

Module 2 : Refraction at Plane and Spherical Surfaces

This module is designed to help you to:

- explain the phenomena of refraction and Snell's law of refraction;
- apply the Snell's law of refraction and concept of refractive index to study the lateral shift of light rays through a glass slab;
- visualize phenomena of total internal reflection as a consequence of refraction from a denser to a rarer medium and conditions involved therein;
- perform activities based on the refraction from spherical surfaces;
- derive relation between object and image distance in terms of refractive index of the medium and radius of curvature of the curved spherical surface.

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9.2.1 Introduction

A straw dipped in a beaker filled with water, appears to be bent, a swimming pool appears shallower than it is, mirages are observed in hot deserts. These are some effects of the refraction of light and the change in the direction of obliquely incident light as it passes from one transparent medium to another. The phenomenon is a consequence of the change in the value of the speed of light as it travels from one medium to another. In the 1600s Dutch mathematician W. Snell developed a law assigning a value called refractive index, later on, another Dutch scientist C. Huygens proposed that the refractive index of a medium is related to the speed at which light travels in the medium.

In this module, we shall obtain the expression for the lateral shift of a ray when it undergoes refraction by way of passing from one medium to another. We shall study the connection between the refractive index of a medium with real and apparent depth. These studies would be restricted to the passage of light from a rarer medium to a denser medium. We shall also analyze the situation when light travels from a denser to a rarer medium, and as a consequence of that, we shall introduce the phenomenon of total internal reflection. We shall also study about refraction of light through a spherical surface,

9.2.2 Refraction

When light passes from one transparent medium to another transparent medium, by way of oblique incidence, the ray of light deviates from its path at the boundary separating the two media. This phenomenon is called refraction.

Laws of refraction

- The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.
- Snell's law, i.e., $\frac{\sin i}{\sin r} = \text{constant}$, where i and r are the angles of incidence and refraction respectively.

This constant is characteristic of the two transparent media involved in refraction and is known as the relative refractive index n_{21} of the second medium with respect to the first medium. n_{21} is the ratio of speed of light in the first medium to that of the second medium, i.e.

$$n_{21} = \frac{v_1}{v_2} \quad (9.2.1)$$

The medium with a larger refractive index is known as an optically denser medium and the one lesser refractive index is known as an optically rarer medium. After defining the refractive index, we may express Snell's law in mathematical form as under:

$$\frac{\sin i}{\sin r} = n_{21} \quad (9.2.2)$$

Refraction at a plane surface

As a consequence of Snell's law ,

- 1) When a ray of light passes from a rarer to a denser medium, it bends towards the normal.
- 2) When a ray of light passes from a denser to a rarer medium, it bends away from the normal.

Activity 9.2.1 Lateral shift of a ray refracted through a rectangular glass slab

Take a rectangular glass slab. Place it on a plane sheet of paper. Draw an incident oblique ray and emergent ray using pins (Fig.9.2.1). Remove the glass slab and draw the refracted ray inside the glass slab. Measure the lateral shift produced by the glass slab by measuring the perpendicular distance between the extended incident ray and the emergent ray.

Discuss the factors on which the lateral shift depends (angle of incidence, thickness of the glass slab, refractive index, wavelength of light).

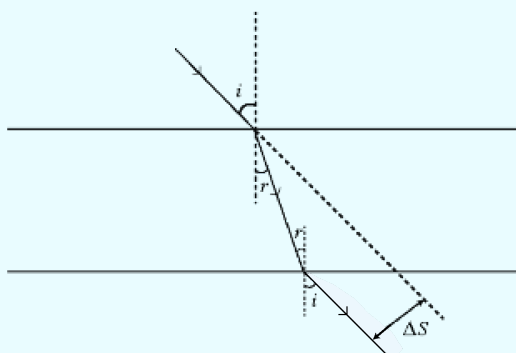


Fig.9.2.1 *Lateral Shift (ΔS) of a ray refracted through a glass slab*

Activity 9.2.2 Deviation of different colors of white light passing through a rectangular glass slab

Take a slab of thickness $d = 4$ cm. Let us assume that the slab material has a refractive index 1.7 for violet light and 1.5 for red light. These numbers are hypothetical and have been just taken for the sake of drawing the ray diagrams (Fig. 9.2.2). In actual practice n , for violet light is only slightly greater than that for red light. Draw a ray of white light (having all colours from violet to red), incident at an angle of incidence of 53° on the slab as shown. Calculate angle of refraction for the violet and red rays, (using Snell's law) and hence draw these rays inside the glass slab. All these rays would emerge at an angle of emergence of 53° in air, but they would be separated. Measure the separation between them. Compare your answer with that obtained from $\Delta S = [(n_V - n_R)][d \tan (r_{\text{mean}})]$

