

Module 3 : Image Formation Due to Lenses

This module is designed to help you to:

- differentiate between different types of lenses and their construction;
- predict the position and nature of image formation for different lenses for different positions of the object using the rules of image formation;
- perform reverse analysis and predict the type and position of the lens for a given position and size of the object and position, size and nature of the image;
- apply the thin lens formula for analyzing the position, size, nature and magnification of the image for the given lens and position and size of the object.

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9.3.1 Introduction

A spherical lens is made of a transparent material with one or both refracting surfaces as part of a larger sphere. A lens is used to make parallel rays converge to a point or to diverge from a point source into a parallel beam. It can also be used to magnify the size of an object. The center of a lens is known as optical center and a line passing through it and perpendicular to the curved surfaces is known as optic axis or principal axis. The focal points (the first and second principal foci) are the key points on the principal axis, about which we shall learn in the module. We shall use the properties of these points to learn about image formation by a lens, and apply the all important lens formula for solving problems.

9.3.2 Types of lenses

Depending upon the nature of two refractive surfaces, one may have different types of lenses (converging or diverging)

- Plano convex
- Double convex
- Plano concave
- Double concave
- Convexo concave
- Concavo convex

9.3.3 Image formation through a lens

For a thin lens, we consider the following points of importance.

- There are two focal points on either side of the lens. These points lie on the principal axis of the lens such that if the light is incident parallel to the principal axis, after refraction, it will either pass through the focal point on the other side or will appear to come from a focal point.
- See Figs 9.3.1(a) and (b)

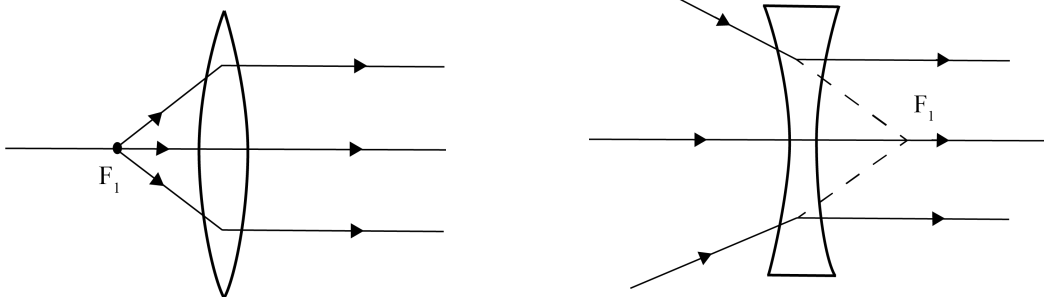
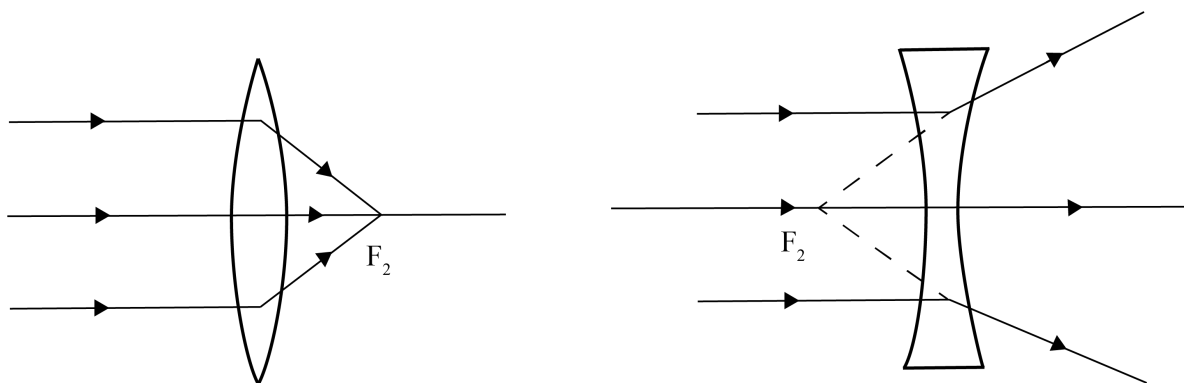


Fig. 9.3.1(a) Converging action of convex lens at F_1

Fig. 9.3.1(b): Converging action of concave lens at F_1

The first principal focus, F_1 , of a lens is that point on its principal axis, such that rays diverging from that point, or appearing to converge to that point after refraction through the lens, are rendered parallel to the principal axis. Observe Figs. 9.3.2 (a) and (b)

The second principal focus, F_2 , of a lens is that point on its principal axis, such that rays travelling parallel to the principal axis, after refraction through the lens, either converge to or appear to diverge from it .

