

Module 3: Single Slit Diffraction

This module is designed to help you to:

- explain the phenomenon of diffraction;
- explain single slit diffraction
- describe the resolving power of an optical instrument;
- justify the validity of ray optics.

Content

10.3.1 Introduction

10.3.2 Single slit diffraction and resolving power

10.3.2.1 Single slit diffraction

10.3.2.2 Resolving power

10.3.3 The validity of ray optics

10.3.4 Summary



10.3.1 Introduction

When light from a narrow slit is incident on the sharp edge of an obstacle, it will be found that there is illumination to some extent within the geometrical shadow of the obstacle. Thus, light can bend round an obstacle and this bending of light round an obstacle is called diffraction of light. The bending is greater for light of longer wavelength and less for light of shorter wavelength. In this module we discuss the phenomenon of diffraction, especially that due to a single aperture (slit). We also discuss the ability of an optical instrument to produce distinct images of two different closely spaced objects, that we call as resolving power.

We also discuss the importance of distance at which diffraction effects from a source become significant and we no longer can treat light as traveling in a straight line.

10.3.2 Single slit diffraction and resolving power

Activity 10.3.1: Demonstration of Diffraction

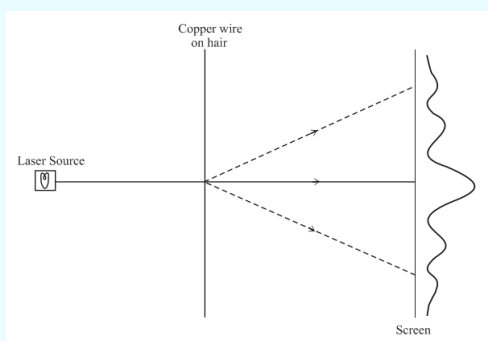


Fig. 10.3.1 *Demonstration of diffraction*

When a beam of laser light is obstructed by a fine object (like copper wire or even hair), the diffraction pattern becomes visible on the screen in the classroom (Fig. 10.3.1).

10.3.2.1 Single slit diffraction

Consider a parallel beam of light of wavelength λ incident on a slit of width a . We want to understand the distribution of light on a screen kept in front of the slit.

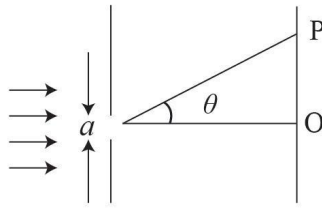


Fig. 10.3.2 *Single slit diffraction*

Let us consider the effect at some point P on the screen, at an angle θ , w.r.t. the perpendicular bisector of the slit.

1. We can imagine the slit divided into two equal halves each of width $a/2$.
2. For each point in the first half, we have a corresponding point in the second half at a distance $a/2$ from it. If waves, from these pair of points, arrive at P, they differ in their path travelled, by a distance $(a/2) \sin \theta$. If this path difference equals $\lambda/2$, then point P will be a dark point i.e.

$(a/2) \sin \theta = \lambda/2$ or $a \sin \theta = \lambda$ is the condition for the first 'dark point'.

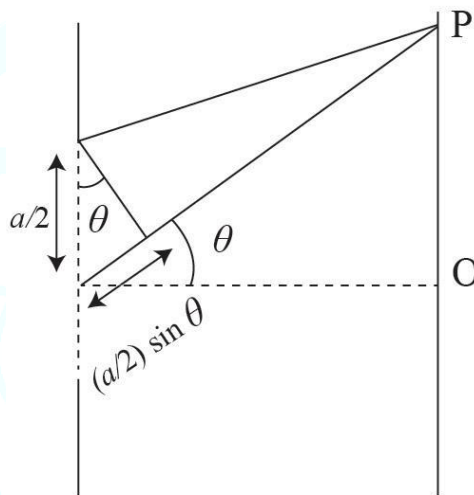


Fig. 10.3.3 *Condition for the first dark point*

3. Similarly, we can divide the slit into 4 equal parts, 6 equal parts, 8 equal parts ..., $2n$ equal parts. We then have respective pairs of points which are separated by $(a/4)$, $(a/6)$, ..., $a/2n$; the path difference (at P) between rays from such pairs will be $(a/4)\sin\theta$, $(a/6)\sin\theta$, ..., $(a/2n)\sin\theta$, respectively. If this path difference

equals $\lambda/2$, the point P, on the screen, $a \sin \theta = n\lambda$ would be a dark point. Convince yourself that this gives the general condition for dark points as

$$a \sin \theta = n\lambda \quad \rightarrow \quad \text{where } n = 1, 2, 3, \dots \quad (10.3.1)$$

