

**Master Lesson Plan**  
**for**  
**Number Systems**

| Board | Standard | Subject     | Chapter        | Language |
|-------|----------|-------------|----------------|----------|
| NCERT | STD IX   | Mathematics | Number Systems | English  |

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|                                                     |                                                                                                                                                                          |
|-----------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>CURRICULAR GOALS</b>                             | <b>CG-1:</b> Understands numbers (natural, whole, integer, rational, irrational, and real), ways of representing numbers, relationships amongst numbers, and number sets |
| <b>COMPETENCIES<br/>AS PER NCF/NIPUN<br/>BHARAT</b> | <b>C-1.1:</b> Develops understanding of numbers, including the set of real numbers and its properties                                                                    |
| <b>SDG(Sustainable<br/>Development Goals)</b>       | <b>SDG4:</b> Quality Education                                                                                                                                           |
| <b>Technology aids</b>                              | PPT                                                                                                                                                                      |
| <b>Cross-Cutting Themes</b>                         | Educational Technology - PPTs for every component<br>Values and Dispositions - VC_Life is like a wave                                                                    |

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## MS\_Objectives\_Number Systems

**Objective:** Teachers will be able to get an overview of the asset objectives in the MLP for the chapter 'Number Systems'.

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**[Notes to the teacher:** This asset lays down the proposed plan for transacting this chapter. It states the asset objectives of the MLP. This asset is for the teacher's reference and need not be taught to the students.]

Students will be able to –

- explore the need for extending the number system beyond rational numbers
- classify the different types of numbers
- solve questions on rational numbers
- define irrational numbers
- imagine the size of the set of natural numbers, the set of real numbers, etc.
- construct the square root spiral
- recognise that life naturally has ups and downs
- write a terminating/non-terminating recurring decimal as a rational number or vice versa
- infer that the decimal expansion of an irrational number is non-terminating/non-recurring
- solve questions on decimal forms of real numbers
- relate to how irrational numbers can be used in our day-to-day life applications
- perform operations like addition, subtraction, multiplication and division on surds, simplify expressions and rationalise the denominators
- apply the Laws of Exponents
- notice the results of arithmetic operations on any two irrational numbers and on a rational number and an irrational number
- locate the point representing  $\sqrt{x}$  for any positive real number  $x$  geometrically
- solve questions on laws of exponents
- summarise the chapter 'Number Systems'.

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## IQ\_Extending the Number System

**Objective:** The student will be able to explore the need for expanding the number system beyond rational numbers.

**Teaching Aids:** PPT

**Notes to the teacher:** The teacher may pose the following questions to the students, allow them to think and come out with as many answers as possible before arriving at a conclusion. If there is any difficulty in getting the answers, the teacher can ask leading questions.

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1. During summers, Bola likes to sleep on the terrace of his house at night. He loves to watch the stars. He knows there are trillions of stars in the sky, but counting them is fun for him.



Boy Star Gazing: commons.wikimedia.org/wiki/File:Martos-Starlight.jpg  
Attribution: Victor

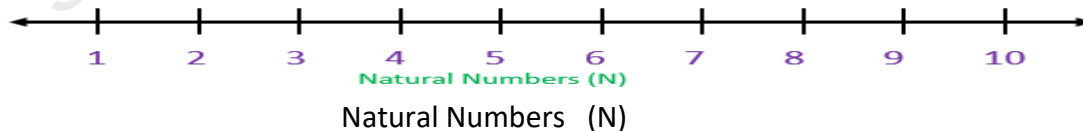
Which numbers in the numbers system does he use to count the stars in the sky?

Why are they called so?

Answers by the students.

**Suggested Answer:**

Bola uses natural numbers to count the stars. They are used naturally to count things in nature, so they get the name 'Natural Numbers' (N).



2. Have you ever thought, if natural numbers can be used in counting things of nature, then why is the number system extended to include a zero?

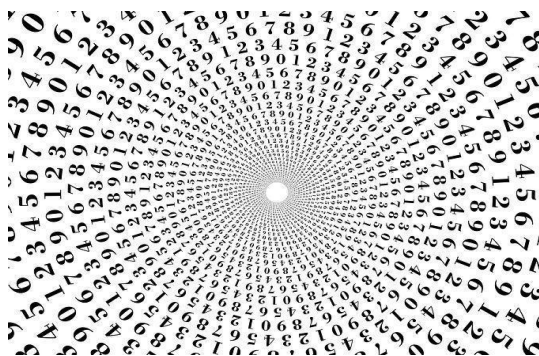


Image Source: pixabay.com/illustrations/pay-digits-number-abundance-series-819587/  
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Give a situation where you realised the importance of extending the number system from natural numbers to 'Whole Numbers' (W) to include 0.

Give a situation where you realised the importance of extending the number system from natural numbers to 'Whole Numbers' (W) to include 0.



Whole Numbers (W)

Answers by the students.

Suggested Answer:

- Nona distributed all the sweets she had with her, among her friends. She has no sweets left for herself. So, the number of sweets left with her is 0.
- Solving equation  $x + 5 = 5$ , gives  $x = 0$

1. The whole numbers are insufficient in some situations. Subtraction of integers, does not always give a whole number,  $6 - 10 = -4$

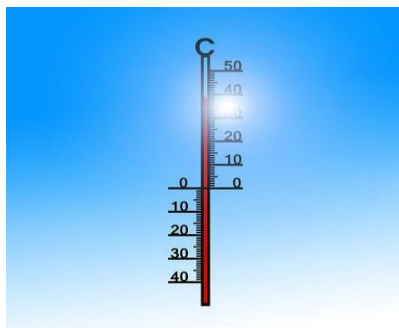
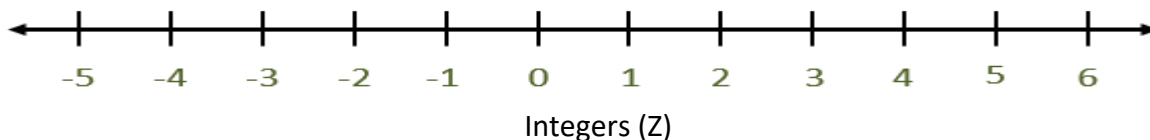


Image Source: pixabay.com/illustrations/thermometer-summer-hot-heat-sun-4353319/  
Attribution: geralt

So, the number system further extended to 'Integers' (Z). Where are negative integers used in real life?



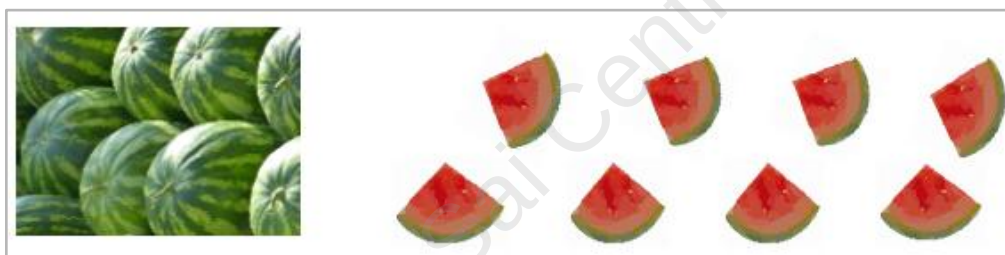
Answers by the students.

Suggested Answers:

The integers are used for:

- For expressing the temperature below the freezing point (zero degree).
- Writing the amount of money to be withdrawn from the account. When ₹ 100 is withdrawn from the account, it is expressed as -100.
- Expressing the height of a place, below the sea level. If the height below the sea level is 8000 metres, it is considered as -8000m.

2. A watermelon is cut into eight equal pieces, each piece is  $\frac{1}{8}$  of the watermelon.



**Watermelon:** pixabay.com/photos/melons-water-melons-fruit-green-197027/

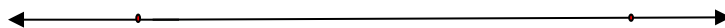
Attribution: LoggaWiggler

**Slices:** pixabay.com/vectors/watermelon-melon-slice-fruit-sweet-295086/

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Dividing an integer by another integer does not always result in an integer. So, the number system extended to 'Rational Numbers' (Q), the numbers of the form  $\frac{p}{q}$ , where  $q \neq 0$ .

Give a few rational numbers lying between 0 and 1. How many such numbers can you find?



Answers by the students.

Suggested Answer:

Some numbers lying between 0 and 1 are  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$  etc. There are infinitely many rational numbers between any two given rational numbers.



3. What is the length of the hypotenuse AC of isosceles right triangle ABC in figure 1? Is the number represented by its length, a rational number?

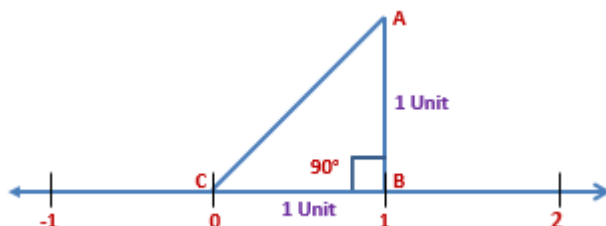


Image Source: OC

Answers from the students.

Suggested Answer:

In right  $\triangle ABC$ ,

$AB^2 + BC^2 = AC^2$  (Pythagoras theorem)

$$\Rightarrow 1^2 + 1^2 = AC^2$$

$$\Rightarrow 2 = AC^2$$

$$\Rightarrow \sqrt{2} = AC$$

The length of the hypotenuse  $AC = \sqrt{2}$  units.

$\sqrt{2}$  is not a rational number, as it can not be expressed in the form  $\frac{p}{q}$ , where  $q \neq 0$ .

Such numbers are called irrational numbers.

**[Notes to the teacher:** The teacher may use a compass to explain to the students that if a point P is marked on the number line, at a distance equal to AC from point C, then  $AC = CP = \sqrt{2}$ .

So, point P represents the number  $\sqrt{2}$  on the number line.

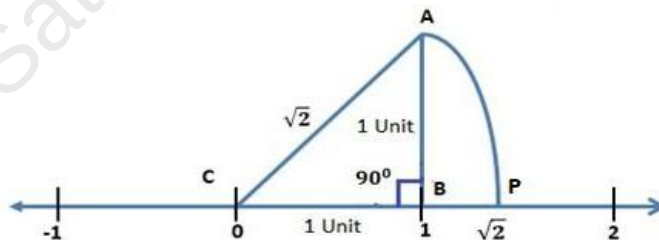


Image Source: OC

Thus,  $\sqrt{2}$  can be represented on the number line.]

So, like the rational numbers, every irrational number also has a place on the number line. You will learn more about them in this chapter.

Rational numbers and irrational numbers together form the collection of 'Real Numbers' (R). And that completes our Number System.

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Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.  
That's why the number line is called the 'Real Number Line'.

| Time to teach | Asset type            | Theme        | Sub Theme        |
|---------------|-----------------------|--------------|------------------|
| 10 minutes    | Inquisitive Questions | Real Numbers | Real number line |

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## MS\_Types of numbers

**Objective:** Students will be able to classify the different types of numbers.

**Teaching Aids:** PPT

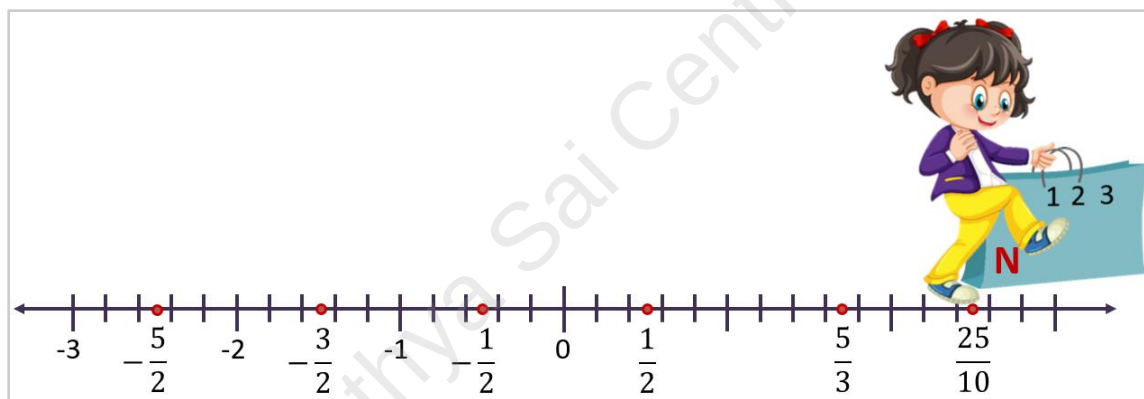
**Notes to the teacher:** The teacher commences the class explaining about the number line and how to represent various numbers on it.

