

Module 4 : Prism and Optical Instruments

This module is designed to help you to:

- analyze the construction of a prism;
- apply the concept of refraction of light in prisms and see the path traced by a light ray and how white light gets split into a spectrum;
- visualize the working of a microscope and calculate its magnifying power;
- describe the working of a telescope;
- discuss the differences between a microscope and telescope.

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9.4.1 Prism

Introduction

A prism is a polyhedron made up of a transparent material with two polygonal faces line in parallel planes with the other faces as parallelograms. An ordinary triangular prism has five faces, out of which two are polished and the other three faces are of ground glass. The dihedral angle between the polished faces is called the angle of prism.

Along with refraction through a prism, we shall study about the operations of an ordinary microscope, a compound microscope, a reflecting and a refracting telescope. It can separate white light into its constituent colors known as spectrum. Each color or wavelength is refracted by a different angle with the longest wavelength bending the least.

Activity 9.4.1

Let us consider an equilateral triangular prism made of a material having a refractive index 1.63. Let a monochromatic ray of light be incident on the first surface at an angle of incidence 37° . Trace the path of this ray and find the angle of deviation.

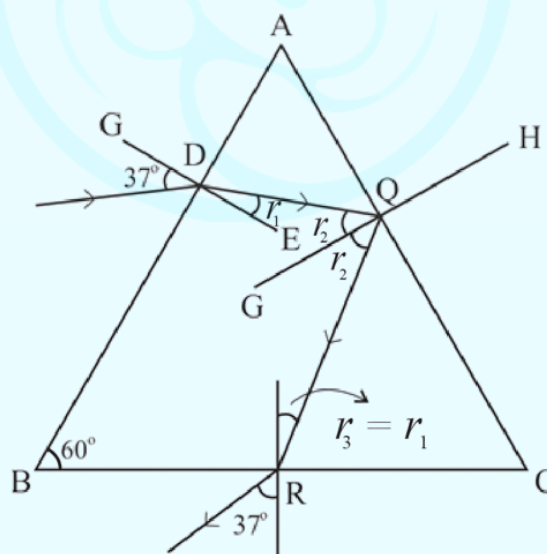


Fig.9.4.1

The ray diagram is shown in Fig 9.4.1

For an equilateral prism $\angle A = 60^\circ$

Angle of refraction at the first surface, $r_1 = \sin^{-1}(\sin i / n) = 21.6670^\circ$

This makes angle of incidence on the second surface, $r_2 = A - r_1 = 38.3329^\circ$

Thus angle of emergence from the second surface, $e = \sin^{-1}(n \sin r_2) = \sin^{-1}(1.0109) \rightarrow$ is indeterminate.

Critical angle $C = \sin^{-1}(1/n) = 37.8427^\circ$

Clearly $r_2 = 38.3329^\circ > C$. This means that total internal reflection would occur at the second surface. In this case there will be no emergent ray at Q. The internally reflected ray will be incident on the third surface at an angle of, $r_3 = A - r_2 = 21.6670^\circ$ to emerge from the third surface at an angle of emergence $e' \approx 37^\circ$.

Thus the deviation of by the incident ray will be (Fig. 9.4.1)

$$\delta = 53 + 60 + 37 = 150^\circ.$$

Activity: 9.4.2

- (a) Fig.9.4.2 shows a cross-section of a light pipe made of a glass fibre of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in Fig. 9.4.2?

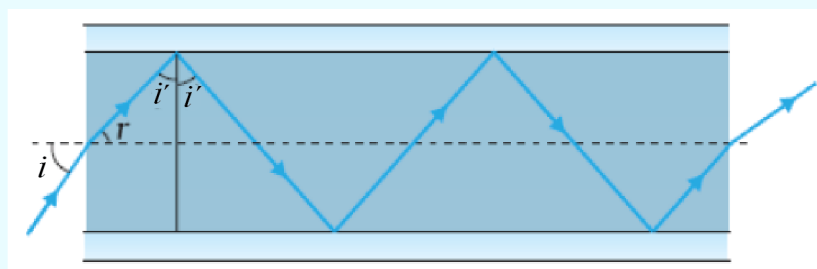


Fig. 9.4.2

- (b) What would the answer be if there is no outer covering of the pipe?
- (c) Draw a ray diagram for both part (a) and (b) to understand the propagation of light through an optical fiber.

