

Module 2: Young's Double Slit Experiment

This module is designed to help you to:

- explain how the arrangement of Young's Double slit presents a situation for obtaining coherent sources.
- derive expression for fringe width in Young's double slit experiment ;
- show that the path difference between any two consecutive bright and dark fringes is equal;
- analyse the conditions for sustained interference.

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10.2.1 Introduction

In the previous module we learnt about the requirement of coherent sources for production of interference fringes. In this module we shall learn further about interference of light especially with the help of Young's double slit experiment. We will also understand the condition that two sources must satisfy in order to produce a clear interference pattern.

10.2.2 Young's double slit experiment

Let S_1 and S_2 be the two point coherent sources of light, the point P on the screen will be a dark or bright fringe depending on the path difference between the two waves from the sources at the point P (Fig. 10.2.1).

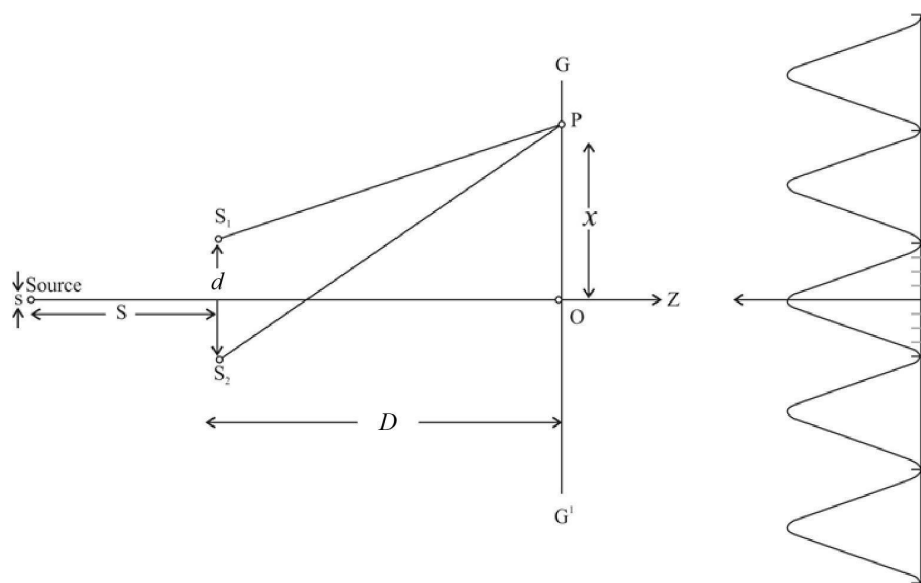


Fig. 10.2.1 Young's double slit experiment

For bright fringes the path difference $\frac{xd}{D}$ must be such that

$$\frac{x_n d}{D} = n\lambda$$

$$x_n = \frac{n\lambda D}{d}, \text{ where } n = 0, 1, 2, 3 \dots \quad (10.2.1)$$

Here x_n is the distance of the n^{th} bright fringe from the centre of the fringe pattern, d is the distance between the two slits, D is the distance between the slits and the screen. The spacing between the n^{th} and $(n + 1)^{\text{th}}$ bright fringes is given by

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} \quad (10.2.2)$$

For dark fringes the path difference $\frac{xd}{D}$ must be such that

$$\frac{x'_n d}{D} = (2n + 1) \frac{\lambda}{2}$$

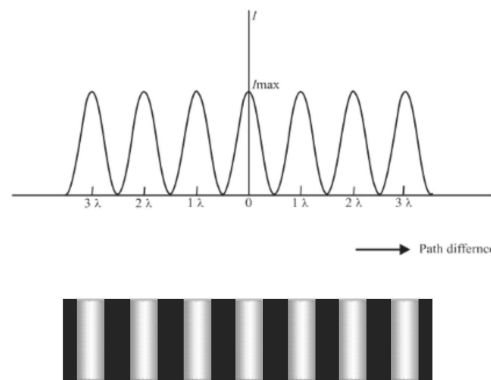
$$x'_n = \frac{(2n+1)\lambda D}{2d} \quad (10.2.3)$$