**Fig 9.3.2(a)** *Converging action of convex lens at F_2* **Fig 9.3.2(b)** *Converging action of concave lens at F_2*

When we say focus only, we mean the *second principal focus*. Thus, the first principal focus is the position of the object, real or virtual, on the principal axis, whose image is at infinity and, the second principal focus is the position of the image, real or virtual, whose object is at infinity. Ideally, the distance between the optical centre and the second principal focus of the lens is the focal length of the lens.

- Optical centre is a point in the lens through which a light ray passes undeviated.
- If a ray of light passes through the focal point, or is directed towards it, it emerges parallel to the principal axis of the lens.

All these results hold under an approximation in which we assume the rays to be paraxial (i.e., the rays make only a small angle with the principal axis) and the lens to be thin (i.e. the thickness of the lens is negligible in comparison with the distances we deal with, for studying the formation of image).

Let us consider two different off axis points, A and B, as shown in the figure. To find the images of these points due to a convex lens. We can draw three rays in the following manner:

- i. Draw a ray, parallel to the principal axis, starting from A/B. This ray after refraction passes through the focal point, F, on the other side (Fig. 9.3.3 (a)).
- ii. A ray starting from A/B, and passing through the optical centre passes undeviated through the lens (Fig. 9.3.3 (b)) .
- iii. A ray from A/B, which passes through the focal point before being incident on the lens, emerges parallel to the principal axis after refraction through the lens.

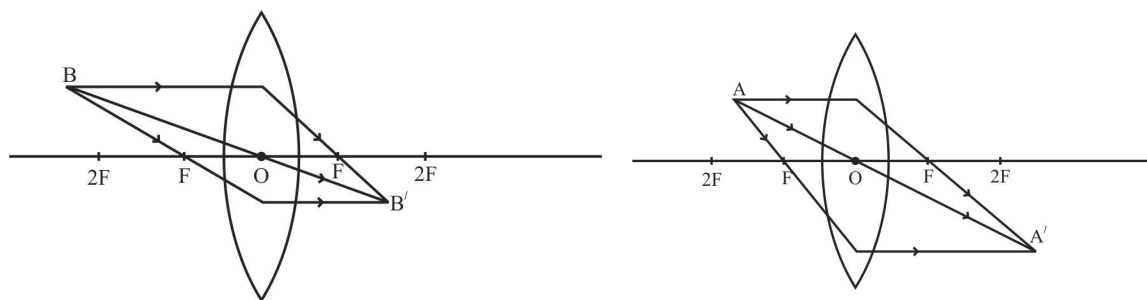


Fig. 9.3.3(a) ray after refraction passes through the focal point, F, on the other side

Fig. 9.3.3 (b) ray passing through the optical centre passes undeviated through the lens

The point where the three rays converge will be the image position $A'B'$ (Fig. 9.3.4). If we have an extended object AB, it can be verified by drawing a ray diagram that the corresponding image formed will be $A'B'$