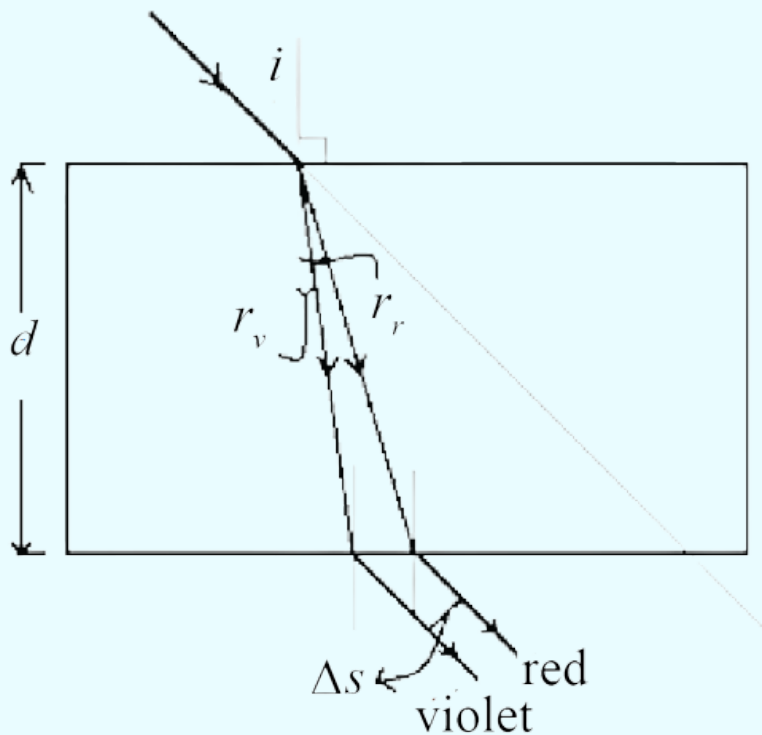


Fig. 9.2.2 Deviation of different colours of light due to refraction through a parallel sided glass slab.

Discuss why, in real life, when we see objects through glass windows but we do not see colors separated out.

9.2.3 Total internal reflection

When light travels from an optically denser medium to a rarer medium, at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. If the angle of refraction is 90° , the refracted ray grazes the interface between the two media.

If the angle of incidence is increased further, refraction is no longer possible and the incident ray is totally reflected back into the denser medium. This phenomenon is known as total internal reflection.

Using Snell's Law,

$$\frac{\sin i}{\sin r} = n_{21}, \text{ when } r = 90^\circ, i = i_c \text{ (critical angle)}$$

$\frac{\sin i}{\sin r} = n_{21} = \frac{n_2}{n_1}$, where n_1 is the refractive index of the denser medium and n_2 is the refractive index of the rarer medium.

$$\therefore \sin i_c = \frac{n_2}{n_1} \quad (9.2.3)$$

Activity 9.2.3 Total internal reflection using laser light

Take a glass beaker filled with water containing a few drops of liquid soap. Stir it well to make the water a little turbid. Shine the beam of laser from the bottom of the beaker and observe the reflected and transmitted beam. Slowly increase the angle of incidence to achieve total internal reflection.

A protractor may be attached to the bottom of the beaker to measure the angle of incidence. If i_c is the angle of incidence for which total internal reflection occurs, then refractive index of the liquid

$$\frac{1}{\sin i_c} = n_{12} \quad (9.2.4)$$

9.2.4 Refraction at a spherical surface

To understand image formation, due to a spherical (refracting) surface separating two (transparent) media, consider a glass sphere ($n = 1.5$) of radius 5 cm and having a tiny air bubble at a distance 2 cm from the centre Fig 9.2.3 (The distances are not to scale and are representative only).