4. Points of maximum intensity are in between two dark points (Fig. 10.3.4), but are not necessarily midway between them. The graph of intensity distribution shown indicates that these are located at $(a/\lambda)\sin \theta = 0, 1.46, 2.43, \dots$ (from exact theory)

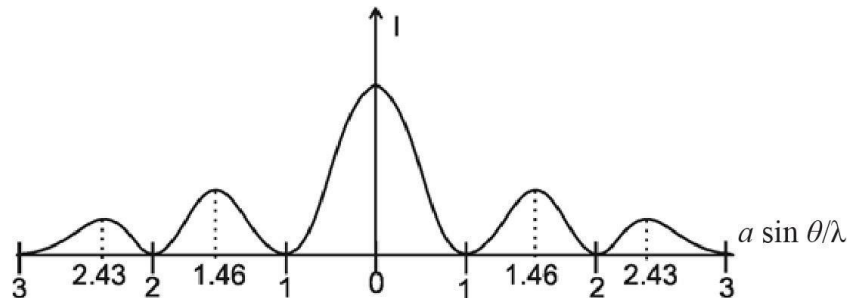


Fig. 10.3.4 showing points of maximum intensity are in between two dark points

5. If $a \approx \lambda \Rightarrow \theta = 90^\circ$ for $n = 1$

This means that if the slit width is of the order of wavelength of light used, the entire screen will get brightly lit.

Example 10.3.1

What happens to single slit diffraction effects, due to each slit, when we observe the double slit interference pattern?

Solution

Intensity in a double slit interference pattern:

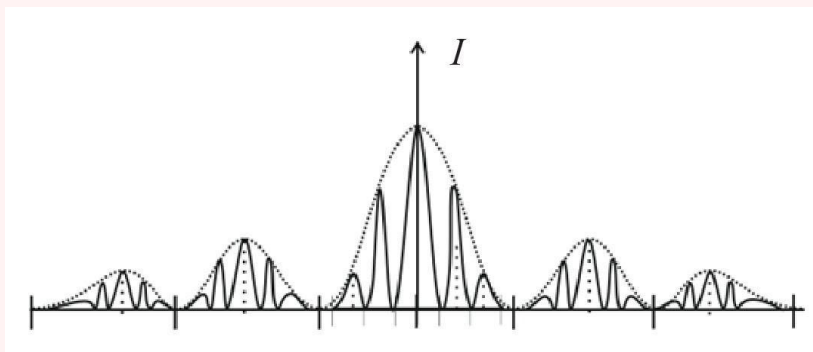


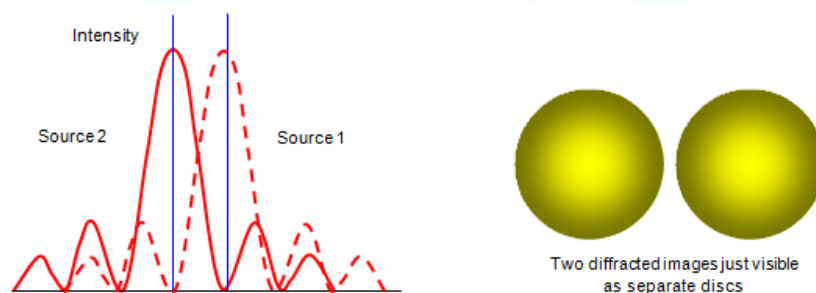
Fig. 10.3.5 *Intensity in a double slit interference pattern*

Refer to Fig. 10.3.5. The intensity distribution of a double slit interference pattern is observed here on the screen. The solid curve represents maximum and minimum due to interference of light coming from two slits. The dotted curve is exactly similar to that of diffraction due to a single slit. In this case the dotted curve represents variation of intensity of maxima due to interference. Thus what we observe is the combined effect of interference and diffraction.

10.3.2.2 Resolving power

Resolving power of an optical instrument is an indicator / measure of its ability to distinguish between two very close objects (*sources of light*) distinctly. Because of diffraction, the beam which is supposed to get focussed at a point, never gets focussed at a point but over a spot of finite area.

For two images to be resolved properly, the central maxima of the diffraction pattern, due to one object, must fall not closer than the first minima of the diffraction pattern due to the other object. This is shown in the Fig. 10.3.6.