### Types of Numbers

Let us take a number line. We know that it extends both ways from 0, going infinitely with various types of numbers on it, as shown below.

Now imagine a young girl walking along the number line, starting from 0, in the positive direction as shown below. All she would see are numbers and numbers - one after the other, going forever.

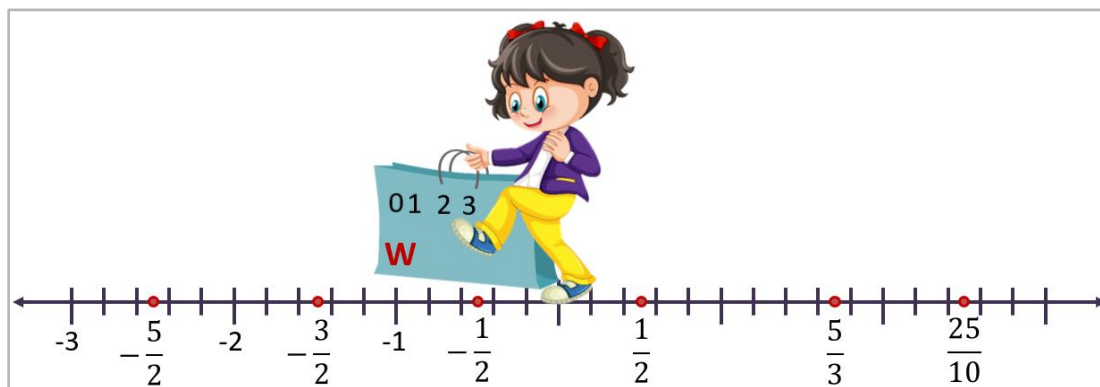
As she is walking along the number line, she notices 1, 2, 3... She starts putting these numbers 1, 2, 3 into a bag, labelled 'N', to denote that these are called 'Natural Numbers'.



Girl : [freepik.com/free-vector/girl-walking-dog-cartoon-character\\_27181710.htm](https://www.freepik.com/free-vector/girl-walking-dog-cartoon-character_27181710.htm) Attribution: brgfx

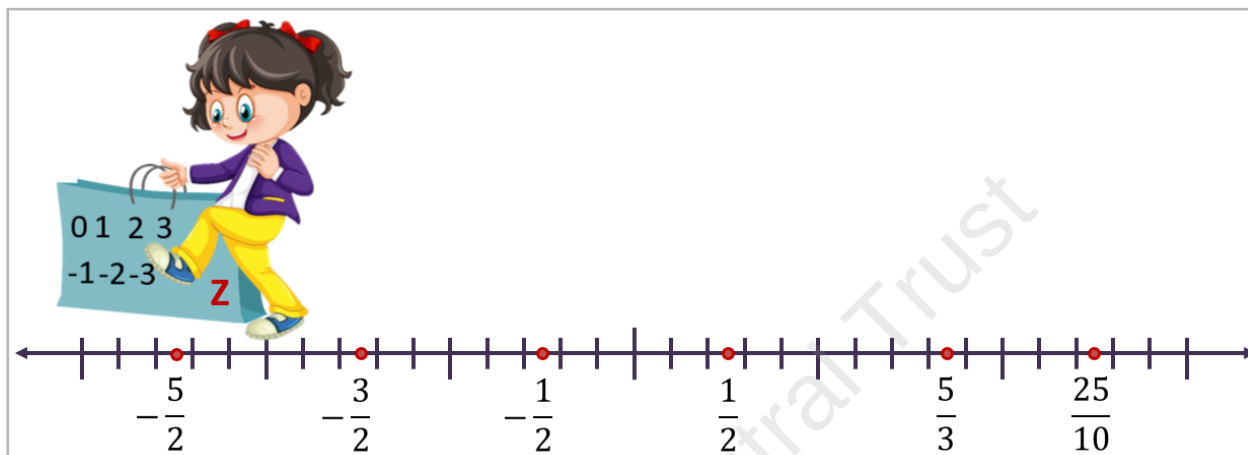
Bag: [pixabay.com/vectors/shopping-shopping-bag-bag-sale-312311/](https://www.pixabay.com/vectors/shopping-shopping-bag-bag-sale-312311/) Attribution: Clker-Free-Vector-Images

Now, let the girl turn back and head towards 0. If she picks up the zero and puts it in the bag, it becomes a bag of 'Whole Numbers'. Now, the bag gets labelled as 'W'.



Girl : [freepik.com/free-vector/girl-walking-dog-cartoon-character\\_27181710.htm](https://www.freepik.com/free-vector/girl-walking-dog-cartoon-character_27181710.htm) Attribution: brgfx

She keeps heading in that direction, she sees a series of negative numbers, one after the other. Let her put all the negative numbers as well into this bag. The collection of all these numbers is now called Integers. Let us label the bag as Z.



Girl : freepik.com/free-vector/girl-walking-dog-cartoon-character\_27181710.htm Attribution: brgfx  
Bag: pixabay.com/vectors/shopping-shopping-bag-bag-sale-312311/ Attribution: Clker-Free-Vector-Images

Now, let us look at what is left on the number line.

We notice numbers like  $\frac{1}{3}$ ,  $-\frac{2}{5}$ ,  $\frac{2001}{3600}$ , etc.

If you put all these numbers as well in the bag, it is now called a bag of 'Rational Numbers' (Rational is derived from the word 'ratio'). Let us label it as **Q** (comes from the word 'quotient').



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Bag: pixabay.com/vectors/shopping-shopping-bag-bag-sale-312311/ Attribution: Clker-Free-Vector-Images

## Summary

- Natural Numbers are 1, 2, 3..
- Whole Numbers are 0, 1, 2, 3, ..
- Integers are negative as well as positive numbers like .... -5, -4, -3, -2, -1, 0, 1, 2, 3, 4,...

- Rational Numbers are everything listed above, and in between, like  $-2, \frac{-3}{2}, -1, \frac{-1}{4}, 0, \frac{3}{4}, 1, ..$

## Rational Numbers

Consider a number like  $\frac{2}{5}$ . This is called a rational number, because it can be written in the form  $\frac{p}{q}$  where p and q are integers and  $q \neq 0$ , i.e.,  $p = 2$  and  $q = 5$ , which are integers and the denominator q (=5) is not a zero.

Every number we listed above can be represented in the form of  $\frac{p}{q}$  where p and q are integers and  $q \neq 0$ .

For example, -36 can be represented as  $\frac{-36}{1}$ . Here,  $p = -36$  and  $q = 1$ .

From this, we can understand that natural numbers, whole numbers and integers may all be called Rational Numbers.

Additionally, rational numbers do not have a unique representation, in the form  $\frac{p}{q}$ , where p and q are integers and  $q \neq 0$ . This means that  $\frac{1}{2} = \frac{2}{4} = \frac{50}{100} = \frac{-8}{-16}$ , and so on. These are called Equivalent Rational Numbers (or fractions).

### Points to remember:

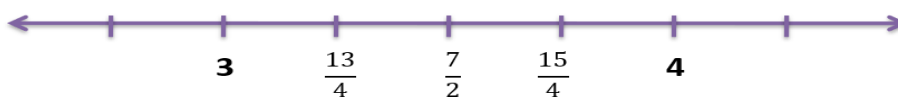
**When we represent  $\frac{p}{q}$  on the number line, or when we say that  $\frac{p}{q}$  is a rational number, there are few conditions:**

1. q, the denominator, is not equal to 0, and
2. p and q do not have any common factor other than 1, that is, p and q are co-prime.

This means that even though there are infinitely many fractions equivalent to  $\frac{1}{2}$  (like  $\frac{1}{2} = \frac{2}{4} = \frac{50}{100} = \frac{-8}{-16}$  and so on), we will choose only  $\frac{1}{2}$  to represent all of them on the number line.

- Between any 2 rational numbers, there are infinitely many rational numbers.

E.g. between 3 and 4, there are other rational numbers like  $\frac{13}{4}, \frac{7}{2}, \frac{15}{4}$  and so on.



| Time to teach | Asset type  | Theme        | Sub Theme        |
|---------------|-------------|--------------|------------------|
| 15 minutes    | Main Script | Real Numbers | Real number line |

### QA\_Questions on Rational Numbers

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**Objective:** Students will be able to solve questions on rational numbers.

**Teaching Aids:** PPT

**Notes to the teacher:** Teacher may present the questions and allow the students to write or say the answers before revealing them to the students.

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1) State whether the following statements are TRUE OR FALSE.

- a. Every rational number is an integer.
- b. Every fraction is a rational number.
- c. Zero is a rational number.
- d. The smallest whole number is 1.
- e. Every integer is a rational number.
- f. The smallest integer is -1.

2) Answer the following questions:-

- (a) Write an integer which is not a natural number.
- (b) Write a rational number which is not an integer.
- (c) Write a whole number which is not a natural number.
- (d) 18 is the successor of which natural number?

3) a) Find five rational numbers between “4 and 5”.

b) Find six rational numbers between  $\frac{3}{7}$  and  $\frac{4}{7}$ .

**[Notes to the teacher:** There could be many answers and methods to find the solution to Q.3.

While only one method is given here in the solutions, the teacher may use her discretion to validate the other answers given by the students.]

**Solution:**

1) a) False, there are infinitely many rational numbers which are not integers.

b) True.

c) True.

d) False, the smallest whole number is 0.

e) True

f) False, there is no smallest integer, given any integer, we can find an integer smaller than it.

2) a) 0

b)  $\frac{1}{3}$

c) 0

d) 17

3) a) Five rational numbers between 4 and 5.

Write both 4 and 5 as fractions, i.e.  $4 = \frac{4}{1}$  and  $5 = \frac{5}{1}$

To find five rational numbers between 4 and 5, multiply both numerator and denominator of the above fractions by 6

$$\text{Thus } 4 = \frac{4}{1} = \frac{4 \times 6}{1 \times 6} = \frac{24}{6} \text{ and } 5 = \frac{5}{1} = \frac{5 \times 6}{1 \times 6} = \frac{30}{6}$$

The numerators are 24 and 30

The five natural numbers between 24 and 30 are 25, 26, 27, 28 and 29

The five rational numbers between 4 and 5 are  $\frac{25}{6}, \frac{26}{6}, \frac{27}{6}, \frac{28}{6}$  and  $\frac{29}{6}$

b) To find six rational numbers between  $\frac{3}{7}$  and  $\frac{4}{7}$

Multiply both numerator and denominator of  $\frac{3}{7}$  and  $\frac{4}{7}$  by 7.

$$\frac{3}{7} = \frac{3 \times 7}{7 \times 7} = \frac{21}{49} \text{ and } \frac{4}{7} = \frac{4 \times 7}{7 \times 7} = \frac{28}{49}$$

The numerators are 21 and 28

The six natural numbers between 21 and 28 are 22, 23, 24, 25, 26 and 27

The six rational numbers between  $\frac{3}{7}$  and  $\frac{4}{7}$  are  $\frac{22}{49}, \frac{23}{49}, \frac{24}{49}, \frac{25}{49}, \frac{26}{49}$  and  $\frac{27}{49}$

| Time to teach | Asset type          | Theme            | Sub Theme                                                                         |
|---------------|---------------------|------------------|-----------------------------------------------------------------------------------|
| 15 minutes    | Questions to Assess | Rational Numbers | Properties of Rational numbers<br>Rational number between two<br>Rational numbers |

## MS\_Irrational numbers

**Objective:** Students will be able to define irrational numbers.

**Teaching Aids:** PPT

**Notes to the teacher:** The teacher commences the class by explaining the types of numbers like natural numbers, whole numbers, integers and rational numbers.