We want to draw a ray diagram to know the position of the image of the bubble when viewed along a diameter passing through the bubble from either side and see where the image is formed. In order to see this we choose a glass sphere consisting of a small air bubble O with a diameter drawn such that it passes through the bubble O with two endpoints on the opposite side of the glass sphere marked as A and C. Let us also consider two points B and D such that rays emanating from O and passing through B or D are not normal to the surface of the glass sphere since they do not lie on the diameter passing through O. As seen from the two different points on the surface of the sphere, rays that reach the surface, and emerge out in the air, bend away from the normal (radial direction), except the rays traveling along the diameter, (which will go undeviated). Thus rays emanating from O and passing through A and C will go undeflected i.e. along the normal direction but those passing through B and D will deviate away from the normal (radial direction). Ray 1 makes an angle of incidence i_1 , and angle of refraction r_1 . Now if the refracted ray at B is produced inside the glass sphere it intersects AC at I_1 . Similarly the ray 2 makes an angle of incidence i_2 , and angle of refraction r_2 . Now if the refracted ray at D is produced inside the glass sphere it intersects AC at I_2 . Thus the

emerging rays, at the two ends, will appear to be coming from I_1 and I_2 , respectively. Hence for an observer for whom O is nearer than C the image of the bubble appears to be formed at I_1 i.e. nearer than the actual position of the air bubble, while if the observer is farther away from C the image of the bubble appears to be formed at I_2 i.e. farther than the actual position of the air bubble.

Use the formula
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad (9.2.5)$$

Here u and v are the object and the image distances respectively, R is the radius of curvature of the refracting surface; n_1 and n are the refractive indices of the two media.

(Take $n_1 = n = 1.5$ and $n_2 = n_{air} = 1$ in this case).

We can use this formula to calculate the positions of the images I_1 and I_2 . Check if your calculated positions match with the schematic diagram positions (**Fig. 9.2.3**).

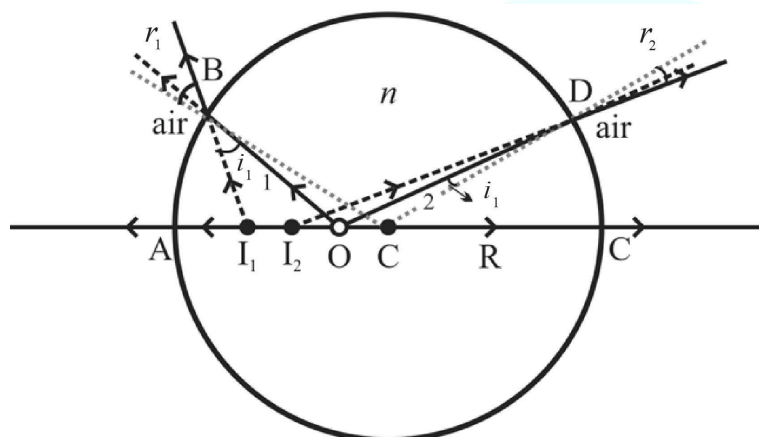


Fig. 9.2.3 Image of an air bubble trapped inside a spherical glass when seen from outside.

Note: We need to use the sign convention for doing our calculations for the first surface (left surface of the glass sphere), rays from the bubble moving from right to left, which therefore, becomes a positive direction. Distances measured, for u and R , are to the right of the surface and hence are to be given a negative sign. The value of v is to be interpreted, accordingly, for finding the position of the image I_1 .

For the second surface (the right surface of the glass sphere), also, we need to interpret and apply the sign convention to calculate the position of the image I_2 .

Activity: 9.2.4

Imagine a spherical fish tank. A fish is looking at an insect that is outside in the air. Take suitable

parameters and draw the ray diagram to locate the position of the insect as seen by the fish. Check the position you obtain by calculating the image position from the equation given above. Take two positions of the insect, one less than R (radius of curvature) and the other greater than $5R$.



Let Us Reflect

Q.9.2.1 Prove that $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ [Eq 9.2.5].

Q.9.2.2 A well is filled with water. An object at the bottom of the well appears to be at a distance h below the water surface. What should be the actual depth of the bottom below the water surface? Take the refractive index of water with respect to air to be μ .

Q.9.2.3 A glass sphere of refractive index $n = 1.5$ and radius R' is used to focus sunlight at a point where will its focal point be?



Answers

A.9.2.1

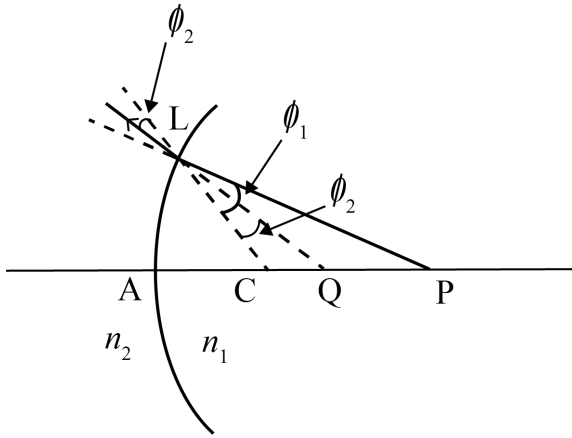


Fig. 9.2.4(a)