9.4.2 Microscope

The simplest and the earliest exposure that we get to a microscope is through a magnifying glass or magnifying lens which is basically a convex lens. A convex lens can be used as a simple magnifier known

as a simple microscope whereas a combination of two convex lenses of suitable focal lengths can be used for a larger magnification. Such a combination is known as a compound microscope.

A simple microscope can be understood as a convex or converging lens of a small focal length. The least distance of distinct vision i.e. the minimum distance upto which a normal human eye can see clearly is around 25 cm also called the near point of the eye. Now if the object is placed between the focus and the optical center of a convex lens then the image formed is magnified, virtual and erect. By choosing the position of the object suitably between the focus and optical center, the image will be formed at a distance 'D' which can be made greater than the near point of the eye (~25 cm) when viewed from the other side of the convex lens. Thus the convex lens under such configuration and placement of the object and the observer (eye) acts as a simple microscope Fig 9.4.3.

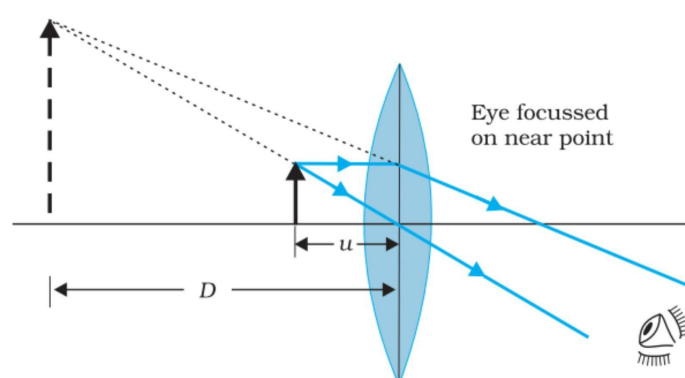


Fig. 9.4.3 Ray diagram of simple microscope

The magnification of the simple microscope can be defined by the equation

$$m = \frac{v}{u} = \left(1 - \frac{v}{f}\right) = \left(1 + \frac{D}{f}\right)$$

Here u = position of the object

v = position of the image which as per sign convention is negative and from Fig 9.4.3 is equal to 'D'

D = position of the image at the near point.

Thus the main purpose of the microscope is to bring the object closer to the observer than the near point 'D' and instead form its magnified image at D .

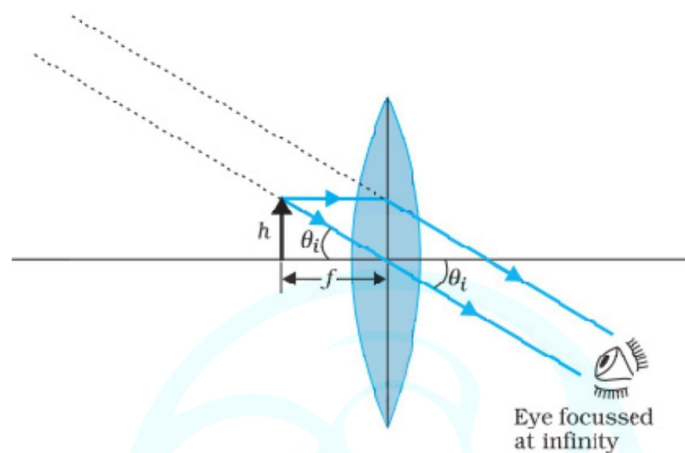


Fig. 9.4.4 Angle subtended by the object of height h at the eye when placed at the focus

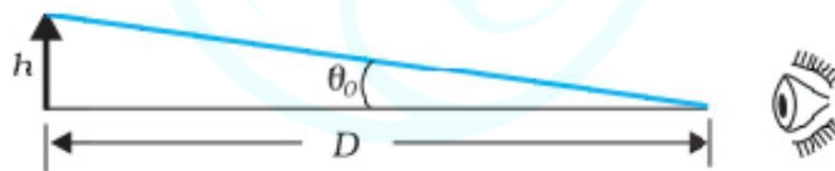


Fig. 9.4.5 Angle subtended by the object of height h at the eye when placed at the near point D