Here x'_n is the distance of the n^{th} dark fringe from the centre of the fringe pattern. The spacing between the n^{th} and $(n+1)^{\text{th}}$ dark fringes, is given by

$$\beta' = x'_{n+1} - x'_n = \frac{\{2(n+1)+1\}\lambda D}{2d} - \frac{(2n+1)\lambda D}{2d} = \frac{\lambda D}{d}$$

$$\text{Thus, } \beta = \beta' \quad (10.2.5)$$

i.e., the spacing between any two consecutive bright or dark fringes is the same. This spacing is called *fringe*



width (Fig. 10.2.2). All bright fringes have equal intensity and all dark fringes have zero intensity.

Fig. 10.2.2 (a) Intensity distribution (b) Interference pattern in Young's double slit experiment

10.2.3 Shape of fringes

Conditions for sustained interference:

- (1) The two interfering waves should be coherent, which means that the phase difference between them must remain constant in time.
- (2) The two waves must have the same frequency.
- (3) The two waves must be in the same state of polarization.

In the double slit experiment shown in Fig 10.2.1, we have taken the source S on the perpendicular bisector of the two slits, which is shown as the line SO.

Now let us suppose that the source is moved slightly to some new point S' and let Q be the midpoint of the line joining S_1 and S_2 . If the angle S'QS is ϕ , as shown in Fig. 10.2.3, the central bright fringe occurs at an angle should be of the same font as that of ϕ in the previous line. i.e. angle OQO', on the other side. This also means that the source S', the mid-point Q and the point O' of the central fringe are in a straight line.

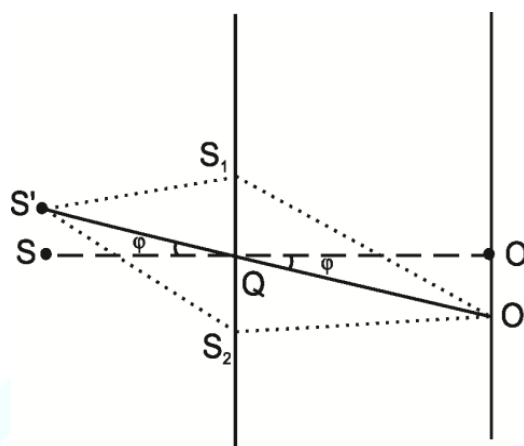


Fig. 10.2.3 *Interference pattern*

Let us consider the following situations and the effect they have on the interference fringes in a Young's double-slit experiment. What will happen if

- The screen is moved away from the plane of the double slit.
- The source slit is moved closer to the double slit plane.
- The monochromatic light is replaced by a source of white light.
- The size of the slit (opening) is made very small
- The two slits are moved very close to each other.

Let us discuss the above situations one by one.

- We know that the angular separation of the fringes given by $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$ (recall Eqn 10.2.2) remains constant since it is independent of the distance of the screen from the slits. However, as the screen is moved away from the plane of the slit, the (linear) separation of the fringes, β , increases in proportion to the increase in the distance of the screen from the plane of the two slits thereby decreasing the distinctness of the fringes. Also the brightness of the fringes decreases. Why?
- The separation of the fringes remains fixed when the source slit is moved closer to the double slit plane, however, the condition, $\frac{s}{S} < \frac{\lambda}{d}$ (where s is the size of the main source and S is its distance from the plane of the two slits) should be satisfied. When the above condition is not satisfied, the main source, of size s , cannot be called a point source; and hence the fringes disappear. Here when the main source is brought closer to the plane of the double slit then S decreases but all other parameters remain unchanged and will result in the overlapping of the interference fringes causing them to become less and less sharp and eventually disappear.
- When a source of white light is used instead of monochromatic light, the fringes are made up of different component colors of white light which overlap incoherently. This is because the different colors of white

light have different wavelengths and the fringe width depends on the wavelength of the light source. This gives us coloured fringes. However the central bright fringe will still appear white since the central fringes for different colors will still be formed at the same position.