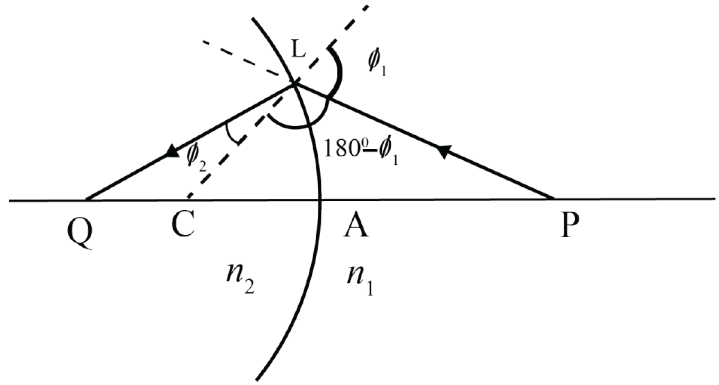


Fig. 9.2.4(b)

(a)Refraction at a concave spherical surface (b)Refraction at a convex spherical surface

n_1 and n_2 are the absolute refractive indices of two media for Fig. 9.2.4(a) and Fig.9.2.4(b) , we have

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

Eq.(9.2.5) is the modified and generalised form of Snell's law. For both the figures , A,C,Q,P are respectively the pole, centre of curvature, position of image and position of object. L is the point of incidence, which is very close to the pole. So the rays are paraxial .Under this condition ,for both the above figures. $LP \approx AP$ and $LQ \approx AQ$

Now, From Fig 9.2.4(a), we get

$$\frac{\sin\phi_1}{\sin\phi_2} = \frac{\sin\angle CLP}{\sin\angle CLQ} = \frac{\frac{\sin\angle CLP}{\sin\angle LCP}}{\frac{\sin\angle CLQ}{\sin\angle LCQ}} \quad (\angle LCP \text{ and } \angle LCQ \text{ are one and the same})$$

$$\text{Or, } \frac{n_2}{n_1} = \frac{\frac{CP}{LP}}{\frac{CQ}{LQ}} \quad (\text{using the properties of triangle})$$

$$\text{Or, } n_2 \left(\frac{CQ}{LQ} \right) = n_1 \left(\frac{CP}{LP} \right)$$

But, $LQ \approx AQ$ and $LP \approx AP$

$$\therefore n_2 \frac{CQ}{AQ} = n_1 \frac{CP}{AP}$$

$$\therefore n_2 \left(\frac{AQ-AC}{AQ} \right) = n_1 \left(\frac{AP-AC}{AP} \right)$$

$$\text{or } n_2 \left(1 - \frac{AC}{AQ} \right) = n_1 \left(1 - \frac{AC}{AP} \right)$$

We shall apply the sign convention, that : all distances are measured from the pole, and all distances measured in the direction of the incident ray are positive.

$$\therefore AC = -R, AQ = -v, AP = -u$$

$$\therefore n_2 \left(1 - \frac{R}{v} \right) = n_1 \left(1 - \frac{R}{u} \right)$$

$$\therefore \frac{n_2}{v} = \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

we write Eq(9.2.5) as, $n_1 \sin(180^\circ - \phi_1) = n_2 \sin \phi_2$

$$\therefore n_1 \sin \angle CLP = n_2 \sin \angle QLC$$

$$\text{or, } n_1 \frac{\sin \angle CLP}{\sin \angle LCP} = n_2 \frac{\sin \angle QLC}{\sin(180^\circ - \angle LCP)} = n_2 \frac{\sin \angle QLC}{\sin \angle LCQ}$$

$$\text{or, } n_1 \frac{CP}{LP} = n_2 \frac{QC}{LQ}; \text{ i.e } n_1 \left(\frac{CA+AP}{AP} \right) = n_2 \left(\frac{QA-AC}{QA} \right)$$

$$\text{or, } n_1 \left(\frac{CA}{AP} + 1 \right) = n_2 \left(1 - \frac{AC}{QA} \right)$$

$$\text{or, } n_1 \left(\frac{R}{-u} + 1 \right) = n_2 \left(1 - \frac{R}{v} \right), \text{ by applying the sign convention}$$

$$\text{or, } n_1 \left(1 - \frac{R}{u} \right) = n_2 \left(1 - \frac{R}{v} \right)$$

$$\text{or, } \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

This equation gives us a relation between object and image distance in terms of the refractive index of the medium and radius of curvature of the curved spherical surface.