In case the image is formed at infinity Fig 9.4.4 then the angular magnification is given by

$$m = \frac{h'}{h} = \frac{\theta_i}{\theta_o} = \frac{D}{f}$$

where h =height of the object

h' =height of the image

θ_i =angle subtended by the object on the eye when it is placed at $u = -f$, Fig 9.4.4

θ_o =angle subtended by the object on the eye when it is placed at the near point D , Fig 9.4.5

For a simple microscope the smaller the focal length the higher is its magnification power for a fixed value of D which is taken as the near point. However, there is a limit to the minimum value of f for a practical convex lens thus putting a limit to the maximum value of the magnifying power of the simple microscope such that $m \leq 9$.

In order to circumvent this problem a compound microscope is used as shown in Fig 9.4.6. As discussed above it uses a combination of two convex lenses with appropriate focal lengths. Here the convex lens which is closer to the object is called the objective and is of smaller focal length, f_o , while the convex lens

which is closer to the observer is called the eyepiece and has larger focal length, f_e . The distance of the object from the objective is u . The objective forms a real, inverted and magnified image, which lies between the objective and the eyepiece with its position either at the focus of the eyepiece or between the focus and optical center of the eyepiece. Here we define a length L , called the tube length of the compound microscope, which is the distance between the second focus of the objective and the first focus of the eyepiece. This position of the first image which serves as the object for the eyepiece ensures that the final image is formed at infinity beyond the objective or at the near point from the eyepiece and it is virtual, inverted and highly magnified.

A schematic diagram of a compound microscope is shown in Fig 9.4.6. The lens near the object, called the objective, forms a real, inverted, magnified image of the object at a distance L , which is the distance between the second focal point of the objective and first focal point of the eyepiece. L is length of the compound microscope. This serves as the object for the second lens, the eyepiece, which functions essentially as a simple microscope or magnifier, producing the final image, which is enlarged and virtual. The first inverted image is thus near (at or within) the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity, or a little closer for image formation at the near point. Clearly, the final image is inverted with respect to the original object.

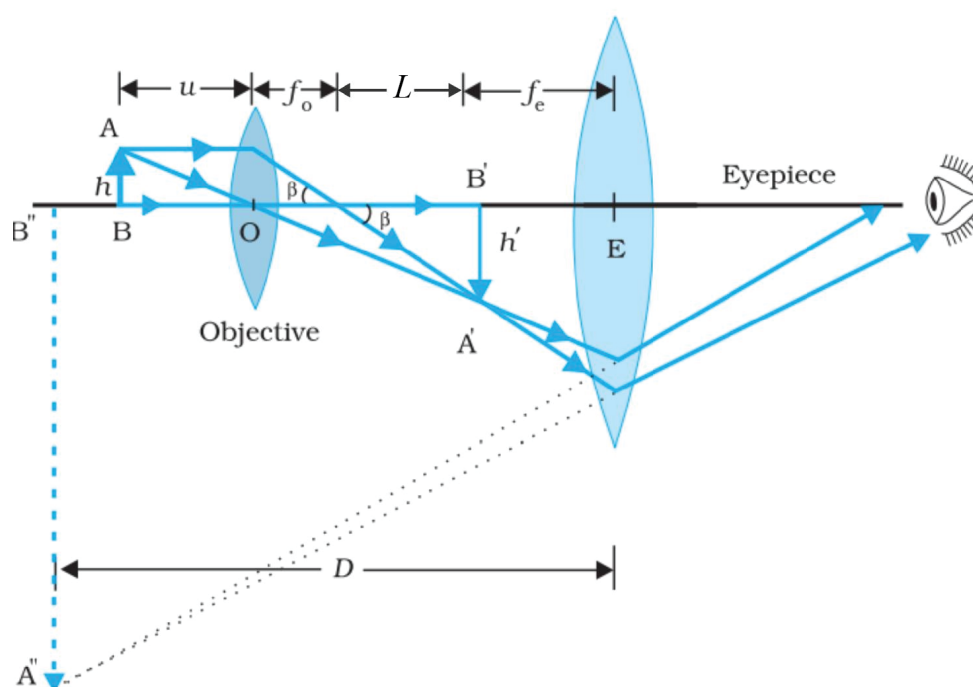


Fig. 9.4.6 Ray diagram of compound microscope

The magnifying power of compound microscope when the final image is formed at distance of distinct vision D from the eyepiece is given by