**Fig. 10.3.6** *Resolving power*

When the central maxima of the diffraction patterns of the two objects overlap, the objects are not resolved and cannot be observed as two distinct objects. This 'resolving criteria' can be derived mathematically as given below:

The general condition for a dark spot is given by (Eq 10.3.1)

$$a \sin \theta = n\lambda$$

For first minima $n=1$

$$a \sin \theta = \lambda,$$

For small angles, this complies with

$$\theta = \lambda/a$$

However, on detailed analysis, the smallest resolvable angular separation (resolving power θ_0) is given by

$$\theta_0 = 1.22 \lambda / a \quad (10.3.2)$$

The origin of the factor (1.22) lies in Bessel Functions, which is beyond the scope of this discussion. Thus, θ_0 can be obtain using Eq(10.3.2), by measuring the smallest value of ‘ a ’ at which the fringes disappear (that is uniform illumination occurs) for the first time.

10.3.3 The validity of ray optics

An aperture (i.e., slit or hole), of size a , illuminated by a parallel beam, sends diffracted light into a cone of angle (approximately) λ / a . This is the angular size of the bright central maximum. In traveling a distance z , the diffracted beam therefore acquires a (linear) width $z\lambda/a$ due to diffraction. It is interesting to look at the value of z at which the spreading, due to diffraction, becomes comparable to the size a of the aperture. We thus need to (approximately) equate $z\lambda/a$ with a . This gives the distance (say Z_E) beyond which divergence, of the initial incident beam of width a , becomes significant. We, therefore, get

$$\theta = \lambda / a$$

This equation defines as quantity, z_F , called the *Fresnel distance*

$$Z_F = a^2 / \lambda \quad (10.3.3)$$

We thus realize that for distances much smaller than z_F , the spreading due to diffraction is small compared to the (initial) size of the beam. It becomes comparable (to this initial size) when the distance approximates z_F . For distances much greater than z_F , the spreading due to diffraction dominates over that given by ray optics (i.e., the size a of the aperture). It also shows that ray optics would remain valid (even for large distances) in the limit of wavelength tending to zero.



Let Us Reflect

Q.10.3.1 What happens to the diffraction pattern when we key on reducing the width of the slit ?

Q.10.3.2 Describe a single slit diffraction pattern with white light.

Q.10.3.3 While viewing a star, using a device where the separation between two mirrors serves the purpose of slit width, it was found that the fringes completely disappear for the first time, when the separation is 122 inches, for a light of wavelength 570 nm. Then what is the angular diameter of the star? (Take 1 inch = 2.5 cm)



A.10.3.1 For 1st minimum of diffraction , we have $a \sin \theta = \lambda$ or $\sin \theta = \lambda/a$

if we keep reducing slit with a , then θ keeps on increasing ,i.e. central maximum becomes broader.

if we keep decreasing $a \rightarrow \lambda$, then $\theta \rightarrow \frac{\pi}{2}$. This means the central maximum fills up the entire forward hemisphere and the diffraction pattern disappears.

A.10.3.2 With a monochromatic light the diffraction pattern would be bright and dark fringes with central fringe being bright .If white light is used , all the colors will have maximum at the center giving white central fringe.

Subsequently ,condition for minima $a \sin \theta = n\lambda$ gives different θ for different wavelengths making the pattern look coloured.

A.10.3.3 The angular diameter will be equivalent to the resolving power, and thus according to Eq(10.3.2), it

$$\text{will be } \theta_0 = \frac{(1.22) \times 570 \times 10^{-9}}{122 \times 2.5 \times 10^{-2}} \approx 2.28 \times 10^{-7} \text{ radian}$$

10.3.4 Summary

- When light passes through a narrow opening (aperture) it spreads. The phenomenon of a part of it entering the geometrical shadow is called diffraction.
- This spread light arriving at a screen from a different part of the aperture produces a pattern that we call a diffraction pattern.
- The smallest angular separation between two closely spaced objects that can be resolved is given by $\theta = \lambda/a$ where a is the size of the aperture of the optical instrument and λ is the wavelength of light used.
- Detailed analysis of the smallest resolvable angular separation turns out to be $\left\{(1.22)\frac{\lambda}{a}\right\}$.
- Propagation of light from aperture can be treated as a ray traveling in straight line upto distance $Z_F = a^2/\lambda$ beyond which diffraction effects become important.

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