You have learnt about different types of numbers such as natural numbers, whole numbers, integers and rational numbers. Let us check if there are any other types of numbers.

Let us look at what is left on the number line again.

Consider one example.

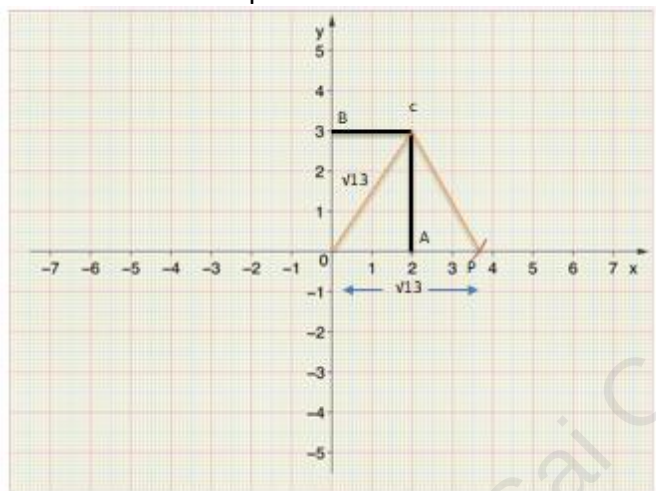


Image source: pixabay.com/illustrations/coordinate-system-four-quadrants-2342859/

Attribution: mbnachhilfe\_de

In the figure, an arc of radius OC, centred at O, cuts x - axis at P.

What is the length of OP?

$OP = OC$  (OP and OC are radii of the same arc)

$OA = 2$  units,  $OB = 3$  units.

$OC^2 = OA^2 + OB^2 = 2^2 + 3^2 = 4 + 9 = 13$  (Using Pythagoras theorem)

$OP = OC = \sqrt{13}$

Thus, the point P on the number line represents the number  $\sqrt{13}$ .

But it's not possible to represent  $\sqrt{13}$  in the form  $\frac{p}{q}$ ,  $p, q$  integers,  $q \neq 0$ . Such numbers are called irrational numbers.

### Examples of irrational numbers:

If a natural number  $n$  is a perfect square OR  $n$  is the quotient of two perfect squares, then its square root is a rational number.

E.g.  $\sqrt{4} = 2$ ,  $\sqrt{9} = 3$ ,  $\sqrt{16} = 4$  are rational numbers.

If  $n = \frac{25}{36}$ , quotient of two perfect squares, then

$\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ , which is a rational number.

The square roots of all natural numbers which are not perfect squares, are irrational

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numbers.

Thus,  $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}$  ....are some examples of irrational numbers.

We know that if  $a > 0$  is a real number, and  $n$  is a natural number,  $\sqrt[n]{a} = b, b > 0$  if  $b^n = a$ .

The symbol ' $\sqrt{\quad}$ ' is called the **radical sign**.

In some cases, we get  $\sqrt[n]{a} = b$ , a rational number. In all other cases it is irrational.

If  $\sqrt[n]{a}$  is an irrational number, then it's called a surd of order  $n$ .

E.g. When  $n = 3$ ,

If the number  $a$  is a cube OR  $a$  is the quotient of two cubes, then its cube root is a rational number.

$\sqrt[3]{8} = 2, \sqrt[3]{27} = 3, \sqrt[3]{125} = 5, \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$  are rational numbers.

In all other cases we get irrational numbers.

$\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5}, \sqrt[3]{10}$ ... are some examples of irrational numbers.

When  $n = 4$

$\sqrt[4]{81} = 3, \sqrt[4]{625} = 5, \sqrt[4]{\frac{16}{81}} = \frac{2}{3}$ , which are rational numbers.

But if  $a$  is not the fourth power of a rational number, we get  $\sqrt[4]{a}$  as an irrational number.

$\sqrt[4]{15}, \sqrt[4]{67}, \sqrt[4]{120}$  are some examples of irrational numbers.

### **Some Special irrational numbers**

#### **The number $\pi$**

For all circles, the ratio of the circumference to the diameter is a constant and it is denoted by  $\pi$ .

We need the value of  $\pi$ , to calculate the circumference or area of a circle when its radius is given. We take the value of  $\pi$  approximately as  $\frac{22}{7}$  or 3.14 in decimal form. Both these numbers are rational numbers.

But  $\pi$  is a special number. When you divide the circumference of a circle by its diameter, you get an infinite number of digits after the decimal point, with a non-repeating pattern. We cannot represent such decimal numbers as fractions. Thus the number  $\pi$  is irrational.

#### **The golden ratio $\phi$ (Phi)**

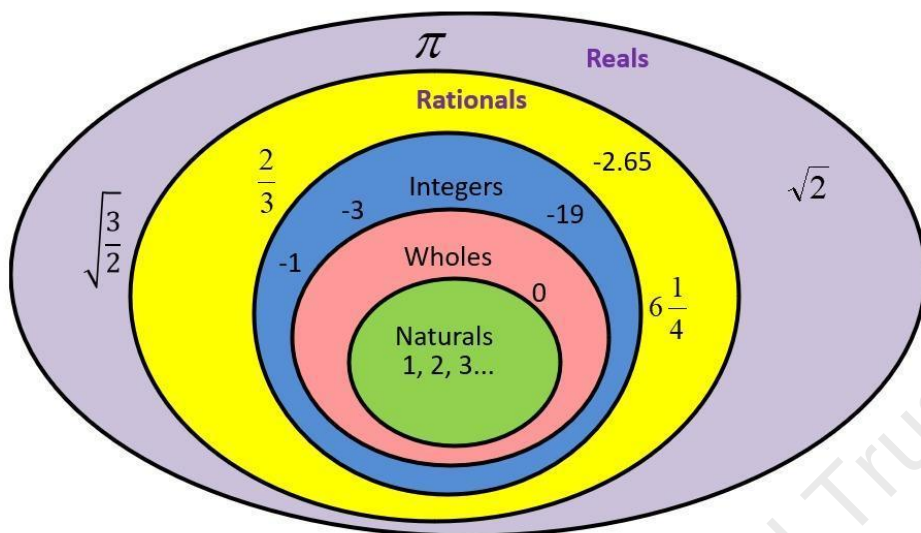
The golden ratio denoted by  $\phi$  has approximate value 1.618, but its exact value is  $\frac{1 + \sqrt{5}}{2}$  which is an irrational number.

The golden ratio is a special number because artists and architects believe that the ratio is aesthetically pleasing. For example if you take the golden rectangle, in which the ratio of the longer side to the shorter is  $\phi$ , then the rectangle looks like a typical frame of painting.

#### **The set of real numbers**

The collection of numbers that include Natural numbers, Whole numbers, Integers, Rational

Numbers and Irrational numbers are called Real Numbers.



The set of real numbers is denoted by R.

Thus,

*Every real number is represented by a unique point on the number line.*

The converse of the above statement is also true.

*Every point on the number line represents a unique real number.*

So, the number line is called the real line.

| Time to teach | Asset type  | Theme        | Sub Theme                  |
|---------------|-------------|--------------|----------------------------|
| 15 minutes    | Main Script | Real Numbers | Definition of Real numbers |

## IA\_Cardinality of sets

**Objective:** Students will be able to imagine the size of the set of natural numbers, the set of real numbers, etc.

**Teaching Aids:** PPT

**Notes to the teacher:** The teacher can keep about seven different things: a pencil, book, pen, eraser, ruler, duster and a chalk, on the table. She asks a student to come forward and count the number of things on the table.

Let the student count by lifting each thing and announcing its order in the process of counting. The teacher asks the remaining students in the class to observe the way of his/her counting

| Numbers used for counting | 1                                                                                 | 2                                                                                 | 3                                                                                 | 4                                                                                 | 5                                                                                 | 6                                                                                  | 7                                                                                   |
|---------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| Things in the set         |  |  |  |  |  |  |  |

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**[Notes to the teacher:** The teacher can explain the cardinality of a finite set using the above example as follows]

**Cardinality of a finite set:** The size or cardinality of a finite set is measured by the number of elements it contains. In this example, {1, 2, 3, 4, 5, 6, 7}.

The cardinality or size of the set is 7.

Thus, cardinality of a set is 7, when

- 1) The total number of things in the set is 7.
- 2) There is a 1 - 1 correspondence between the set and the set {1, 2, 3, 4, 5, 6, 7}

**Cardinality of an infinite set:**

**[Notes to the teacher:** The teacher may pose the question, “What is infinity?” She allows the students to think and come out with as many answers as possible.]

She explains ‘infinity’ is not a number, it is an idea of something without a bound or without an end.

E.g. The supergiant galaxy IC 1101 is one of the largest known galaxies and its halo extends 20 lakhs light years where one light year is about 9,500,000,000,000 km. This number looks tremendously large, still it has an end. So, it is not infinity.

The set  $N$  of all natural numbers, is an infinite set. As there is no end to the set, we can’t write all the elements of the set. The cardinality of  $N$  is denoted by  $\aleph_0$ .

- **In general, if some finite/ininitely many elements are added or deleted from an infinite set, the cardinality of the set does not change.**

Let us consider an example to show this.

**Hilbert’s Infinite Hotel Paradox:** Imagine there is a hotel with an infinite number of rooms numbered as 1, 2, 3,....

The rooms are fully occupied by an infinite number of guests, say  $g_1, g_2, g_3, \dots$

|              |       |       |       |       |       |       |       |       |       |          |          |      |
|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|------|
| Room numbers | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10       | 11       | .... |
| Guests       | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ | .... |

Suppose that the hotel manager has to accommodate 5 new guests A, B, C, D and E.

He can do it as follows.

Shift  $g_1$  to room no. 6,  $g_2$  to room no. 7 and so on. Since there is no last room, this shifting continues. The first 5 vacant rooms are occupied by the new guests A, B, C, D and E.

|              |   |   |   |   |   |       |       |       |       |       |       |      |
|--------------|---|---|---|---|---|-------|-------|-------|-------|-------|-------|------|
| Room numbers | 1 | 2 | 3 | 4 | 5 | 6     | 7     | 8     | 9     | 10    | 11    | .... |
| Guests       | A | B | C | D | E | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | .... |

How can you find the size or cardinality of infinite sets?

- **In infinite sets the test for determining cardinality is the existence of a 1 - 1 correspondence relation between the sets.**

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E.g. if there is a 1 - 1 correspondence between a set and  $\mathbb{N}$  then the cardinality of the set is the same as  $\aleph_0$ , the cardinality of  $\mathbb{N}$ .

**[Notes to the teacher:** The teacher may pose the question, is the size of all infinite sets the same? She allows the students to think and come out with as many answers as possible.]

The set  $\mathbb{R}$  of all real numbers contains all irrational numbers, which have non-terminating, non-recurring decimal expansion. It's not possible to establish a 1 - 1 correspondence between irrational numbers and  $\mathbb{N}$ (Natural numbers). Thus, the cardinality of the set of all real numbers is greater than  $\aleph_0$ , the cardinality of  $\mathbb{N}$ .

Thus, the concept of cardinality is very interesting in Set Theory because

- It leads to the awareness that some sets differ from other sets by containing more/less elements.
- It has been used to prove that some infinite sets are larger in size than some other infinite sets.

| Time to teach | Asset type         | Theme        | Sub Theme              |
|---------------|--------------------|--------------|------------------------|
| 8 minutes     | Interesting Asides | Real Numbers | Axioms on real numbers |

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### SA\_Square root spiral

**Objective:** Students will be able to construct the square root spiral.

**Teaching Aids:** PPT

**Notes to the teacher:** Before starting the activity, the teacher can explain that a square root spiral is formed by right angled triangles, with hypotenuse as  $\sqrt{n}$ ,  $n = 2, 3, 4$ , etc.