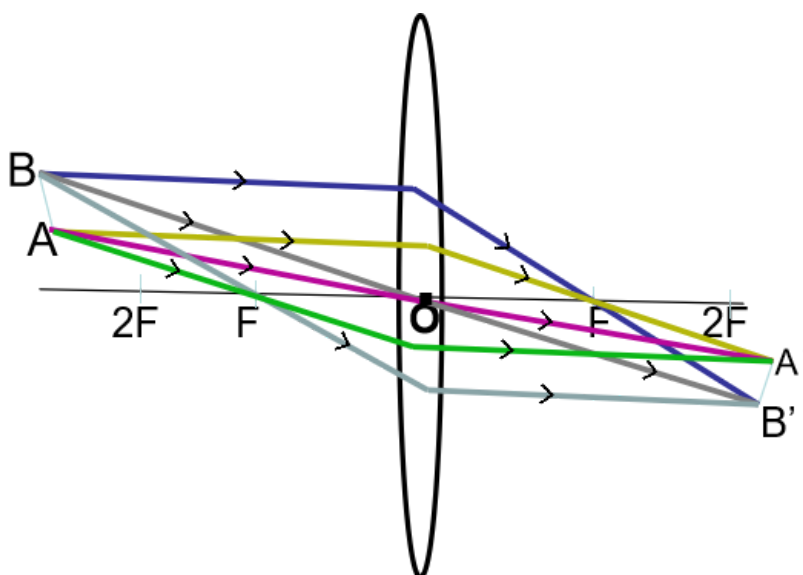


Fig. 9.3.4 Infinite rays starting from AB and converging to $A'B'$

Note: We can have infinite rays starting from A/B and converging to A'/B' . The rules for ray tracing, for all the rays, except the three mentioned above, are not so obvious. However, three rays are sufficient to locate

the position of the image. If for some reason, a part of the lens is covered, we could still have a full image (only weaker in intensity) as, in reality, an infinite number of rays start from A, and pass through the lens to form the image at A'. Blocking of a part of the lens results only in blocking some of the rays from this infinite set of incident rays.

Activity 9.3.1 Carry out a similar ray tracing for a concave lens for different positions of objects.

Activity 9.3.2 Draw similar ray diagrams for a convex lens when the object is kept at distances for which (a) $u \rightarrow \infty$ (b) $u > 2f$ (c) $u = 2f$ (d) $f < u < 2f$ (e) $u = f$ (f) $u < f$
(Here u = distance of (point) object from the optical centre of the lens and f = magnitude of the focal length of the convex lens.)

Activity 9.3.3 The image of an object, AB, is shown in Fig 9.3.5.

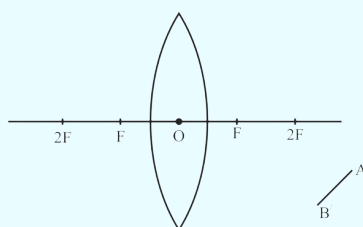


Fig. 9.3.5 Showing image of an object AB

Consider now A'B' as the object. Draw the ray diagram to get its image and verify that it coincides with AB, the (original) object.

9.3.4 Using a ray diagram to find position of the lens

We have noticed how one can use the ray diagram rules, given above, to find the details of the image of an object, when it is put before a given lens. Let us now see how we can use these rules in the reverse manner.

Suppose we are given diagrammatic information about the object and its image, as formed by a lens, whose principal axis is also shown on the diagram. We are required to find the location of the lens and the position of its (second) focus. How do we go about it?

Activity 9.3.4 Let it be given that the principal axis of the lens is PP' . Let $A'B'$ be the image for the object AB . Where has the lens been put and where is its focus?

We join B to B' . The point O where this line cuts the principal axis has to be (as per the rule for optical centre) the optical centre of the lens (Fig. 9.3.6).

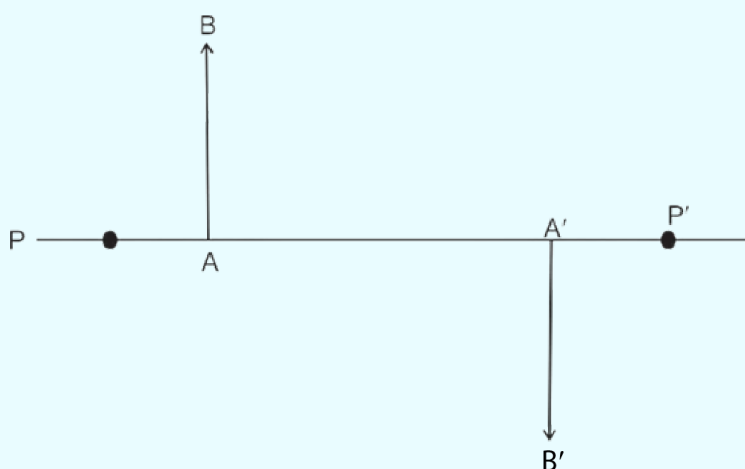


Fig. 9.3.6 Solving reverse problem

Having located the position of the lens, we now draw a ray BN , parallel to the principal axis. We now join N and B' . The point F_1 , where this line cuts the principal axis has to be (as per the rule for focus), the focus of the lens (Fig. 9.3.7).