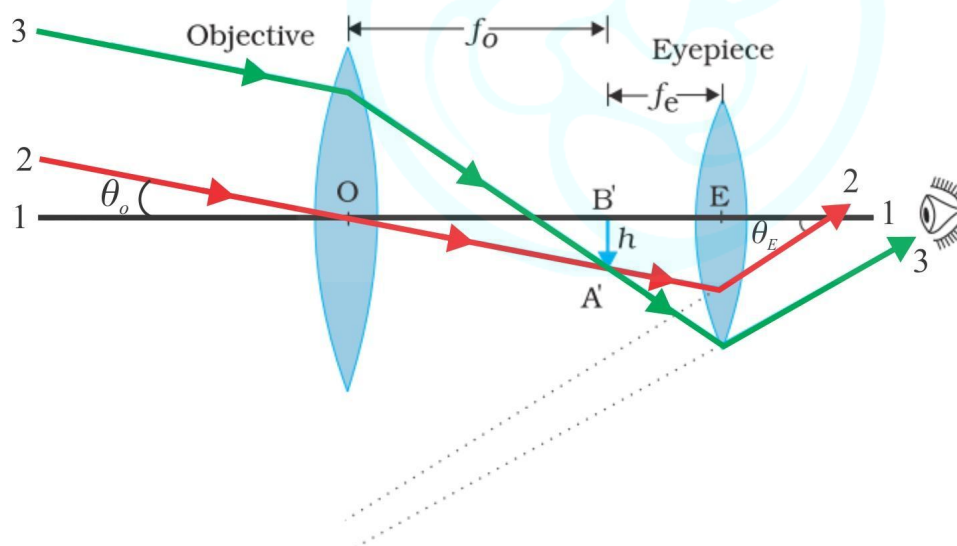
$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(1 + \frac{D}{f_e} \right)$$

where f_o and f_e are the focal length of the objective and eyepiece, respectively. If the final image is formed at infinity then the magnification is given by

$$m = m_o m_e = \left(\frac{L}{f_o} \right) \left(\frac{D}{f_e} \right)$$

9.4.3 Telescope

Distant objects are not seen clearly by the naked eye. In order to increase the visibility and size of distant objects, we use a combination of lenses in an arrangement known as a telescope. Different types of telescopes such as reflecting type or refracting type are available.


Fig. 9.4.7 Ray diagram of a refracting telescope

Consider two rays (rays as 2 and 3 in Fig.9.4.7) coming from an astronomical object. These two rays subtend an angle θ_o at the naked eye and hence at the objective of the telescope. Ray 1 is along the principal axis of the telescope lens system and hence passes undeviated. Rays 2 and 3, after emerging out of the telescope, subtend angle θ_e at the eyepiece. Thus an astronomical object, which was subtending an angle θ_o

at the naked eye, now subtends an angle θ_E at the eye (as a result of the telescope) and appears to be magnified by a factor θ_E / θ_0 . Ray 2 and 3 show that the image formed by the objective is in the focal plane of both the objective and the eyepiece, and hence the final image formed is at infinity.

Activity 9.4.3 Draw a ray diagram for a telescope, for two rays incident parallel to the principal axis. (say ray 1 and one more ray, say ray 4). Using just these two rays, can you find the formula for magnification?

Activity 9.4.4 (a) A giant refracting telescope, at an observatory, has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what would be the angular magnification of the telescope? (b) This telescope is used to view the moon. What is the diameter of the image of the moon formed by the objective lens? Given that the diameter of the moon is 3.48×10^6 m, and the radius of lunar orbit is 3.8×10^8 m.

Activity 9.4.5 Draw the ray diagram of inverted telescope and inverted microscope. Find if they can achieve the usual results of microscope and telescope respectively.

Example 9.4.1 Combination of lens and mirror

A convex lens of focal length 10 cm is placed in front of a convex mirror. Both are coaxial and the lens is 10 cm from the pole of the mirror. When an object is placed on the axis at a distance of 12 cm from the lens, it is found that its image (I) (formed by this lens mirror combination) coincides with the object. Calculate the focal length of the convex mirror.

