or 126. For this reason, these numbers have been given the name magic numbers in nuclear physics.

Nuclei with any of the magic number protons i.e. with

$Z = 2, 8, 20, 28, 50, 82$

or, those with a magic number of neutrons $N = 2, 8, 20, 28, 50, 82$ and 126 are found to be very stable.

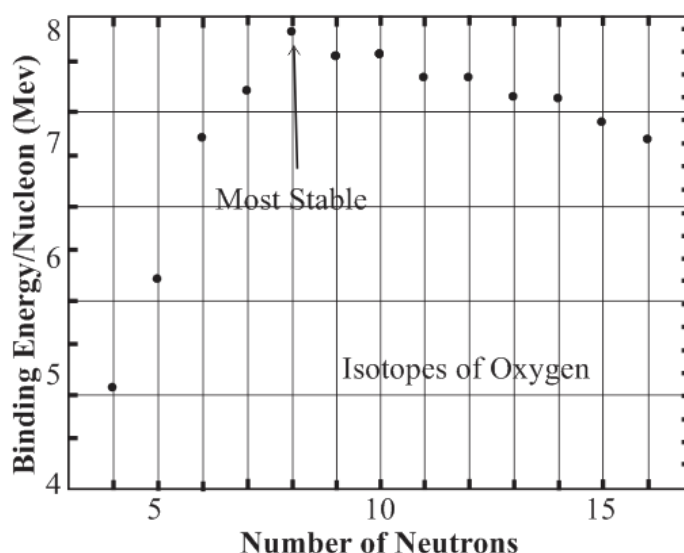


Fig. 13.2.4: Binding energy/nucleon of the isotopes of oxygen. The most stable nuclide (shown by an arrow) is the one with both proton number and neutron number having the value 8, a magic number



Let Us Reflect

[Use: $m_n = 1.008665 \text{ u}$, $m_p = 1.007276 \text{ u}$ & $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}$]

Q.13.2.1 Obtain the binding energy (in MeV) of a nitrogen nucleus.

[Given: $m(^{14}\text{N}) = 14.00307 \text{ u}$]

Q.13.2.2 Calculate the binding energy per nucleon of the nuclei ^{56}Fe and ^{209}Bi from the following data:

$m(^{56}\text{Fe}) = 55.92068 \text{ u}$ and $m(^{209}\text{Bi}) = 208.980399 \text{ u}$.

Which one has greater B.E.per nucleon?

Q.13.2.3 A given coin has a mass of 3.0 g. Calculate the energy that would be required to separate all the neutrons and protons from one another in the nuclei of all the atoms making up this coin. Assume that the coin is entirely made of $^{63}_{29}\text{Cu}$ atoms each of mass 62.92960 u.

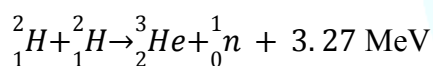
Q.13.2.4 If 200 MeV energy is released in the fission of a single U^{235} nucleus, How many fissions are required per second to produce 1 kilowatt power? (Given $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$)

Q.13.2.5 A 1000 MW fission reactor uses U-235 as fuel and consumes half of its fuel in five years. How much did it contain initially? Assume

- (i) that the reactor operated 80% of the time,
- (ii) that all the energy generated arises from the fission of U-235, and
- (iii) that this nuclide was consumed only by the fission process.

[Given: Energy released per fission: 200 MeV]

Q.13.2.6 How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as



Answers

A.13.2.1 number of protons in Nitrogen atom = 7

number of neutrons in Nitrogen atom = 7

$$\text{B.E.} = \Delta m c^2$$

$$[(Zm_p + Nm_n) - m_{\text{tot}}] c^2$$

$$= [(7 \times 1.007276 + 7 \times 1.008665) - 14.00307] \times 931.5 \text{ MeV}$$

$$= 101.1 \text{ MeV}$$

A.13.2.2 B.E. per nucleon of the nuclei ^{56}Fe :

number of protons in $\text{Fe} = 26$

number of neutrons in $\text{Fe} = 30$

$$\text{B.E. / nucleon} = \Delta m c^2 / A$$

$$= [(Zm_p + Nm_n) - m_{\text{tot}}] c^2 / A$$

$$= [26 \times 1.007276 + 30 \times 1.008665] - 55.92068 \times 931.5 \text{ MeV} / 56$$

$$= [8.79] \text{ MeV}$$

B.E. per nucleon of the nuclei of ^{209}Bi :

number of protons in $\text{Bi} = 83$

number of neutrons in $\text{Bi} = 126$

$$\text{B.E. / nucleon} = \Delta m c^2 / A$$

$$= [(Zm_p + Nm_n) - m_{\text{tot}}] c^2 / A$$

$$= [83 \times 1.007276 + 126 \times 1.008665] - 208.98 \times 931.5 \text{ MeV} / 83$$

$$= [7.65] \text{ MeV}$$

Hence, ^{56}Fe has greater B.E. per nucleon.