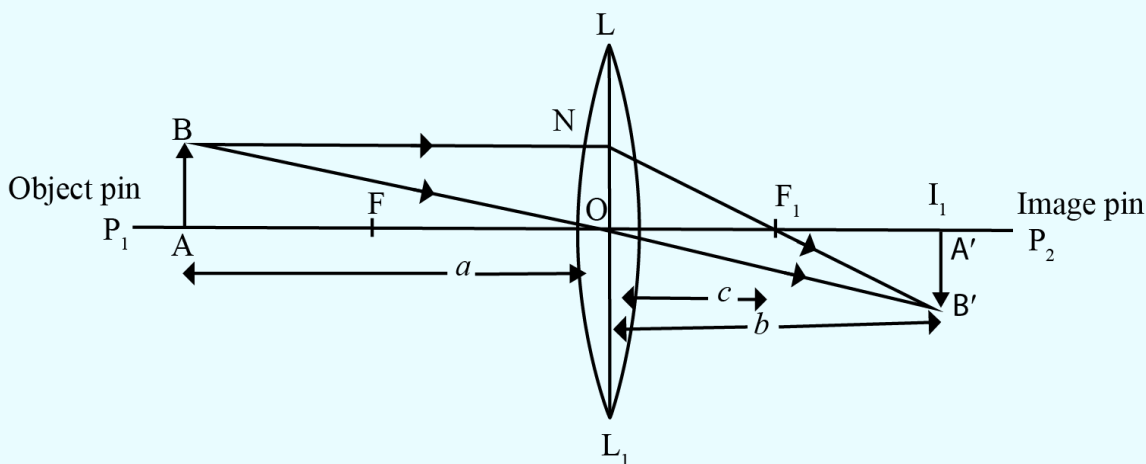


Fig. 9.3.7

We have thus used our conceptual understanding, of the laws for refraction, to solve the reverse problem, i.e. finding the position of the lens and its focus, from the given position of the object and its image.

Activity 9.3.5 The teacher may ask the students to solve this reverse problem, when the object and image position are as shown in Fig 9.3.8 and Fig 9.3.9 (converging or diverging lenses may be used).

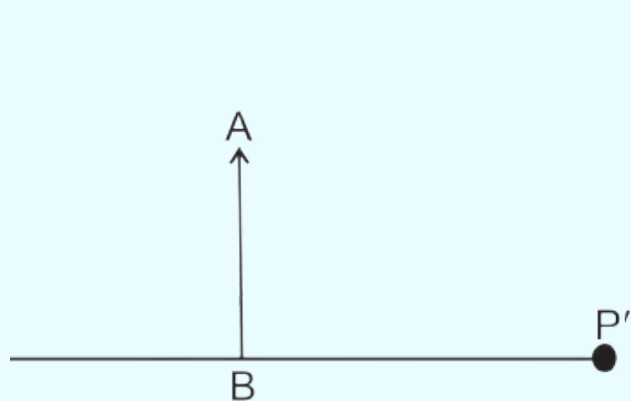


Fig. 9.3.8

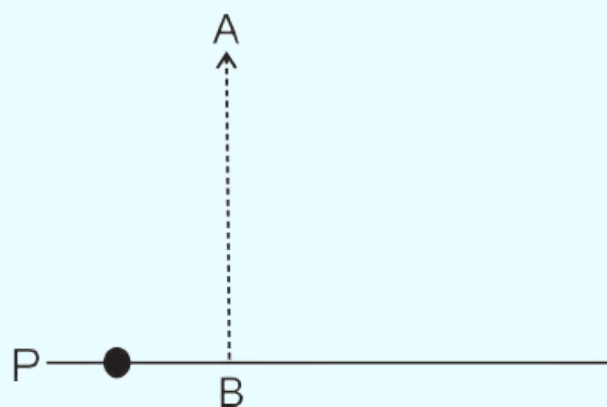


Fig. 9.3.9

9.3.5 Thin lens formula

The thin lens formula aims to provide us with a simple formula for finding the position of the image, formed by the lens for a given position of the object.