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#### **Aim:**

Each student will draw the spiral with points  $O, P_1, P_2, P_3, P_4, P_5, \dots$  such that  $OP_1, P_1P_2, P_2P_3, P_3P_4, P_4P_5, \dots$  are of length one unit, also  $OP_2 = \sqrt{2}$ ,  $OP_3 = \sqrt{3}$ ,  $OP_4 = \sqrt{4}$ ,  $OP_5 = \sqrt{5}$  and so on.

**Resources Required:** A large sheet of paper, pencil, protractor, compass, scale/ruler

**Setting for the Activity:** Indoor

**Type of Activity:** Project

**Preparation of Activity:** Distribution of material

**Role of the Teacher:** Facilitator

#### **Procedure:**

The teacher gives the following instructions to students:

Step 1: On the sheet of paper, draw a straight horizontal line. Place a point  $O$  at the centre. Mark a point  $P_1$  one inch away from the centre  $O$ , as shown below.



Step 2: Mark a point  $P_2$  perpendicular to  $P_1$  such that  $OP_1 = P_1P_2 = 1$  inch. Join the points  $O$  and  $P_2$ .

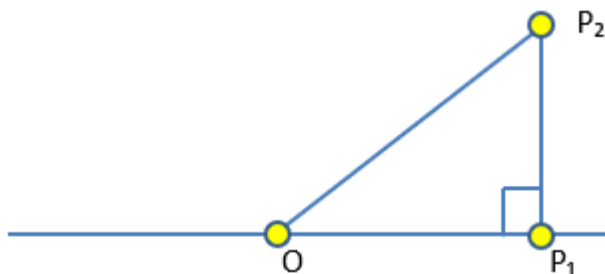


Image source: PPT Shapes

Since  $OP_1 = P_1P_2 = 1$  inch, the hypotenuse  $OP_2 = \sqrt{2}$

Explanation:  $(OP_2)^2 = (OP_1)^2 + (P_1P_2)^2$   
 $(OP_2)^2 = 1^2 + 1^2 = 2$   
 $OP_2 = \sqrt{2}$

Step 3: With  $OP_2$  as base, mark a point  $P_3$  such that  $P_2P_3$  is perpendicular to  $OP_2$  and  $P_2P_3 = 1$  inch.

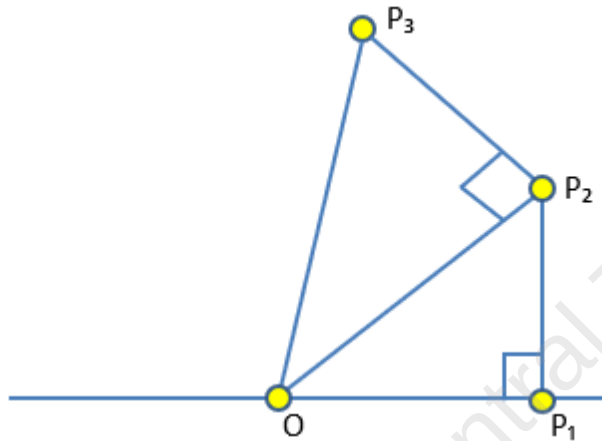


Image Source: PPT shapes

$OP_2 = \sqrt{2}$  ;  $P_2P_3 = 1$  inch; hence the hypotenuse  $OP_3 = \sqrt{3}$

Step 4: Repeat this process by using the hypotenuse as base, mark a point perpendicular to the hypotenuse, 1 inch long. Join that point to the point O.

### **Observation:**

It would start looking like this:

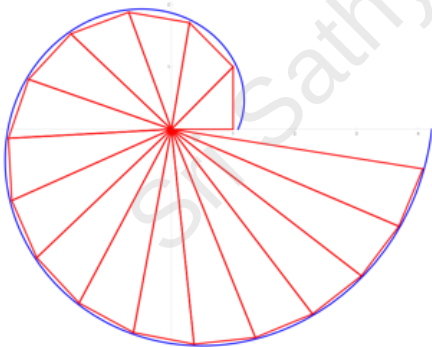


Image source: [commons.wikimedia.org/wiki/File:Theodorus\\_vs\\_archimedean\\_spiral.svg](https://commons.wikimedia.org/wiki/File:Theodorus_vs_archimedean_spiral.svg)  
 Attribution: User:Rectas

**Conclusion:** A square root spiral is a spiral with many right angled triangles, hypotenuse of the triangles  $\sqrt{n}$ ,  $n = 2, 3, 4 \dots$  as the radii of the spiral.

### **Follow-up activity:**

Now, the students may get creative and see how they may extend this drawing into a snail, an umbrella, a bird or a circular stairway. Here are a few ideas:



**Hurricane:** pixabay.com/photos/hurricane-isabel-tropical-cyclone-74031/

Attribution: Wikilimages

**Snail:** pexels.com/photo/orange-and-white-seashell-on-white-surface-33234/

Attribution: pixabay

**Plants:** pixabay.com/photos/plant-nature-spirals-1158445/

Attribution: mish258

**Snail Drawing:** pixabay.com/vectors/snail-animal-slow-spiral-1293083/

Attribution: OpenClipart-Vectors

**Bird:** pixabay.com/de/vectors/truthahn-das-erntedankfest-gef%C3%BCgel-1299176/

Attribution: OpenClipart-Vectors

**Umbrella:** freepik.com/free-photo/red-floral-wagasa-umbrella-studio-high-angle\_32483679.htm

Attribution: freepik

| Time to teach | Asset type         | Theme        | Sub Theme                                                      |
|---------------|--------------------|--------------|----------------------------------------------------------------|
| 20 minutes    | Suggested Activity | Real Numbers | Square root spiral(locating irrational numbers on number line) |

## VC\_Life Is Like A Wave

**Objective:** The students will be able to recognise that life naturally has ups and downs.

**Concept:** The number line is incomplete without negative integers, irrational numbers, etc. Life is similar. With very few exceptions, life is undulating; successes and failures, happiness and sorrow, pleasant and unpleasant events alternate to create a rewarding and complete life. We

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should be able to take things as they come and face situations with patience and determination.

**Notes to the teacher:**

1. Please narrate the following story and ask the students to consider the 'Discussion Questions'.
2. Let the students volunteer to answer each question.
3. More than one student can express her/his answer to any question.
4. After all the questions are answered, you could summarise the discussion and provide them points to reflect upon.

**Story**

Here is a story that teaches us an important life lesson.

Michael, an American person of colour, created a record when he got the rare honour of being selected for triple board residency last year. This is extraordinary, as getting into residency requires years of hard work after graduating from medical school.

Getting into medical school in the USA is not easy either! It requires excellent grades in undergraduate degree of four years in Science. That requires a tremendous amount of hard work plus sufficient financial resources. Most students struggle to get sufficient bank loans to finance their education.

Was Michael born rich, born to highly educated parents or blessed with an environment that made the efforts easier? Far from it. Michael was raised by a single mother who was extremely poor and survived by picking leftover food from garbage dumpsters. He used to help his mother in the task as it was a constant struggle to find a few morsels of food.

Asked about his remarkable achievement despite the extreme hardship, Michael said, "I could survive and educate myself thanks to the opportunities that came my way. I had several setbacks and impediments, but I chose to take them by their horns and move on in my task. Opportunities were always followed by obstacles but I had to ride over them and continue. When I secured the job of a security guard outside the major hospital in my city, I could cultivate friendship with students and professors. That was such a big positive! They were most helpful with words of encouragement and also books and other help.

I wanted to enter the portals of that hospital to train and become a doctor. All my contacts were sympathetic and gave me the right advice and necessary support. All I had to do was work hard and study day and night. Life was full of ups and downs, but I learned to cope with the downs and focus on the ups. As day follows night and calm follows the storm, I am certain that

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difficulties will be followed by success. If life were too smooth with no challenges, you wouldn't learn much".

### Discussion Questions

1. Do you think someone can have uninterrupted success with talent and some luck? Can you name any famous personality in sports or arts or science, who had such success?

**Expected answer:** Talent and luck may be useful to have, but not enough for continued success. Building on failures makes one more perfect and long lasting.

2. Could Michael have risen to such heights if he did not get the support of the hospital staff and students?

**Expected answer:** He probably would not have. However, someone who has a positive attitude will always find help from unexpected quarters.

### Points to Ponder/reflect

Quote:

**Life is an undulating sine wave; that starts at zero, goes through peaks and troughs and eventually ends back at zero. - Saad Shakeel**

**(Source: Ownquotes.com/quote/854)**

| Time to teach | Asset type    | Theme | Sub Theme     |
|---------------|---------------|-------|---------------|
| 12 minutes    | Value Content | Peace | Understanding |

### MS\_Rational Decimal conversion

**Objective:** Students will be able to write a terminating/non-terminating recurring decimal as a rational number or vice versa.

**Teaching Aids:** PPT

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**[Notes to the teacher:** The teacher can start the class by explaining the types of decimals with examples.]

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**Types of decimal representation:**

Consider the following examples of decimal numbers

1.25, 0.3333333..., 5.764, 6.85237237237.....

Can you find any difference in these decimals?

**Terminating decimals:**

In the examples given, the decimal numbers 1.25 and 5.764 have a finite number of digits after the decimal point. Such decimals are called terminating decimals.

**Non-terminating decimals:**

In the decimal numbers 0.3333333 ... and 6.85237237237...we can find infinitely many decimals. Such decimals are called non-terminating decimals.

**Recurring or repeating decimals:**

A non-terminating decimal representation of a number whose digits repeat at regular intervals is called recurring or repeating decimal.

In 0.3333333..., the digit 3 is repeated indefinitely.

In the non-terminating decimal 6.85237237237..., from the third decimal place, the sequence 2,3,7 of digits are repeated indefinitely.

Thus 0.3333333... and 6.85237237237... are examples of non-terminating recurring decimals.

**Decimal representation of rational numbers**

As rational numbers are expressed in the form  $\frac{p}{q}$ ,  $p, q$  integers,  $q \neq 0$ , they can also be represented in decimal form. We use the long division method to represent  $\frac{p}{q}$  in decimal form.

Let us find the decimal expansion of the rational numbers  $\frac{7}{40}$ ,  $\frac{29}{25}$ ,  $\frac{13}{7}$  and  $\frac{17}{90}$  using the long division method.

|                                                                                                                                                                 |                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                              |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 0.175 \\ 40 \overline{) 7} \\ \underline{70} \\ 40 \\ \underline{400} \\ 280 \\ \underline{200} \\ 200 \\ \underline{200} \\ 000 \end{array}$ | $\begin{array}{r} 1.16 \\ 25 \overline{) 29} \\ \underline{25} \\ 40 \\ \underline{25} \\ 150 \\ \underline{150} \\ 000 \end{array}$ | $\begin{array}{r} 1.8571428571 \dots \\ 7 \overline{) 13} \\ \underline{7} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \dots \end{array}$ | $\begin{array}{r} 0.18888 \dots \\ 90 \overline{) 17} \\ \underline{170} \\ 90 \\ \underline{800} \\ 720 \\ \underline{800} \\ 720 \\ \underline{800} \\ 720 \\ \underline{800} \\ 720 \\ \dots \end{array}$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

In these long divisions, we have two cases.

**Case 1) The remainder becomes 0.**

In  $\frac{7}{40}$  and  $\frac{29}{25}$ , the remainder becomes 0 after some steps. Therefore the decimal representation of the fraction terminates.

Thus we get,  $\frac{7}{40} = 0.175$  and  $\frac{29}{25} = 1.16$

**Case 2) The remainder never becomes 0.**

In the long division of  $\frac{13}{7}$ , the divisor is 7 and the expected remainders are 1, 2, 3, 4, 5, 6, but the remainder never becomes 0. Also the other remainders repeat in the order, 6, 4, 5, 1, 3, 2, 6, 4, 5, 1, 3, 2 ... as a result, in the quotient, we get a repeated string or block of digits 857142 after the decimal point.