(d) $\beta = \frac{\lambda D}{d}$, now as d decreases; β increases. As slits are moved closer, fringe width starts increasing.

In interference of light, light energy is redistributed. If it gets reduced in one region, (producing a dark fringe), it increases in another region, (producing a bright fringe). There is no gain or loss of energy which is consistent with the principle of conservation of energy.

Let a_1 and a_2 be the amplitude and I_1 and I_2 be the intensities of the two interfering light waves respectively.

Then

$$\{a_1 = a \rightarrow I_1 = a_1^2 = a^2\}$$

$$\{a_2 = a \rightarrow I_2 = a_2^2 = a^2\}$$

$$\text{Constructive interference : } I = (a_1 + a_2)^2 = 4a^2$$

$$\text{Destructive interference : } I = (a_1 - a_2)^2 = 0$$

$$\text{Average} = 2a^2$$

The intensity that would arise from the two waves if they had arrived independently without interference, is also $2a^2$.

Example 10.2.1

In a double slit experiment using sodium light of wavelength 589 nm, the angular width of a fringe formed on a distant screen is $\frac{\pi}{900}$ rad. Find the spacing between the two slits?

Solution: We know that the angular width of a fringe is $\frac{\lambda}{d}$

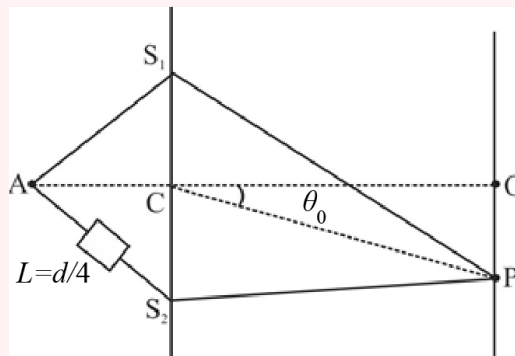
Given angular width = $\frac{\pi}{900}$ radian

$$\therefore \frac{\lambda}{d} = \frac{\pi}{900}$$

$$\frac{589 \times 10^{-9}}{d} = \frac{\pi}{900}$$

$$d = \frac{589 \times 10^{-9} \times 900}{\pi} \text{ m}$$

$$d = 1.69 \times 10^{-4} \text{ m}$$

Example: 10.2.2

Fig. 10.2.5

$$CO = D$$

$$S_1C = S_2C = d \ll D$$

A small transparent slab of length L , made of a material of $\mu = 1.5$ is placed along AS_2 (Fig. 10.2.4). Find the distance from O , of the central maxima, and of the first minima, on either side of the central maxima, obtained in the absence of the glass slab. Take $d = \lambda$ and $L = d/4$?

Solution:

$$\text{Optical path difference} = 2d \sin \theta + (\mu - 1)L$$

For central maxima,

$$2d \sin \theta_0 + 0.5L = 0$$

$$\sin \theta_0 = -\frac{L}{4d} = -\frac{1}{16} \quad \left(\text{since } L = \frac{d}{4} \right)$$

$$\therefore OP = D \tan \theta_0 \approx -\frac{D}{16}$$

For the first minima:

$$\therefore 2d \sin \theta_1 + 0.5L = \pm \frac{\lambda}{2}$$

$$\sin \theta_1 = \frac{\pm \lambda/2 - 0.5L}{2d} = \frac{\pm \lambda/2 - \lambda/8}{2\lambda} = \frac{1}{4} - \frac{1}{16}$$

$$\text{On the positive side: } \sin \theta_1 (+) = \frac{3}{16}$$

$$\text{On the negative side: } \sin \theta_1 (-) = -\frac{1}{4} - \frac{1}{16} = -\frac{5}{16}$$