An apparently simple way of specifying the position of the object and the image would be to just know their respective distances from the lens. These distances, being magnitudes only, would be expressed through positive numbers with appropriate values as per the unit of length used.

Spherical lenses, however, have interesting characteristics viz-a-viz the images formed by them. The image formed, as we know, can be a real and diminished one; a real and magnified one; a virtual and magnified one; and a virtual and diminished one. The first three cases correspond to a convex lens, the last case corresponds to a concave lens.

When we derive the lens formula, for the different types of images formed, we get three different formulae in terms of the object and image distances and the magnitude of the focal length. Using these three different formulae would obviously be inconvenient, and very much prone to error. This is because we would need to remember as to which formula is relevant for a real image, which for a virtual magnified image, and which for a virtual diminished image.

This confusing and inconvenient situation gets simplified and streamlined by adopting a sign convention. We find that we can talk of object and image positions (rather than object and image distances) and agree (in accordance with an adopted standard sign convention) to treat these positions (as well as the focal length of mirror/lens) as algebraic quantities, having a magnitude as well as sign (+ve or –ve) associated with them. In terms of these positions, it then turns out that the three different formulae, all reduce to the mirror formula ($\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$) or the lens formula ($\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$).

The terms u , v and f in the mirror/lens formulae are thus algebraic quantities which have both a magnitude and a sign. While substituting the values for these quantities (in any given (numerical) problem/situation) we have to substitute their algebraic values (i.e. values with a +ve or –ve sign, according to the sign convention) and not just their magnitudes.

We have to remember that

- 1) The mirror/lens formulae are in terms of POSITIONS which are algebraic quantities.
- 2) When we are using a formula, we have to substitute the magnitudes as well as sign of the quantities concerned.

It is, therefore, necessary to attach a sign to u , v and f (in accordance with the sign convention) when we are substituting their values in a given problem. The +ve/–ve sign, with the calculated value of the unknown quantity, would then give us its position, i.e., its distance from the mirror/lens as well as its location (towards right or left) with respect to the mirror/lens.

Activity: 9.3.6

Draw ray diagrams for a convex lens forming a (i) real (ii) virtual image. Also draw the ray diagrams for a concave lens. Use these diagrams to derive lens formulas in terms of object and image distance and the magnitude of the focal length of the lens.

Hint: Refer to Fig. 9.3.10

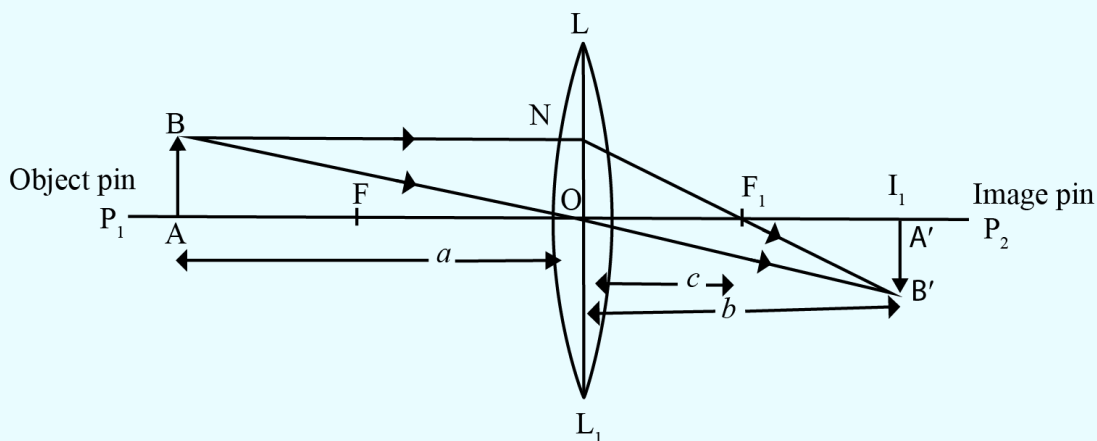


Fig. 9.3.10 Ray diagram for a real image formed by a convex lens

You will get the relation using a pair of similar triangles between a , b and c as $\frac{1}{b} + \frac{1}{a} = \frac{1}{c}$.