Thus we have a non-terminating repeating decimal expansion of  $\frac{13}{7} = 1.857142857142857142 \dots$  and we write it as  $1.\underline{857142}$

Similarly, in the long division of  $\frac{17}{90}$ , the remainder never becomes 0. Starting from second division, we get the remainder and quotient both as 8 and the decimal expansion of  $\frac{17}{90}$  is

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$$0.188888 \dots = 0.\underline{18}$$

Thus the decimal expansion of a rational number is either terminating or non-terminating and repeating.

Is the converse of the above statement true?

This means on the number line if you come across a terminating decimal like 0.75 or a non-terminating repeating decimal  $1.666 \dots$  can you conclude that it is a rational number?

Yes, we can express terminating or non-terminating recurring decimals in the  $\frac{p}{q}$  form. In other words, such decimals are rational numbers.

### To express terminating decimals as rational numbers:

[Notes to the teacher: The teacher can explain as follows:

In such examples we can write the decimal numbers in the form  $\frac{p}{q}$ , where  $q = 10, 100, 1000$ , etc.]

E.g. Express 0.75, 19.2573 in the form  $\frac{p}{q}$ ,  $p, q$  integers,  $q \neq 0$

**Solution:**  $0.75 = \frac{75}{100} = \frac{3}{4}$  after simplification

$$19.2573 = \frac{192573}{10000}$$

### To express non-terminating recurring decimals as rational numbers:

[Notes to the teacher: The teacher can explain as follows:

In such examples,

i) Assume that the given decimal number is  $x$

ii) Find the number of digits in the repeating sequence of digits in  $x$ .

If there is one digit in this string, multiply  $x$  by 10, if there are two digits multiply  $x$  by 100 and so on.]

E.g. Express a)  $1.6666\dots$  b)  $29.\underline{90}$  c)  $32.43565656\dots$  in the form  $\frac{p}{q}$ ,  $p, q$  integers,  $q \neq 0$

**Solution:**

a) Let  $x = 1.6666\dots$  (i)

The repeating sequence contains only one digit 6.

Multiplying both sides of equation (i) by 10,

$$10x = 16.6666\dots \text{ (ii)}$$

Subtracting equation (i) from equation(ii)

$$10x - x = 16.6666\dots - 1.6666\dots$$

$$9x = 15$$

$$x = \frac{15}{9} = \frac{5}{3}$$

$$\text{Hence, } 1.\underline{6} = \frac{5}{3}$$

b) Let  $x = 29.\underline{90} = 29.909090.....$  (i)

The repeating sequence contains two digits 9 and 0.

Therefore multiply both sides of equation (i) by 100.

$$100x = 2990.9090..... \text{ (ii)}$$

Subtracting equation (i) from equation (ii)

$$100x - x = 2990.9090..... - 29.9090.....$$

$$99x = 2961$$

$$x = \frac{2961}{99} = \frac{329}{11}$$

c) Let  $x = 32.43565656.....$  (i)

The digits 4,3 after the decimal point, do not repeat. The repeating sequence '5,6' contains 2 digits.

So, multiplying equation (i) by 100,

$$100x = 3243.565656..... \text{ (ii)}$$

Subtracting equation (i) from (ii) we get,

$$100x - x = 3243.565656..... - 32.43565656..... = 3211.13 = \frac{321113}{100}$$

$$99x = \frac{321113}{100}$$

$$x = \frac{321113}{100 \times 99} = \frac{321113}{9900}$$

Thus if the decimal expansion of a number is either terminating or non-terminating and repeating, then the number is a rational number.

**[Notes to the teacher:** The teacher may pose the following questions to the students and allow them to think.]

Given a fraction  $\frac{p}{q}$ ,  $p, q$  integers,  $q \neq 0$ . Without using long division, can you find out what type of decimal representation it has? Terminating or non-terminating and repeating?

Expected Answer:

- If the denominator has either 2 or 5 (or both) as prime factors then the number has terminating decimal representation, i.e. all fractions of the type  $\frac{p}{2^m 5^n}$  have terminating decimal expansion.
- If the denominator has prime factors other than 2 and 5, then the number has a non-terminating, repeating decimal expansion.

E.g. Consider  $\frac{11}{20}$

The denominator is 20.

$$20 = 2 \times 2 \times 5$$

Thus, the prime factors of 20 are 2 and 5.

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Hence,  $\frac{11}{20}$  has terminating decimal representation.

Consider  $\frac{29}{25}$

The denominator 25 contains only 5 as a prime factor.

The decimal representation of the number is terminating.

Consider the number  $\frac{47}{60}$

The denominator  $60 = 2 \times 2 \times 3 \times 5$  contains a prime factor 3, other than 2 and 5.

Hence,  $\frac{47}{60}$  has non-terminating repeating decimal representation.

| Time to teach | Asset type  | Theme        | Sub Theme                                       |
|---------------|-------------|--------------|-------------------------------------------------|
| 20 minutes    | Main Script | Real Numbers | Terminating & Non-terminating/Recurring numbers |

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### MS\_Irrationals Decimal form

**Objective:** Students will be able to infer that the decimal expansion of an irrational number is non-terminating/non-recurring.

**Teaching Aids:** PPT

**[Notes to the teacher:** The teacher can commence the class by explaining the decimal representation of a rational number.]

---

#### Decimal representation of Irrational Numbers:

**[Notes to the teacher:** Teacher may pose the following question to the students, allow them to think and come out with as many answers as possible before arriving at a conclusion.

What is the pattern of an irrational number in decimal form?

If there is any difficulty in getting the answers, the teacher can give the hint that decimal representations of rational numbers are

1) Either terminating OR 2) Non-terminating and recurring.

The teacher can tell them that decimal representation of irrational numbers is the opposite of the above two patterns.]

#### Expected Answer:

The decimal representation of an irrational number is non-terminating and non-recurring. Also, the converse of the above result is true. A non-terminating and non-recurring decimal number cannot be expressed in the form of  $\frac{p}{q}$ , where  $p, q$  are integers, and  $q \neq 0$ . In other words, it is an irrational number.

**[Notes to the teacher:** The teacher can give the following examples of non-terminating and non-recurring decimal numbers]

The numbers with decimal representation

1.2401001000100001..... and 0.23223222322223.....are irrationals.

- The square root of every non-perfect square natural number is irrational. Examples :  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ ,  $\sqrt{7}$ , etc. are irrational numbers and their decimal representations are non-terminating and non-recurring.
- The decimal representation of  $\pi$  is 3.141592... which is non-terminating and non-recurring. Thus,  $\pi$  is irrational, but in calculations we take the value of  $\pi$  as  $\frac{22}{7}$  which is a rational number.

1) Find the decimal representation of  $\sqrt{5}$  upto three digits after the decimal point.

|   |            |
|---|------------|
|   | 2.236      |
| 2 | 5.00 00 00 |

---

|      |       |
|------|-------|
| +2   | 4     |
| 42   | 100   |
| +2   | 84    |
| 443  | 1600  |
| +3   | 1329  |
| 4466 | 27100 |
| +6   | 26796 |
| 4472 | 304   |
|      | .     |
|      | .     |
|      | .     |

The decimal representation of  $\sqrt{5}$  upto three digits after the decimal point is 2.236.

2) Find the decimal representation of  $\sqrt{7}$  upto four digits after the decimal point.

|       |            |
|-------|------------|
|       | 2.6457     |
| 2     | 7.00 00 00 |
| +2    | 4          |
| 46    | 300        |
| +6    | 276        |
| 524   | 2400       |
| +4    | 2096       |
| 5285  | 30400      |
| +5    | 26425      |
| 52907 | 397500     |
| 7     | 370349     |
|       | .          |
|       | .          |
|       | .          |

The decimal representation of  $\sqrt{7}$  upto four digits after the decimal point is 2.6457.

**Finding irrational numbers between the two given numbers.**

---

Given two numbers, between them we can find infinitely many numbers with non-terminating and non-recurring decimal expansion. Thus, there are infinitely many irrational numbers between two numbers.

E.g. 1) Find four irrational numbers between 3 and 4.

We have  $3^2 = 9$  and  $4^2 = 16$

The natural numbers 10, 11, 12, 13, 14 and 15 between 9 and 16 are not perfect squares, hence, the square roots of these numbers are irrational numbers between 3 and 4.

Thus 4 irrational numbers between 3 and 4 are  $\sqrt{10}$ ,  $\sqrt{11}$ ,  $\sqrt{12}$  and  $\sqrt{13}$

E.g. 2) Find two irrational numbers between  $\sqrt{3}$  and  $\sqrt{7}$ .

The average between 3 and 7 is  $\frac{3+7}{2} = 5$ , which is not a perfect square.

Now  $\sqrt{5}$  is an irrational number between  $\sqrt{3}$  and  $\sqrt{7}$ .

We have  $\sqrt{3} < \sqrt{5} < \sqrt{7}$

$\frac{\sqrt{3}+\sqrt{5}}{2}$  is another irrational number between  $\sqrt{3}$  and  $\sqrt{5}$

Thus,  $\sqrt{3} < \frac{\sqrt{3}+\sqrt{5}}{2} < \sqrt{5} < \sqrt{7}$

The two irrational numbers between  $\sqrt{3}$  and  $\sqrt{7}$  are  $\frac{\sqrt{3}+\sqrt{5}}{2}$  and  $\sqrt{5}$ .

E.g. 3) Find two irrational numbers between 0.3 and 0.4

**[Notes to the teacher:** The teacher can explain the method of finding terminating decimals between two decimal numbers.

Between 0.5 and 0.6, we have terminating decimals, 0.51, 0.52, 0.53, 0.54...0.59

E.g. between 0.06 and 0.07, we have terminating decimals, 0.061, 0.062, 0.063, 0.064...0.069.]

To find two irrational numbers between 0.3 and 0.4,

0.31 and 0.32 are two terminating decimals (rational numbers) between 0.3 and 0.4.

Thus,  $0.3 < 0.31 < 0.32 < 0.4$

The numbers  $x = 0.31010010001.....$  and  $y = 0.31020020002....$  are two irrational numbers between 0.31 and 0.32

Also, these irrational numbers  $x$ ,  $y$  lie between 0.3 and 0.4

E.g.4) Find two irrational numbers between  $\frac{1}{11}$  and  $\frac{1}{9}$

Using long division method, find the non-terminating, repeating decimal expansion of  $\frac{1}{11}$  and  $\frac{1}{9}$

$\frac{1}{11} = 0.090909....$  and  $\frac{1}{9} = 0.1111....$

0.09010010001... and 0.09020020002.... are two irrational numbers between  $\frac{1}{11}$  and  $\frac{1}{9}$ .

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| Time to teach | Asset type  | Theme        | Sub Theme                                       |
|---------------|-------------|--------------|-------------------------------------------------|
| 15 minutes    | Main Script | Real Numbers | Terminating & Non-terminating/Recurring numbers |

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classroom. The teacher may work out the first sum in the class if the students need guidance.]

**(III) Insert a rational and an irrational number between the following:-**

1. 2.537 and 3.211
2.  $\sqrt{3}$  and  $\sqrt{5}$
3. 0.15 and .16

**Solutions:**

1. 2.547, 2.60660666066660.....
2. 2.112123123412345.....
3. 0.156, 0.1501001000100001.....

| Time to teach | Asset type          | Theme        | Sub Theme                         |
|---------------|---------------------|--------------|-----------------------------------|
| 10 minutes    | Questions to Assess | Real Numbers | Decimal expansion of real numbers |

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## DD\_Real numbers in daily life

**Objective:** Students will be able to relate to how irrational numbers can be used in our day-to-day life applications.

**Teaching Aids:** PPT

**Notes to the teacher:** There are two examples provided to allow students to understand how irrational numbers are helpful in our day-to-day activities. The first one describes usage of  $\pi$ , while the second example is use of irrational numbers in encryption.

---

### Usage of Pi ( $\pi$ ) in daily Life

We already know how pi  $\pi$  is useful in calculations involving circles - like determining the diameter and circumference. Now let us see how we can apply this concept in a real life application. Assume that a farmer would like to estimate the cost to build a wall around a well.