The first principal maxima on the positive side is at distance

$$D \tan \theta_1 (+) = D - \frac{3}{\sqrt{16^2 - 3^2}} \text{ above } O, \text{ i.e. } \left(D - \frac{3}{\sqrt{247}} \right) = \left(D - \frac{3\sqrt{247}}{247} \right) \text{ above } O$$

$$\text{On the -ve side, the distance will be } D \tan \theta_1 (-) = D - \frac{5}{\sqrt{16^2 - 5^2}} \text{ below } O = D - \frac{5}{\sqrt{231}}$$

$$= (D - \frac{5\sqrt{231}}{231}) \text{ below O.}$$

Example 10.2.3

A beam of light consists of two wavelengths, one of them is 420 nm, being used to obtain interference fringes in a Young's double-slit experiment with distance between slits being 0.2 mm and distance of the screen from the plane of the two slits being 2 m.

(a) Find the distance of the fourth bright fringe on the screen from the central maxima for wavelength 420 nm.

(b) Find the wavelength of the second beam if its third bright fringe from the central maxima coincides with the fourth bright fringe of the 420 nm wavelength.

Solution : (a) 1.68 cm (b) 560 nm

Solution: (a) The distance of the n^{th} bright fringe from central maxima is given by

$$x_n = \frac{n\lambda D}{d}$$

For $\lambda = 420 \text{ nm}$

$$x_4 = \frac{4 \times 420 \times 10^{-9} \times 2}{2 \times 10^{-4}} = 0.0168 \text{ m} = 1.68 \text{ cm}$$

(b) Now $n = 3$ and $x_3 = 1.68 \text{ cm}$

$$\lambda = \frac{x_n d}{nD} = \frac{1.68 \times 10^{-2} \times 2 \times 10^{-4}}{3 \times 2} = 560 \text{ nm}$$

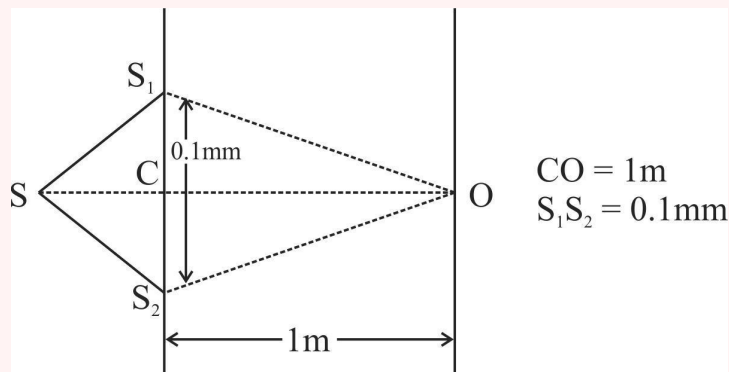
Example 10.2.4

In a double-slit experiment the wavelength of light used is 500 nm, the screen is placed 1 m away from the plane of the slits and the distance between the slits is 0.1 mm. Find the distance of the first and dark bright fringe from the principal maxima, the fringe width and angular width of the fringe. If the entire experiment were to be repeated in water then what would be the effect on the position of the principal maxima, distance of the first dark and bright fringe from the principal maxima, the fringe width and angular width of the fringe. (Take the refractive index of water to be 4/3).

Solution :

Fig.10.2.7

Refer to Fig.10.2.7



Distance of the n^{th} bright fringe

$$x_n = \frac{n\lambda D}{d} \text{ for } n = 1, \lambda = 500 \text{ nm}, D = 1 \text{ m and } d = 0.1 \text{ mm}$$

$$x_1 = \frac{\lambda}{d} = \frac{500 \times 10^{-9} \times 1}{0.1 \times 10^{-3}} = 5 \times 10^{-3} \text{ m} = 5 \text{ mm}$$