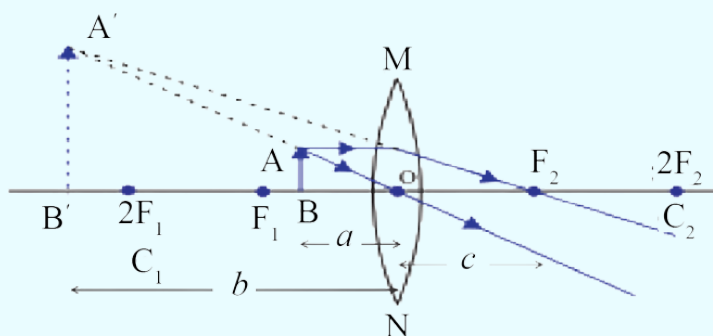


Fig. 9.3.11 Ray diagram for a virtual image formed by a convex lens

Relation between a , b and c is now found to be $\frac{1}{b} + \frac{1}{a} = \frac{1}{c}$

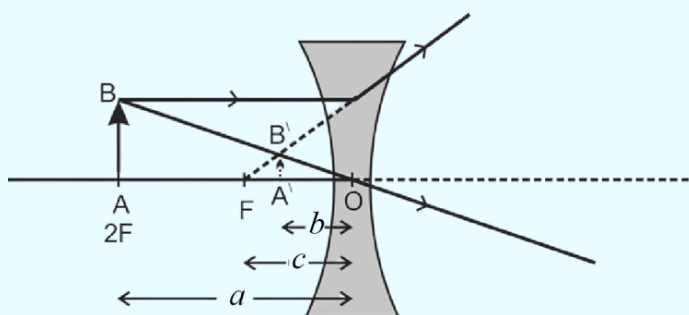


Fig. 9.3.12 Ray diagram for a virtual image formed by a concave lens

A relation between a , b , and c would now found to be $\frac{1}{b} - \frac{1}{a} = \frac{1}{c}$

As per Sign Convention

(i) Object distance (a); object position (u)

$$u = -a \text{ (For all the three cases)}$$

(ii) Image distance (b); Image position (v)

$$v = +b \text{ (For Fig. 9.3.10)}$$

$$v = -b \text{ (For Fig. 9.3.10 and 9.3.11)}$$

Focus distance (c); focal length (f)

$$f = +c \text{ (For Fig. 9.3.8 and 9.3.9)}$$

$$f = -c \text{ (For Fig. 9.3.10)}$$

Using the sign convention, we get the same formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ For all the three cases.} \quad (9.3.1)$$

Eq(9.3.1) can be arrived at geometrically. It can also be obtained from the lens-makers formula, which is as under:

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (9.3.2)$$

where n is the refractive index of the material of the lens. The radii of curvature of the two sides of the lens are R_1 and R_2 .

Let us now derive lens makers formula using the Eq $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ [refer Eq(9.2.5)]

Refer to Fig.9.3.13. We have taken the example of a convex lens, the refractive index of the material of the lens is n . The outside medium is air, where the refractive index is 1.

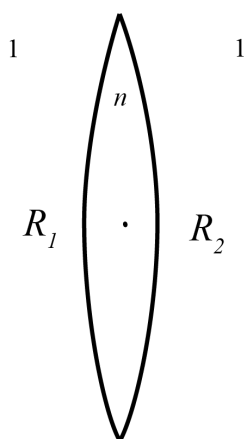


Fig.9.3.13 A thin lens of thickness t , where t is much less than the object and image distances concerned, so much so that it can be neglected in comparison to them.