- Cost of building the wall = Rs.y per sq. ft.
- The wall needs to be 2 ft. tall. Now, he would need to determine the length of the wall, to estimate the cost.
- The diameter of the well (d ft.) can be determined by measuring the centre post holding the pulley (as in the picture below).



commons.wikimedia.org/wiki/File:Po%C3%A7o.jpg Attribution: User:Juntas

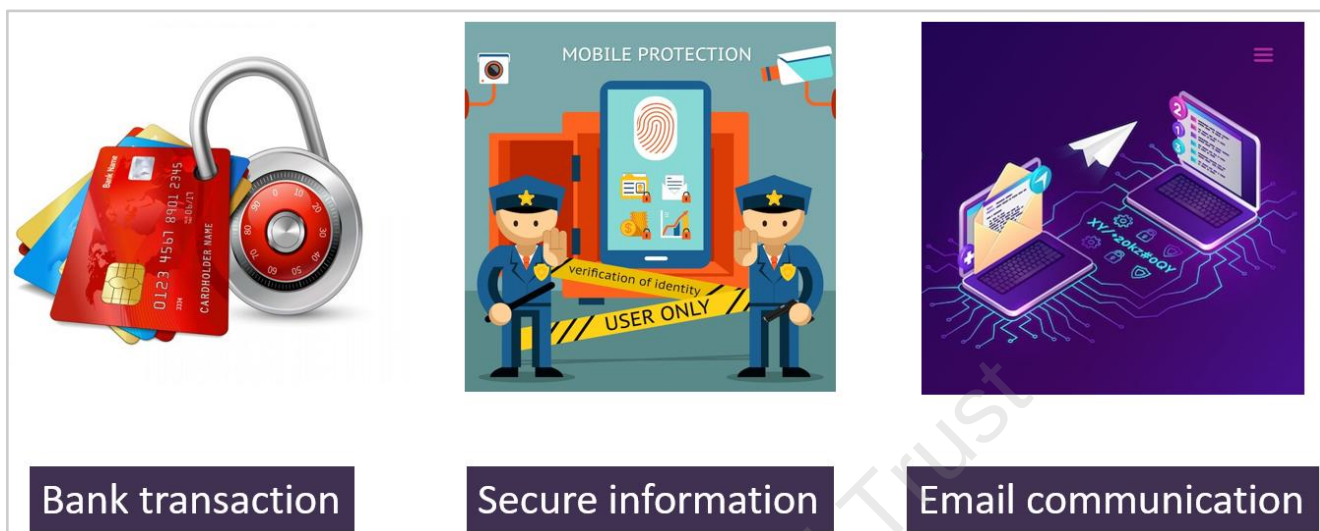
- The circumference or the length of the wall =  $\pi d = 3.14d$  ft.
- Area of the wall = 2 ft. x  $3.14 \times d$  ft. =  $6.28d$  sq. ft.
- Cost of building the wall = Rs. ( $y \times 6.28d$ )

### Usage of Irrational Numbers in daily life

Irrational numbers are used extensively in encrypting data.

Encryption is a method to convert information into a secret code, to hide the true meaning.

Encryption is used extensively in our day-to-day activities like securing email communication, bank transactions, etc.



**Credit cards:** [freepik.com/free-vector/set-secure-credit-cards-with-chips\\_4560852.htm](https://www.freepik.com/free-vector/set-secure-credit-cards-with-chips_4560852.htm)

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Encryption key is an essential part to encrypt and decrypt the information. Encryption keys are randomly generated strings of bits, using complex algorithms. Producing a high security encryption algorithm, largely depends on the irregularity of the numbers generated. Making use of the irregularity of irrational numbers, instead of prime numbers, as the key, produces more rapid encryption.

| Time to teach | Asset type           | Theme        | Sub Theme |
|---------------|----------------------|--------------|-----------|
| 5 minutes     | Day-to-day Relevance | Real Numbers | Surds     |

## MS\_Operations on Real numbers

---

**Objective:** Students will be able to perform operations like addition, subtraction, multiplication and division on surds, simplify expressions and rationalise the denominators.

**Teaching Aids:** PPT

**Notes to the teacher:** The teacher commences the class by defining similar or like surds with examples and the operations of addition and subtraction of irrational numbers as follows.

---

## How to perform operations on irrational numbers

### 1. Addition of irrational numbers

E.g: Add  $(7\sqrt{6} + 3\sqrt{5})$  and  $(2\sqrt{6} - \sqrt{5})$

Just like in algebraic expressions, here too, we can only add similar surds. We can add similar surds by adding their coefficients.

In the sense, while adding  $(7x+3y)$  and  $(2x-y)$ , we would add the coefficients of  $x$  and  $y$  separately, to result in

$$\begin{aligned}(7x+3y) + (2x-y) \\ &= (7+2)x + (3+(-1))y \\ &= 9x + 2y.\end{aligned}$$

Similarly, adding  $(7\sqrt{6} + 3\sqrt{5})$  and  $(2\sqrt{6} - \sqrt{5})$  would be:

$$\begin{aligned}(7\sqrt{6} + 3\sqrt{5}) + (2\sqrt{6} - \sqrt{5}) \\ &= (7 + 2) \sqrt{6} + (3+(-1))\sqrt{5} \\ &= 9 \sqrt{6} + 2 \sqrt{5}\end{aligned}$$

**Point to remember:** The first step, always, is to analyse the given number and identify if it is in simplified form. Else, we will have to first simplify it as shown below.

E.g.  $(7\sqrt{12} + 3\sqrt{45})$  and  $(2\sqrt{3} - \sqrt{20})$  would first have to be simplified as

$$\begin{aligned}&= (7\sqrt{4 \times 3} + 3\sqrt{9 \times 5}) + (2\sqrt{3} - \sqrt{4 \times 5}) \\ &= ((7 \times 2\sqrt{3}) + (3 \times 3 \sqrt{5})) + (2\sqrt{3} + (-1 \times 2) \sqrt{5}) \\ &\text{(And now, add the like surds as shown below)} \\ &= (14 \sqrt{3} + 9 \sqrt{5}) + (2\sqrt{3} - 2 \sqrt{5}) \\ &= ((14 + 2) \sqrt{3} + (9 - 2) \sqrt{5}) \\ &= (16 \sqrt{3} + 7 \sqrt{5})\end{aligned}$$

The above described logic applies to subtraction as well, as shown below.

### 2. Subtraction of irrational numbers

$(8\sqrt{6} + 3\sqrt{175}) - (2\sqrt{24} - 4\sqrt{7})$  would first have to be simplified as

$$\begin{aligned}&= (8\sqrt{6} + 3\sqrt{25 \times 7}) - (2\sqrt{4 \times 6} - 4\sqrt{7}) \\ &= (8 \sqrt{6} + (3 \times 5 \sqrt{7}) - ((2 \times 2) \sqrt{6} - 4 \sqrt{7}))\end{aligned}$$

---

(And now, subtract the like terms as shown below)

$$\begin{aligned} &= (8\sqrt{6} + 15\sqrt{7}) - (4\sqrt{6} - 4\sqrt{7}) \\ &= ((8 - 4)\sqrt{6} + (15 - (-4))\sqrt{7}) \\ &= (4\sqrt{6} + 19\sqrt{7}) \end{aligned}$$

### 3. Multiplication of irrational numbers

There are 2 important properties to remember with respect to multiplication of surds.

- $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$

When  $n = 2$ ,  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

- $a\sqrt[n]{b} \times c\sqrt[n]{d} = ac\sqrt[n]{bd}$

E.g. multiply  $8\sqrt{6}$  and  $9\sqrt{2}$

$$\begin{aligned} &8\sqrt{6} \times 9\sqrt{2} \\ &= 8\sqrt{6} \times 9\sqrt{2} \\ &= 8 \times 9 \sqrt{6 \times 2} \\ &= 8 \times 9 \sqrt{3 \times 2 \times 2} \\ &= 8 \times 9 \times 2\sqrt{3} = 144\sqrt{3} \end{aligned}$$

Similarly, multiply  $\sqrt{5}$  and  $\sqrt{7} = \sqrt{5 \times 7} = \sqrt{35}$

### 4. Division of irrational numbers

There are 2 important properties to remember with respect to division of surds.

- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

When  $n = 2$ ,  $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

- $\frac{a\sqrt[n]{b}}{c\sqrt[n]{d}} = \frac{a}{c} \sqrt[n]{\frac{b}{d}}$

E.g. divide  $20\sqrt{35}$  by  $\sqrt{125}$

$$\begin{aligned} 20\sqrt{35} \div \sqrt{125} &= \frac{20\sqrt{35}}{\sqrt{5 \times 5 \times 5}} = \frac{20\sqrt{7 \times 5}}{5\sqrt{5}} \\ &= \frac{20\sqrt{7}\sqrt{5}}{5\sqrt{5}} = \frac{20}{5} \times \frac{\sqrt{7} \times \sqrt{5}}{\sqrt{5}} = 4\sqrt{7} \end{aligned}$$

---

Let us consider a few facts here and verify those statements with an example.

**Fact #1: The sum or difference of a rational number and an irrational number is irrational.**

Consider calculating the sum of a rational number 7 and irrational number  $\sqrt{5}$

$\sqrt{5}$  is an irrational number with a non-terminating, non-recurring decimal expansion, whose value is 2.236067977499789.....

When you add a rational number like 7 to this irrational number, it yields a non-terminating and non-recurring decimal as follows :

$$7 + \sqrt{5} = 7 + 2.236067977499789... = 9.236067977499789... \text{ goes on.}$$

Likewise,  $7 - \sqrt{5}$  has non-terminating, non-recurring decimal expansion and is irrational.

From this example, it's clear that the sum or difference of a rational number and an irrational number has non-terminating and non-recurring decimal expansion, and hence irrational number.

**Fact #2: The product or quotient of a non-zero rational number with an irrational number is irrational.**

Consider a non-zero rational number 5 and an irrational number  $2\sqrt{5}$ .

Their product  $\Rightarrow 5 \times 2\sqrt{5} = 10\sqrt{5}$ , which is irrational.

Similarly consider a non-zero rational number 14 and an irrational number  $\sqrt{7}$

Their quotient  $\frac{14}{\sqrt{7}} = \frac{2 \times 7}{\sqrt{7}} = 2 \times \sqrt{7}$ , which is irrational too.

**Fact #3: If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.**

- $\sqrt{5} + (-\sqrt{5}) = 0$ , which is rational
- $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$  which is irrational

Same can be applied to subtraction, multiplication and division and arrive at results as shown below:

- $\sqrt{5} - \sqrt{5} = 0$ , which is rational
- $\sqrt{5} - 2\sqrt{5} = -\sqrt{5}$  which is irrational
- $\sqrt{5} \times \sqrt{5} = 5$ , which is rational
- $\sqrt{5} \times \sqrt{7} = \sqrt{35}$  which is irrational

- $\frac{\sqrt{5}}{\sqrt{5}} = 1$ , which is rational
- $\frac{\sqrt{35}}{\sqrt{5}} = \frac{\sqrt{7 \times 5}}{\sqrt{5}} = \frac{\sqrt{7} \times \sqrt{5}}{\sqrt{5}} = \sqrt{7}$  which is irrational

| Properties                                                                                                                                                         | For Rational Numbers                                                                                                                                                                                                                                                                                             | For Irrational Numbers                                                                                                                                                                                                                                                                      |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Commutative law for addition and multiplication<br>$a + b = b + a$ and $a \times b = b \times a$ for any $a, b$ in the set                                         | Holds good                                                                                                                                                                                                                                                                                                       | Holds good                                                                                                                                                                                                                                                                                  |
| Associative law for addition and multiplication<br>$a + (b + c) = (a + b) + c$ and<br>$a \times (b \times c) = (a \times b) \times c$ for any $a, b, c$ in the set | Holds good                                                                                                                                                                                                                                                                                                       | Holds good                                                                                                                                                                                                                                                                                  |
| Distributive law for addition and multiplication<br>$a \times (b + c) = a \times b + a \times c$ for any $a, b, c$ in the set                                      | Holds good                                                                                                                                                                                                                                                                                                       | Holds good                                                                                                                                                                                                                                                                                  |
| With respect to the operations addition, subtraction, multiplication, division                                                                                     | Rational numbers are 'closed', i.e.<br>Rational + Rational = Rational<br>Rational - Rational = Rational<br>Rational $\times$ Rational = Rational<br>Rational / Rational (Not 0) = Rational<br><br>If we add, subtract, multiply or divide (except by zero) two rational numbers, we still get a rational number. | However, operation between 2 irrational numbers need not yield an irrational number, e.g. the following operations do not yield an irrational number:<br>$\sqrt{5} + (-\sqrt{5}) = 0$<br>$\sqrt{27} - \sqrt{27} = 0$<br>$\sqrt{2} \times \sqrt{2} = 2$<br>$\frac{\sqrt{41}}{\sqrt{41}} = 1$ |

Let us try to review and understand what we already know about rational numbers. And use those facts to relate to how irrational numbers behave in those conditions:

### Simplify the expressions

Before we get into understanding how to simplify the expression, let us understand a few concepts and properties.