Distance of the n^{th} dark fringe after 1st bright fringe

$$x'_n = \frac{(2n+1)\lambda D}{2d}$$

For 1st dark fringe after principal maxima we have $n = 0$

$$x'_0 = \frac{1 \times 500 \times 10^{-9} \times 1}{2 \times 1 \times 10^{-4}} \text{ m} = 250 \times 10^{-5} \text{ m} = 2.5 \text{ mm}$$

$$\text{fringe width } \beta = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 1}{1 \times 10^{-4}} = 5 \text{ mm}$$

Angular separation of the fringe is

$$\theta = \frac{\beta}{D} = \frac{5 \times 10^{-3}}{1} = 5 \times 10^{-3} \text{ rad}$$

Refractive index of water is $\mu = \frac{4}{3}$

The wavelength in water changes to λ' where

$$\lambda' = \frac{\lambda}{\mu} = \frac{500 \times 10^{-9}}{4/3} \text{ m} = 375 \times 10^{-9} \text{ m}$$

$$\Rightarrow \lambda' = 375 \text{ nm}$$

The position of the principal maxima does not change and is still formed at O.

The distance of the 1st bright fringe from principal maxima is now

$$x'_1 = \frac{\lambda' D}{d} = \frac{375 \times 10^{-9} \times 1}{1 \times 10^{-4}} = 3.75 \text{ mm}$$

The distance of the 1st dark fringe from central maxima is

$$x'_0 = \frac{1 \times 375 \times 10^{-9} \times 1}{2 \times 1 \times 10^{-4}} \text{ m} = 187.5 \times 10^{-5} \text{ m} = 1.875 \text{ mm}$$

$$\text{Fringe width } \beta' = \frac{\lambda D}{d} = \frac{375 \times 10^{-9} \times 1}{1 \times 10^{-4}} \text{ m} = 3.75 \times 10^{-5} \text{ rad}$$

$$\Rightarrow \beta' = 3.75 \text{ mm}$$

Angular separation of the fringe is

$$\theta' = \frac{\beta}{D} = \frac{375 \times 10^{-5}}{1} = 3.75 \times 10^{-3} \text{ rad}$$

Example 10.2.5

In two slit interference arrangements (Fig. 10.2.8) the distance of the screen from the plane of the two slits, D , is 100 times the distance between the slits, d . Obtain the value of d in terms of λ such that the first minimum on the screen falls at a distance $10d$ from the centre O .

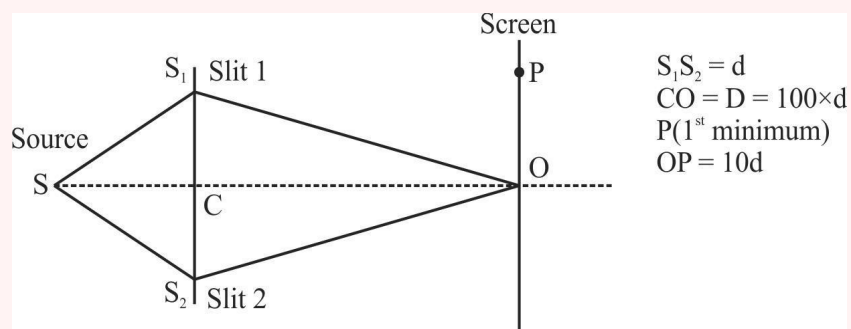


Fig.10.2.8

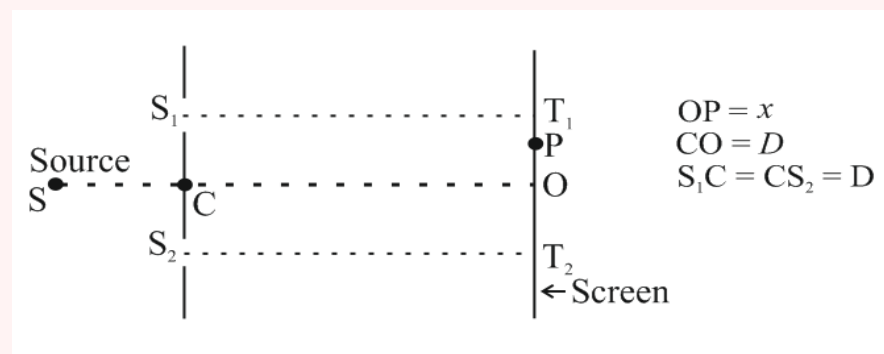


Fig. 10.2.9

Solution: Refer to Fig.10.2.9 The distance of the first minima from the principal maxima is given by