A.13.2.3 atom $\rightarrow {}^{63}_{29}\text{Cu}$

No. of atoms in 3g of coin

$$= \frac{6.023 \times 10^{23} \times 3}{63}$$

$$= 2.868 \times 10^{22}$$

No. of protons in $\text{Cu} = 29$

No. of neutrons, in $\text{Cu} = 34$

$$\text{Mass defect} = 29 \times 1.00783 + 34 \times 1.00867 - 62.929 = 0.59225 \text{ u}$$

$$\begin{aligned}\text{Total mass defect of all atoms} &= 0.59225 \times 2.868 \times 10^{22} \\ &= 1.6985 \times 10^{22} \text{ u}\end{aligned}$$

$$\begin{aligned}\text{So nuclear energy required} &= 1.6985 \times 10^{22} \times 931 \text{ Mev} \\ &= 1.58 \times 10^{25} \text{ Mev}\end{aligned}$$

A.13.2.4 Since 200 MeV energy is released per fission, i.e

$$\begin{aligned}200 \text{ MeV} &= 200 \times 1.6 \times 10^{-13} \\ &= 3.2 \times 10^{-11} \text{ J}\end{aligned}$$

$\therefore$ No. of fission per second

$$n = \frac{\text{Total energy released per second}}{\text{Energy released per fission}} \quad \text{or } n = \frac{10^3}{3.2 \times 10^{-11}} = 3.125 \times 10^{13}$$

A.13.2.5 In the fission of one nucleus of ${}_{92}\text{U}^{235}$ = energy generated is 200 MeV. Energy generated

$$\begin{aligned}\text{in fission of 1 kg of } {}_{92}\text{U}^{235} &= 200 \times \frac{6 \times 10^{23}}{235} \times 1000 \text{ MeV} \\ &= 5.106 \times 10^{26} \text{ MeV} \\ &= 8.17 \times 10^{13} \text{ J}\end{aligned}$$

Since the reactor operate 80% of time , So time of operation is $\frac{80}{100} \times 5 = 4$ years

$$\text{Total energy generated in 5 Years} = 1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4 \text{ J}$$

$$\begin{aligned}\therefore \text{Amount of energy consumed in 5 Years} &= \frac{1000 \times 10^6 \times 60 \times 60 \times 24 \times 365 \times 4}{8.17 \times 10^{13}} \text{ kg} \\ &= 1544 \text{ kg}\end{aligned}$$

So , initial amount of ${}_{92}\text{U}^{235} = 2 \times 1544 \text{ kg}$
 $= 3088 \text{ kg}$

A.13.2.6 No. of atoms in 2 kg deuterium

$$= \frac{6.023 \times 10^{23} \times 2000}{2} = 6.023 \times 10^{26}$$

Energy released when 2 atoms fuse = 3.27 Mev

$$\text{Thus , Total energy released} = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ Mev}$$

$$= 15.75 \times 10^{13} \text{ J}$$

Energy consumed by the bulb Per second = 100 J

$$\text{Thus, time for which the bulb glows} = \frac{15.75 \times 10^{13}}{100} \text{ s}$$

$$= 15.75 \times 10^{11} \text{ s}$$

$$= 4.99 \times 10^4 \text{ years}$$

13.2.6 Summary

- The energy required to break a nucleus into its constituent nucleons is called its binding energy.
- Average binding energy per nucleon (B.E./nucleon) is the binding energy divided by the mass number of the nucleus.
- When plotted as a function of the mass number, B.E./nucleon increases rapidly at first, reaches a maximum at $A \approx 60$ and then remains almost constant for A up to 170.
- A nucleus with the maximum B.E./nucleon is the most stable nucleus.
- Near constancy of B.E./nucleon for $30 \leq A \leq 170$ indicates that the nuclear forces are short range forces.
- Lighter nuclei can fuse, producing nuclei with higher B.E./nucleons, which are more stable than the original nuclei. In the process of fusion, energy is released.
- Heavier nuclei can undergo fission breaking into smaller nuclei which are more stable than the original nuclei. In the process of fission, energy is released.
- It is observed that there are certain peaks in the B.E./nucleon versus mass number curve. A close scrutiny reveals that these peaks occur for nuclei whose proton or neutron number or both have the values 2, 8, 20, 28, 50, 82 or 126. For this reason, these numbers have been given the name magic numbers in nuclear physics.

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