A.9.2.2 Referring to Fig.9.2.5, object O at the bottom appears to be at O'

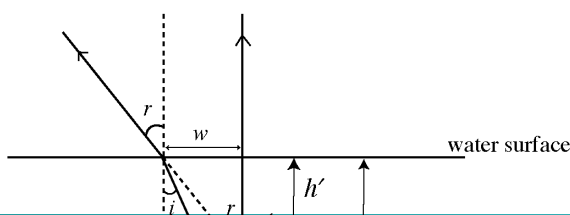


Fig.9.2.5

and at this stage we take account of the fact that the object at the bottom of the well is viewed from above, and that the line of sight is perpendicular to the surface of separation, and so the incidence is near normal, due to which the angles of incidence and refraction are very small.

and

$$\tan i = \frac{w}{h} \text{ and } \tan r = \frac{w}{h'}$$

for small angle θ : $\tan \theta \approx \sin \theta$

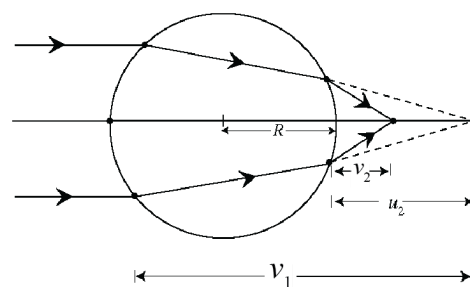
$$\therefore \sin i = \frac{w}{h} \text{ and } \sin r = \frac{w}{h'}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{w/h}{w/h'}$$

$$\therefore \frac{1}{\mu} = \frac{h'}{h}$$

or $h = \mu h'$ is the actual depth.

A.9.2.3 Refer Fig.9.2.6


Fig.9.2.6

For first surface, $u_1 = \infty$ and $R_1 = R'$

$$\therefore \frac{n}{v_1} - \frac{1}{u_1} = \frac{n-1}{R'} \text{ gives}$$

$$\frac{n}{v_1} = \frac{n-1}{R'} \text{ or } v_1 = \frac{nR'}{(n-1)}$$

$$\begin{aligned} \therefore \text{Image is at a distance } u_2 = v_1 - 2R' &= \left[\frac{n}{n-1} - 2 \right] R' \\ &= \left(\frac{2-n}{n-1} \right) R' \text{ from the second surface.} \end{aligned}$$

$$\therefore \text{For second surface } R_2 = -R'$$

$$\frac{1}{v_2} - \frac{n}{u_2} = \frac{1-n}{R_2}$$

$$\frac{1}{v_2} = \frac{n(n-1)}{(2-n)R'} + \frac{1-n}{-R'}$$

$$= \frac{(n-1)}{R'} \left\{ \frac{n}{2-n} + 1 \right\} = \left(\frac{n-1}{2-n} \right) \times \frac{2}{R'}$$

$$\therefore v_2 = \frac{(2-n)R'}{2(n-1)} = \frac{0.5}{0.5} \frac{R'}{2} = \frac{R'}{2}$$

$\therefore$ Final image or focal point will be at a distance of $\frac{R'}{2}$ from the second surface. First spherical surface forms a virtual image which acts as an object for the second spherical surface.

9.2.5 Summary

- Laws of refraction (1) The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane . (2) Snell's law, i.e., $\frac{\sin i}{\sin r} = \text{constant}$, where i and r are the angles of incidence and refraction respectively.
- Refraction at a plane surface follows Snell's law $\frac{\sin i}{\sin r} = n_{21}$, where i is the angle of incidence measured in first medium and r angle of refraction measured in second medium and n_{21} is the

refractive index of medium 2 with respect to medium 1. $n_{21} = \frac{v_1}{v_2}$ where v_1 and v_2 are the speed of light in medium 1 and 2, respectively.

- When light travels from an optically denser medium to a rarer medium, at the interface, it is partly reflected back into the same medium and partly refracted to the second medium. If the angle of refraction is 90° , the refracted ray grazes the interface between the two media. If the angle of incidence is increased further, refraction is no longer possible and the incident ray is totally reflected back into the denser medium. This phenomenon is known as total internal reflection.
- When light passes from a denser medium to a rarer medium, the angle of incidence i_c for which total internal reflection takes place is $\frac{1}{\sin i_c} = n_{12}$
- When light is refracted from a curved surface separating two media it follows the equation $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ where n_1 and n_2 are refractive indices of medium 1 and 2, respectively, R is the radius of curvature of a curved surface at the point of incidence, u is the object distance and v is the image distance.

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