$$x_n = \frac{(2n+1)\lambda D}{2d} \text{ where } n=0$$

Here $D = 100 d$ and $x = 10 d$

$$10d = \frac{\lambda \times 100d}{2d}$$

$$d = 5\lambda$$



Let Us Reflect

Q.10.2.1 As shown in the Fig.10.2.10 light derived from same source are polarized in two perpendicular directions before reaching S_1 and S_2 . What will happen to the interference pattern?

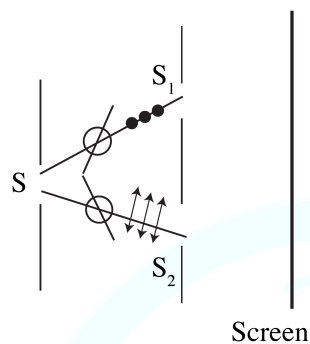


Fig.10.2.10

Q.10.2.2 Fig.10.2.11 shows a standard two slit arrangements with slits S_1 and S_2 . P_1 and P_2 are the two minima points on either side of P.

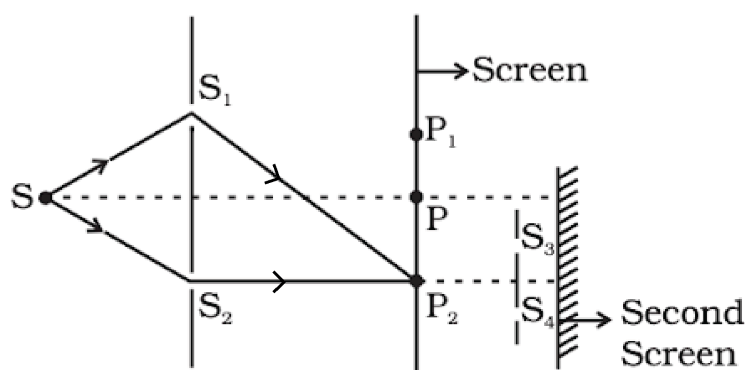


Fig.10.2.11

At P_2 on screen, there is a hole and behind P_2 is a second two slit arrangement with slits S_3 , S_4 and a second screen behind them. What will be observed on the second screen?



A.10.2.1 Light is a transverse wave. Two perpendicular vectors can never add to give a resultant vector of zero magnitude. Thus in this case there will be no interference pattern.

A.10.2.2 As P_2 is the dark point, no energy propagates through P_2 . Hence no interference pattern is formed on the second screen.

10.2.5 Summary

- Light from two sources if obtained from the same original source are coherent, i.e. they differ in phase by constant value.
- Coherent light from two different sources can produce interference patterns.
- In Young's double slit experiment, two slits receive light from the original source and hence are coherent.
- The spacing between any two consecutive bright or dark fringes is the same. This spacing is called fringe width.
- Fringe width, $\beta = \frac{\lambda D}{d}$, where d is the distance between the two slits, D is the distance between the slits and the screen.
- **Conditions for sustained interference:**
 - (i) The two interfering waves should be coherent, which means that the phase difference between them must remain constant in time.

(ii) The two waves must have the same frequency.

(iii) The two waves must be in the same state of polarization.

- The double slits can produce clear interference patterns if they are narrow and very closely spaced.
- On insertion of a thin plate of thickness t and of material of refractive index μ , in the path of one of the interfering rays introduces an additional path difference, equal to $(\mu - 1)t$.

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