Proof of lens makers formula:

Let the object and image distances from refraction at the first surface be u and v' , respectively. then, from

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad , \text{ we get}$$

$$\frac{n}{v'} - \frac{1}{u} = \frac{n-1}{R_1} \quad (9.3.3)$$

Since, the thickness is t , the object distance with reference to the second surface is $(v' + t)$.

the final image distance is v . so, we have by using Eq(9.2.5) again,

$$\frac{1}{v} - \frac{n}{v' + t} = \frac{1-n}{R_2} \quad (9.3.4)$$

Adding Eqns. (9.3.3) and (9.3.4), and using $v' + t \simeq v'$, we get

$$\frac{1}{v} - \frac{1}{u} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (9.3.5)$$

from definition of focal length, we know

$$v \rightarrow f, \text{ when } u \rightarrow \infty \text{ or } \frac{1}{u} \rightarrow 0$$

$$\therefore \text{ From (9.3.5); } \frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (9.3.6)$$

which is the lens- makers formula. $\frac{1}{f}$ is the power of the lens. A lens-maker makes adjustments with the values of n , R_1 and R_2 to arrive at the desired value of the power.

Now, by combining Eqns.(9.3.5) and (9.3.6), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ which is Eq(9.3.1).}$$

Activity: 9.3.2 Draw ray diagrams for a concave mirror forming a (i) real and (ii) virtual image. Also draw the ray diagram for a convex mirror. Use these diagrams to derive the mirror formula in terms of object and image distance and the magnitude of the focal length of the mirror.

Hint: Proceed as in activity 9.3.1 and observe the effect, of ‘sign convention’ on the three different formulae.



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Q.9.3.1 Prove Eqn(9.3.1) geometrically for a convex lens, forming a real image as shown in Fig.9.3.10.

Q.9.3.2 A concave lens of focal length 10 cm is kept to the right of a convex lens of focal length 10 cm. Where will the effective focal point of the combination w.r.t concave lens (Fig. 9.3.14) ?

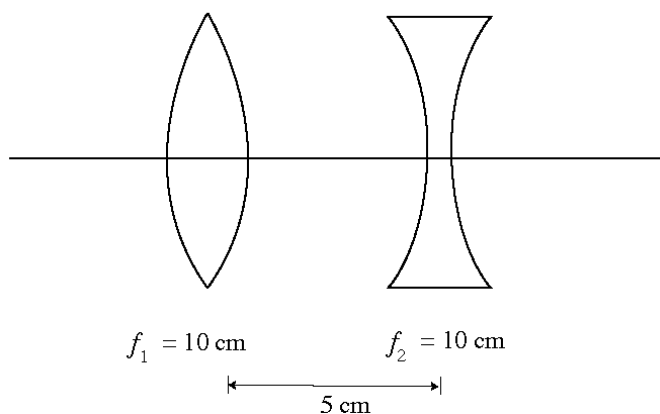


Fig. 9.3.14

Q.9.3.2 Consider a thin lens made of material of refractive index 1.5. Its first surface is convex with radius of curvature 50 cm and second surface is concave with radius of curvature 25 cm. What will be the power of such lens? Will it be used to correct nearsightedness or farsightedness?