- $(\sqrt{a} + \sqrt{b})(\sqrt{c} + \sqrt{d}) = \sqrt{ac} + \sqrt{ad} + \sqrt{bc} + \sqrt{bd}$
- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

Reason:  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

[Based on  $(x + y)(x - y) = x^2 - y^2$ , where x and y are any real numbers]

Similarly  $(a + \sqrt{b})(a - \sqrt{b}) = a^2 - b$

- $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{ab} + b$

Reason:  $(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2(\sqrt{a}\sqrt{b}) + (\sqrt{b})^2 = a + 2\sqrt{ab} + b$

[Based on  $(x + y)^2 = x^2 + 2xy + y^2$ , where x and y are any real numbers]

### Rationalise the denominator

**[Notes to the teacher:** To introduce the concept the teacher can start the class with an example as follows.

Trying to estimate the position of say,  $\sqrt{\frac{2}{3}}$  on a number line would be easier, if we could remove the irrational number in the denominator or as we call it, rationalise the denominator.

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \text{ — [Because } \frac{\sqrt{3}}{\sqrt{3}} = 1 \text{ ]}$$

$$= \sqrt{\frac{2 \times 3}{3 \times 3}}$$

$$= \frac{\sqrt{6}}{3}$$

We know that  $\sqrt{2}$  is approximately 1.414 and  $\sqrt{3}$  is approximately 1.732

So, we can estimate that  $\frac{\sqrt{6}}{3} = \frac{\sqrt{2} \times \sqrt{3}}{3} = \frac{(1.414 \times 1.732)}{3} = \frac{2.449}{3}$  is definitely less than 1 and greater than 0.5, in the number line.]

So, when the denominator of an expression contains a term with a square root (or a number under a radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called **rationalising the denominator**.

E.g.

Rationalise the denominator of  $\frac{1}{\sqrt{7} - \sqrt{6}}$

$$\frac{1}{\sqrt{7} - \sqrt{6}} = \frac{1}{(\sqrt{7} - \sqrt{6})} \times \frac{(\sqrt{7} + \sqrt{6})}{(\sqrt{7} + \sqrt{6})} = \frac{(\sqrt{7} + \sqrt{6})}{(\sqrt{7})^2 - (\sqrt{6})^2} = \frac{(\sqrt{7} + \sqrt{6})}{7 - 6} = (\sqrt{7} + \sqrt{6})$$

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| Time to teach | Asset type  | Theme        | Sub Theme                 |
|---------------|-------------|--------------|---------------------------|
| 20 minutes    | Main Script | Real Numbers | Operation on real numbers |

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## MS\_Laws of Exponents

**Objective:** Students will be able to apply the Laws of Exponents.

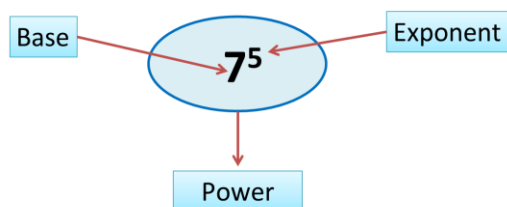
**Teaching Aids:** PPT

**Notes to the teacher:** The teacher commences the class by revising the concept of exponent and the laws of exponents for natural numbers.

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### Recap: What is an exponent?

Exponent is a type of notation in mathematics that tells us about specific ways of using the numbers in calculations. Numbers can be expressed as  $a^m$  where,  $a$  = base and  $m$  = exponent. The exponent tells us the number of times the base is to be multiplied by itself. 'Power' represents a number which is raised to another number (called exponent).



### Laws of Exponents for natural numbers

If  $a$ ,  $m$  and  $n$  are natural numbers, where  $a$  is the base, and  $m$  and  $n$  are exponents, then the laws of exponents you have learnt are:

1. **Product rule – Same base:**  $a^m \times a^n = a^{(m+n)}$

2. **Product rule – Same exponent:**  $a^m \times b^m = (ab)^m$

3. **Quotient Rule – Same base:**  $\frac{a^m}{a^n} = a^{(m-n)}$

4. **Quotient rule – Same exponent:**  $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

5. **Power of a power rule:**  $(a^m)^n = a^m \times a^n$

6. **Zero exponent rule:**  $a^0 = 1$

The teacher then proceeds to explain how these laws of exponents can be extended to understand the negative exponent rule and the fractional exponent rule.

### Rule 7: Negative exponent rule

For any base with a negative exponent, the value is equal to the reciprocal of the base raised to the positive exponent.

$$a^{-m} = \frac{1}{a^m}$$

**Proof:**

Negative exponent rule can be understood by combining the Quotient rule - Same base and the Zero exponent rule

Quotient rule – Same base:  $\frac{a^m}{a^n} = a^{(m-n)}$

Zero exponent rule:  $a^0 = 1$

For example,  $a^{(-5)}$  can be written as,  $a^{(0-5)} = \frac{a^0}{a^5}$  [Quotient rule]

Further,  $\frac{a^0}{a^5} = \frac{1}{a^5}$  [because  $a^0 = 1$ ]

Hence,  $a^{-5} = \frac{1}{a^5}$

By observing the pattern also we can prove the negative exponent rule.

Observe table 1 for values of  $5^4$ ,  $5^3$  and  $5^2$ . Reducing the exponent by 1 is the same as dividing the expression by 5. To get the pattern, at every step divide the previous answer with the base 5 starting from S.No.2

**Table 1: Power Pattern Table**

| S.No | Expression | Pattern    | Expansion                                       | Answer          |
|------|------------|------------|-------------------------------------------------|-----------------|
| 1    | $5^4$      |            | $5 \times 5 \times 5 \times 5$                  | 625             |
| 2    | $5^3$      | $5^{4-1}$  | $5 \times 5 \times 5$ [625 / 5]                 | 125             |
| 3    | $5^2$      | $5^{3-1}$  | $5 \times 5$ [125 / 5]                          | 25              |
| 4    | $5^1$      | $5^{2-1}$  | 5 [25 / 5]                                      | 5               |
| 5    | $5^0$      | $5^{1-1}$  | 5 [5/5]                                         | 1               |
| 6    | $5^{-1}$   | $5^{0-1}$  | $\frac{5^0}{5^1}$ $\left[\frac{1}{5}\right]$    | $\frac{1}{5}$   |
| 7    | $5^{-2}$   | $5^{-1-1}$ | $\frac{5^0}{5^2}$ $\left[\frac{1/5}{5}\right]$  | $\frac{1}{25}$  |
| 8    | $5^{-3}$   | $5^{-2-1}$ | $\frac{5^0}{5^3}$ $\left[\frac{1/25}{5}\right]$ | $\frac{1}{125}$ |

**Examples:**

1.  $5^{-2} = ?$        $5^{-2} = \frac{1}{5^2}$

2.  $8^{-6} = ?$        $8^{-6} = \frac{1}{8^6}$

3.  $3^{-3} = ?$        $3^{-3} = \frac{1}{3^3}$

4.  $9^{-5} = ?$        $9^{-5} = \frac{1}{9^5}$

**Rule 8: Fractional Exponent Rule**

Recollect the definition of square root and cube root.

$y$  is the square root of  $x$ , if  $y^2 = x$ , that is  $\sqrt{x} = \sqrt{y^2} = y$ .

Using the exponent  $\frac{1}{2}$  for  $x$ , we see that  $x^{1/2} = (y^2)^{1/2} = (y)^{2 \times (1/2)} = y = \sqrt{x}$ .

Thus,  $\sqrt{x} = x^{1/2}$ .

Similarly for cube root  $\sqrt[3]{x} = x^{1/3}$ .

Hence,  $n$ th root of a number in exponent form is  $x^{1/n}$ .

Further, we know that if  $a > 0$  be a real number and  $n$  be a positive integer, then

$\sqrt[n]{a} = b$ , if  $bn = a$ , and  $b > 0$ . In an exponential expression, it is stated as  $\sqrt[n]{a} = a^{1/n}$ .

For example,  $9^{5/2}$  can be indicated in two ways:

$$[9^{1/2}]^5 = 3^5 = 243$$

$$[9^5]^{1/2} = [59049]^{1/2} = 243$$

From this we can deduce the following definition:

Let  $a > 0$  be a real number. Let  $m$  and  $n$  be integers such that  $m$  and  $n$  have no common factors other than 1, and  $n > 0$ . Then,  $a^{(m/n)} = \sqrt[n]{a^m}$

Fractional exponential rules can be extended from this. In an exponential expression of  $am/n$ , ' $a$ ' is the base and ' $m/n$ ' is the exponent and is called a fractional exponent.

Fractional exponents represent powers and roots together.

The fractional exponent rule is  $a^{(m/n)} = \sqrt[n]{a^m}$  [ $n$ th root of  $am$ ]

Rules involving fractional exponents are used along with the basic exponent rules. They are:

- $a^{1/m} \times a^{1/n} = a^{(1/m + 1/n)}$  [Product rule – Same base]
- $a^{1/m} \times b^{1/m} = (ab)^{1/m}$  [Product rule – Same exponent]
- $\frac{a^{1/m}}{a^{1/n}} = a^{(1/m - 1/n)}$  [Quotient rule – Same base]
- $\frac{a^{1/m}}{b^{1/m}} = (a/b)^{1/m}$  [Quotient rule – Same exponent]
- $a^{-(m/n)} = 1/a^{m/n}$  [Negative exponent rule]