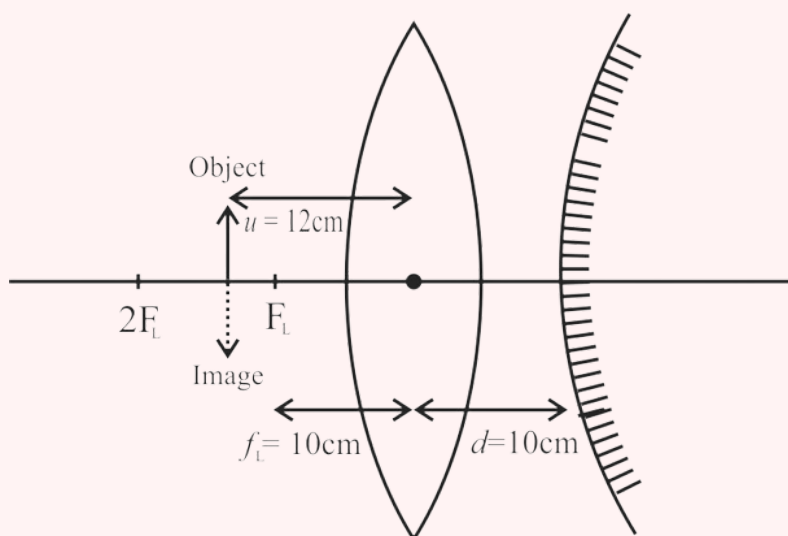


Fig. 9.4.8

Solution: For the lens $f_L = 10$ cm and $u = -12$ cm. Using the lens formula, we get $v = 60$ cm (to the right of the lens). This image acts as a virtual object for the mirror.

For the mirror, object distance $u_m = 50$ cm (since the distance between lens and mirror is 10 cm).

The image formed by the mirror acts as an object for the lens which forms its final image.

Since the final image for the lens is to its left at 12 cm, the appropriate sign convention gives this object distance for the lens as $+60$ cm. This means that the image formed by the mirror was at 50 cm to the right of the mirror.

Thus, for the mirror the object and image are at the same location. This means for the mirror $2f = 50$ cm and its focal length is 25 cm.

The best way to realize this, is to construct a ray diagram as shown. The ray diagram shows that the image I' (formed by the lens) must be at the centre of curvature of the mirror.

Hence the distance $PI' = R = 50$ cm. This gives the focal length of the mirror as 25 cm ($f = \frac{R}{2}$).