Answers

A.9.3.1 Refer to Fig.9.3.15

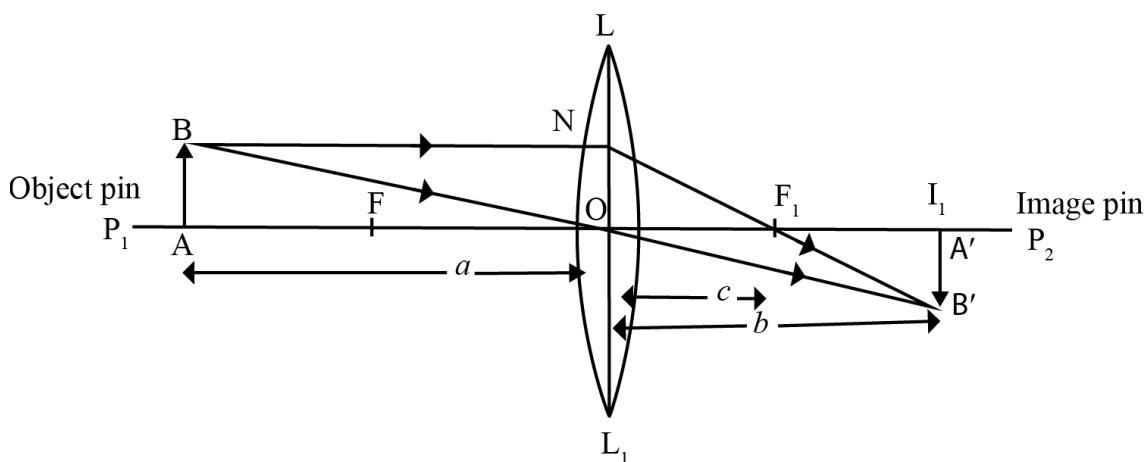


Fig.9.3.15

The diagram is exactly similar to Fig.9.3.10. AB and A'B' are the object and the image. F is the focus. O is the optical centre.

Applying the knowledge of similar triangles, we get

$$\frac{AB}{A'B'} = \frac{ON}{A'B'} = \frac{OF}{FA'} = \frac{OA}{OA'}$$

$$\text{or } \frac{OA}{OA'} = \frac{OF}{OA' - OF}$$

Now, applying the sign convention, we get

$$\frac{-u}{v} = \frac{f}{+v - (+f)} \Rightarrow \frac{-u}{v} = \frac{f}{v - f} \Rightarrow fv = -vu + uf$$

or, $vu = uf - f$, Dividing both sides by uvf ,

$$\text{we get } \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

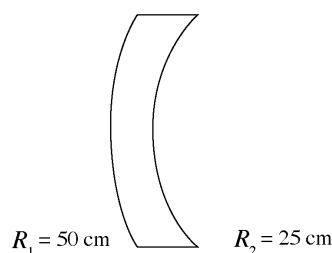
A.9.3.2 If rays are incident from infinity to the left of the convex lens, they will converge towards the focal point of the convex lens to the right which will become an object for the concave lens.

Thus for concave lens $f_2 = -10$ cm and $u = +5$ cm

$$\therefore \frac{1}{v} = \frac{1}{f_2} + \frac{1}{u} = \frac{-1}{10} + \frac{1}{5} = \frac{1}{10} \Rightarrow v = 10 \text{ cm}$$

the effective focal point will be at 10cm to the right of the concave lens.

A.9.3.3 $h = 1.5$ cm, $R_1 = 50$ cm, $R_2 = 25$ cm

**Fig.9.3.16**

R_1 and R_2 are positive as shown in Fig.9.3.16 as center of curvature lies on the right.

$$\begin{aligned}\therefore p = \frac{1}{f} &= (n - 1) \left\{ \frac{1}{R_1} - \frac{1}{R_2} \right\} \\ &= 0.5 \left\{ \frac{1}{50} - \frac{1}{25} \right\} = - \frac{1}{100 \text{ cm}} = - 1 \text{ m}^{-1}\end{aligned}$$

$$\therefore P = - 1 \text{ Diopter}$$

Negative power indicated that this is a diverging lens and will be used for nearsightedness.

9.3.6 Summary

A spherical lens is made of a transparent material with one or both refracting surfaces as part of a larger sphere. A lens is used to make parallel rays converge to a point or to diverge from a point source into a parallel beam. It can also be used to magnify the size of an object.

- Focal length of a lens is given by

$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$ where n is the refractive index of material of lens and R_1 and R_2 are radii of curvature of two surfaces of the lens.

- Thin lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

where u is the object distance and v is image distance measured from the optic center of the lens.

A converging lens has positive focal length and a diverging lens has negative focal length. While using lens formula sign conversion for u , v , f must be taken into consideration.

- Magnification produced by a lens, $m = \frac{v}{u}$

- For a combination of convex lens and a concave lens, if rays are incident from infinity to the left of the convex lens, they will converge towards the focal point of the convex lens to the right side which will become an object for the concave lens.

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