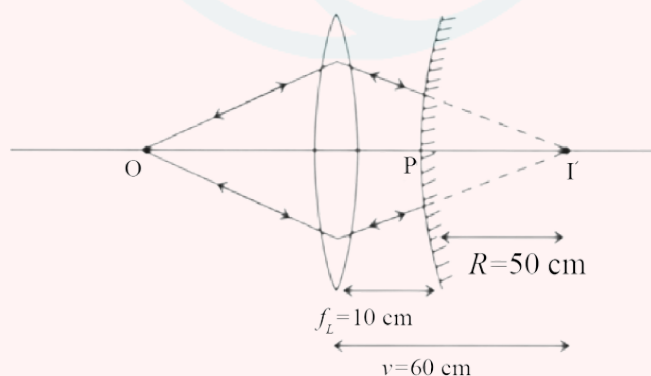


Fig. 9.4.9

Reflecting telescope	Refracting telescope
As the name suggests this telescope uses the principle of reflection to focus light rays from far off objects	This telescope uses the principle of refraction for focussing light rays from far off objects
It has a single or a combination of curved (mainly concave) mirrors to focus light coming from distant objects and form an image	It uses a combination of two convex (an objective lens having a larger focal length and an eyepiece having a smaller focal length) lens to focus light and form an image
It does not suffer from chromatic aberration	It suffers from chromatic aberration i.e. different coloured lights having different wavelengths getting split from each other and arriving to focus at slightly different angles
It is suitable for astronomical viewing as even large aperture mirrors (which can collect more light) are relatively easy to make and are less expensive	Generally the lenses used in it are of smaller aperture and hence do not give a very good view of astronomical objects. It is hugely expensive to construct very large aperture lenses (similar aperture size as reflecting telescope) which are free from any defects
It has an open tube design and hence easily collects dust and humidity and is hence difficult to maintain	Due to its closed tube design it is easy to maintain and remains dust and humidity free.
Reflecting telescopes suffer from a defect called coma which is that since these telescopes use parabolic or concave mirrors hence different points on these mirrors have different focal lengths and for images that are far from the center of the field of view they appear elongated and also fuzzy.	Refracting telescopes do not suffer from this defect
These telescopes do not have good contrast and sharpness and are not as suitable for astrophotography	These telescopes have good contrast and sharpness and have no image distortion hence are suitable for astrophotography
They are heavy and bulky and cannot be transported easily	They are light and compact and hence are very portable



Q.9.4.1 Can a microscope be inverted to use it as a telescope and can a telescope be inverted to be used as a microscope?

Q.9.4.2 A ray of light is incident on an equilateral prism of refractive index 1.5 at an angle of 30° . By what angle will this ray be deviated as it emerges from the prism? Show this with an appropriate ray diagram.



Answers

A.9.4.1 There is a basic difference between the way a microscope is constructed and a telescope is constructed. A microscope and a telescope both have an objective and an eyepiece. However, the objective and eyepiece of a microscope have small focal lengths while the objective of a telescope has a large focal length and larger aperture than its eyepiece. Hence a microscope cannot be inverted to make it work like a telescope. Similarly, the focal lengths and aperture size of the objective and eyepiece of a telescope are chosen to perform the following:

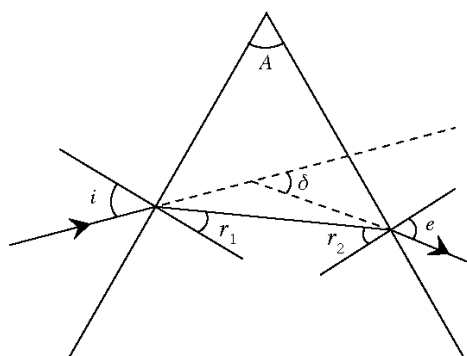
- (a) To collect more light to view faint objects clearly
- (b) To facilitate resolution of close objects
- (c) To facilitate magnification of distant objects

Since a microscope is used to only magnify nearby objects which cannot be achieved if the telescope is inverted as it will not be able to focus on small nearby objects and also since it will make the object look even smaller.

$$\text{A.9.4.2 } \sin r_1 = \frac{\sin i}{h} = \frac{\sin 30^\circ}{1.5} = \frac{1}{3}$$

$$\therefore r_1 = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$$

$$r_2 = A - r_1 = 40.53^\circ$$


Fig.9.4.10

$$\sin e = h \sin r_2 = 1.5 \times \sin 40.53^\circ = 1.5 \times 0.6498 = 0.9747$$

$$\therefore e = \sin^{-1}(0.9747) = 77.10^\circ$$

$$\delta = i + e - A = 30 + 77.10^\circ - 60$$

$$= 47.10^\circ$$

9.4.4 Summary

- When a ray of light passes through a prism of refracting angle A , the emergent ray makes an angle δ with incident ray such that
 $i + e = A + \delta$ where i = angle of incidence, e = angle of emergence.
- A ray of light while passing through the prism suffers minimum deviation when $i = e$. Therefore angle of minimum deviation $\delta_m = 2i - A$ where i is the angle of incidence for which minimum deviation expresses.
- Prism formula

$$\mu = \frac{\sin(A + \delta_m)}{\sin\left(\frac{A}{2}\right)}$$
- A convex lens of small focal length can be used as a simple microscope. A compound microscope has two converging lenses as an objective and an eye-piece. The objective has a smaller diameter and the eyepiece has a larger diameter.

The magnifying power of compound microscope when the final image is formed at distance of distinct vision D is given by

$$m = \frac{L}{f_0} \left(1 + \frac{D}{f_e} \right)$$

- A telescope consists of two converging lenses. Objective lens has a larger diameter and the eyepiece has a smaller diameter. Length of telescope $L = f_0 + f_e$.

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