### Examples:

$$1. (3^{2/5}) \times (3^{3/5}) = 3^{(2/5 + 3/5)} = 3^{5/5} = 3^1 = 3$$

$$2. (3^{2/5}) \times (5^{2/5}) = (3 \times 5)^{2/5} = 15^{2/5} = \sqrt[5]{15^2} \text{ [5th root of } 15^2]$$

$$3. \frac{3^{3/5}}{3^{2/5}} = 3^{(3/5 - 2/5)} = 3^{(1/5)} = \sqrt[5]{3} \text{ [5th root of 3]}$$

$$4. \frac{3^{2/5}}{5^{2/5}} = (3/5)^{2/5} = \sqrt[5]{(3/5)^2} \text{ [5th root of } (3/5)^2]$$

$$5. 5^{-(3/5)} = 1/5^{(3/5)} = 1/\sqrt[5]{5^3} \text{ [1 / 5th root of } 5^3]$$

**[Notes to the teacher:** The teacher can summarise the laws of exponents as shown in Table 2]

**Table 2 - Summary of Laws of Exponents**

| S.No. | Rule Name | Law | Example | Base/s | Exponent/s |
|-------|-----------|-----|---------|--------|------------|
|       |           |     |         |        |            |

|   |                               |                                                |                                                |                |         |
|---|-------------------------------|------------------------------------------------|------------------------------------------------|----------------|---------|
| 1 | Product rule – Same base      | $am \times an = a^{(m+n)}$                     | $6^5 \times 6^2 = 6^{(5+2)}$                   | 6              | 5 and 2 |
| 2 | Product rule – Same exponent  | $am \times bm = (ab)m$                         | $(\sqrt{2})^{1/3} \times 5^{1/3}$              | $\sqrt{2}$ & 5 | 1/3     |
| 3 | Quotient rule – Same base     | $\frac{a^m}{a^n} = a^{(m-n)}$                  | $7^7 \div 7^4 = 7^{(7-4)} = 7^3$               | 7              | 7 & 4   |
| 4 | Quotient rule – Same exponent | $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ | $\frac{2^2}{5^2} = \left(\frac{2}{5}\right)^2$ | 2 & 5          | 2       |
| 5 | Power of a power rule         | $(am)^n = a(m \times n)$                       | $(5^3)^5 = 5^{(3 \times 5)}$                   | 5              | 3 & 5   |
| 6 | Zero exponent rule            | $a^0 = 1$                                      | $5^0 = \frac{5}{5} = 1$                        | 5              | 0       |
| 7 | Negative exponent rule        | $a^{-m} = 1/am$                                | $5^{-2} = \frac{1}{5^2}$                       | 5              | -2      |
| 8 | Fractional exponent rule      | $a^{(m/n)} = \sqrt[n]{a^m}$                    | $3^{(3/5)} = \sqrt[5]{3^3}$                    | 3              | 3/5     |
| 9 | Exponent one rule             | $a^1 = a$                                      | $20^1 = 20$                                    | 20             | 1       |

| Time to teach | Asset type  | Theme                | Sub Theme         |
|---------------|-------------|----------------------|-------------------|
| 15 minutes    | Main Script | Exponents and Powers | Laws of exponents |

### SA\_Properties of Numbers

**Objective:** Students will be able to notice the results of arithmetic operations on any two irrational numbers and on a rational number and an irrational number.

---

**Teaching Aids:** PPT and Task Sheets

**Notes to the Teacher:** The teacher may introduce the activity to the students as follows:

You are familiar with the four operations on numbers which are, addition, subtraction, multiplication and division. Now let's explore what is the outcome of these operations on any two irrational numbers and also on rational and irrational numbers.

---

**Aim:** To establish the result of mathematical operations on two irrational numbers and on rational and irrational numbers.

**Resources Required:** Pen and a notebook for calculations.

**Setting for the Activity:** Indoor (Classroom)

**Type of Activity:** To be conducted in pairs of students

**Preparation of Activity:** The teacher explains the activity to the students and distributes two task sheets to each pair, one for each student.

[**Notes to the teacher:** The teacher must ensure that there is least change in the seating arrangement in the classroom for conducting the activity, so as to save time.]

**Role of the Teacher:** Facilitator

**Procedure:** The students write the rational and irrational numbers in the space provided in their task sheet. Then they perform the required mathematical operation on these numbers and discuss the results with their partner to write their observation/conclusion on the activity. After completion of the task sheet by each pair, the teacher encourages the students to share their results with the class by asking questions related to the outcome of the operations on the numbers. The result of the activity is summarised by the teacher on the black/white board on the basis of the observations/conclusions of the students.

[**Notes to the teacher:** In the task sheet, some cells of the table have already been filled, to help the students. The teacher may draw a table on the black/white board to explain how to complete the table, if required.]

**Observation:** The mathematical operations on irrational numbers and on rational and irrational numbers give the result as a rational number or an irrational number.

| Number I | Number II | Sum | Difference |
|----------|-----------|-----|------------|
|----------|-----------|-----|------------|

|            |            |                        |                        |
|------------|------------|------------------------|------------------------|
| Rational   | Irrational | Irrational             | Irrational             |
| Irrational | Irrational | Rational or Irrational | Rational or Irrational |

| Number I          | Number II  | Product                | Quotient               |
|-------------------|------------|------------------------|------------------------|
| Non-Zero Rational | Irrational | Irrational             | Irrational             |
| Irrational        | Irrational | Rational or Irrational | Rational or Irrational |

### **Conclusion:**

1. The sum or difference of a rational number and an irrational number is irrational.
2. The product or quotient of a non-zero rational number with an irrational number is irrational.
3. If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.

**Follow-up activity:** The corrected/graded task sheets of the students can be pasted in their notebooks by the students and the result of the activity be displayed on the class bulletin board during the topic.

**Precautions:** The observations/conclusions shared by the students with the class are based on the calculations done by them. So, while sharing with the class, their inputs should be supported by examples of the numbers considered and their results of the mathematical operations, so that the teacher can rectify if there's a mistake.

### **Troubleshooting:**

- ✓ The teacher may write a few examples of rational and irrational numbers on the black/white board to assist the students to start.
- ✓ If the teacher does not want to allow discussion among the students in pairs, then they may instead be asked to exchange their task sheets after completion, for cross checking and writing the result.

| Time to teach | Asset type         | Theme        | Sub Theme                 |
|---------------|--------------------|--------------|---------------------------|
| 15 minutes    | Suggested Activity | Real Numbers | Operation on real numbers |

### **SA\_Represent sqrt x geometric**

**Objective:** Students will be able to locate the point representing  $\sqrt{x}$  for any positive real

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number  $x$  geometrically.

**Teaching Aids:** PPT

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**Aim:** To explain the geometrical method of representing  $\sqrt{x}$  for a given positive real number  $x$  (with the explanation how the method gives  $\sqrt{x}$ ). Develop a project idea for students to propose a solution for stadium seating or some other life problem using the computation required for identifying  $\sqrt{x}$

**Resources Required:** Thick coloured paper, cut into a semicircle of diameter 17 cm (for each group)  
Pen, Sketch pens, Pencil, Eraser, Compass (each student)

**Setting for the Activity:** Indoor

**Type of Activity:** Project

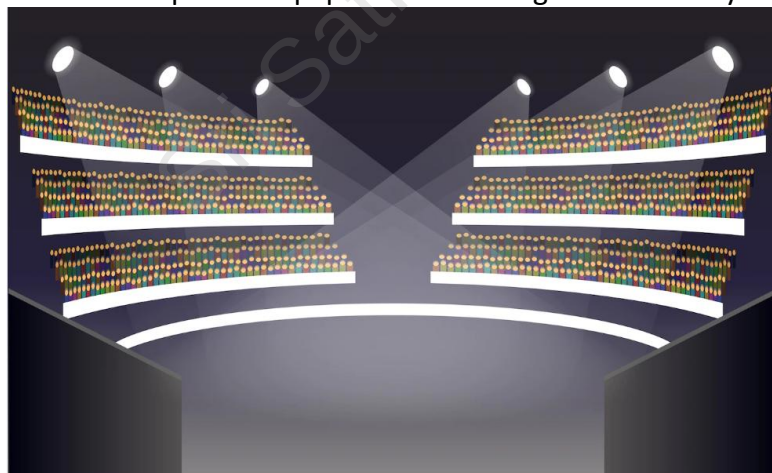
**Preparation of Activity:** Distribute a semicircular piece of paper with diameter 17 cm for each group.

**Role of the Teacher:** Observer

**Procedure:**

The class is divided into groups of three students. The groups are made to sit separately and a semicircular piece of paper of diameter 17cm is distributed to each group.

Each group creates a simple model of the amphitheatre as shown below with coloured semicircular pieces of paper in which lengths of stairways are 4 cm, 3 cm and 2 cm respectively.



Source: [freepik.com/free-vector/coliseum-arena-stage-auditorium-amphitheatre-balcony-tribune-ceremony-building-theater-illustration\\_11059122.htm](https://www.freepik.com/free-vector/coliseum-arena-stage-auditorium-amphitheatre-balcony-tribune-ceremony-building-theater-illustration_11059122.htm)  
Attribution: macrovector

The teacher gives the following instructions and sees that each student in the group takes part in the activity.

One student in each group performs the following constructions.

Step1:

- A. Mark the end points of the diameter of the semicircle as A, B and the centre of the semicircle (midpoint of AB) as O.
- B. Choose a point Y on AB, such that  $YB = 1$  cm.

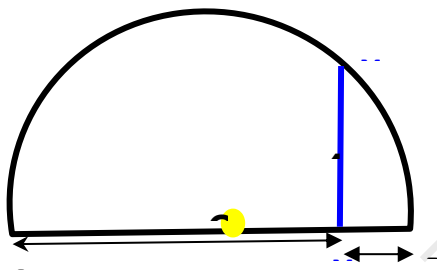


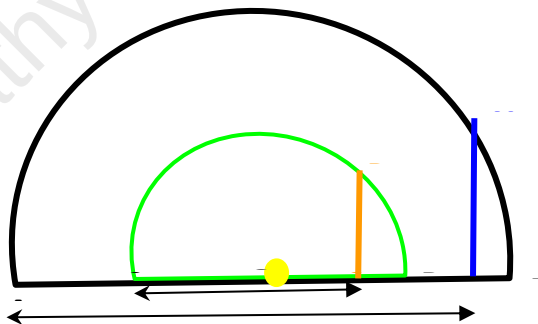
Fig (1)

- C. Draw a perpendicular to AB at Y which meets the semicircle at X. Join XY and measure the length of XY.

Step2:

The second student in the group continues the activity as per the following instructions:

- A. Draw a semicircle of radius 5 cm with O as centre which cuts diameter AB at C, D.
- B. On CD, mark a point Q such that  $QD = 1$  cm.



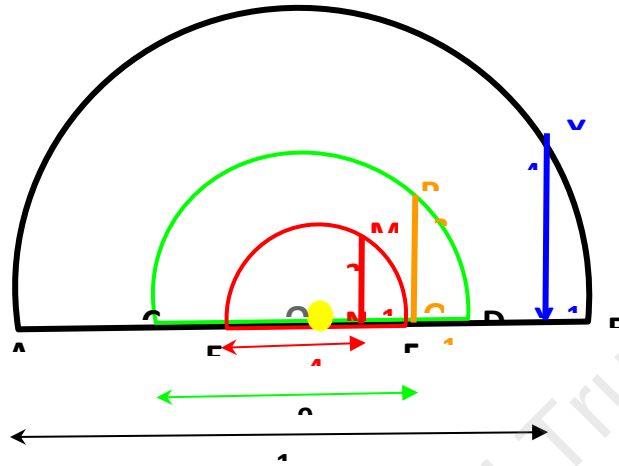
Fig(2)

- C. Draw a perpendicular to CD at Q which meets the semicircle at P. Join PQ and measure the length of PQ.

Step3:

The third student in the group completes the activity as per the following instructions:

- A. Draw a semicircle of radius 2.5 cm with O as centre which cuts diameter AB at E, F.  
 B. On EF, mark a point N such that NF = 1 cm.



Fig(3)

- C. Draw a perpendicular to EF at N, which meets the semicircle at M. Join MN and measure the length of MN.

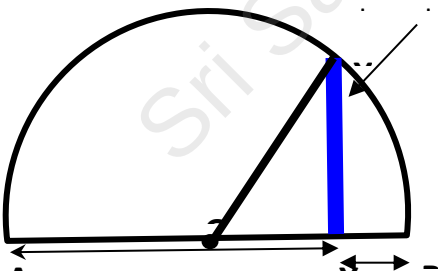
**Observation:**

Did you notice that the length of the stairway  $XY = 4 = \sqrt{16} = \sqrt{AY}$  (Fig 1)?

Similarly, the length of the stairway  $PQ = 3 = \sqrt{9} = \sqrt{CQ}$  (Fig(2)).

The length of the stairway  $MN = 2 = \sqrt{4} = \sqrt{EN}$  (Fig(3)).

**Conclusion:**



In the figure, length of the perpendicular  $XY = \sqrt{AY}$

[**Notes to the teacher:** The teacher can explain the proof of the above result as follows]

Now, let us understand XY in terms of AY  $\Rightarrow XY = \sqrt{AY}$

Let  $AY = y$

AB (diameter) =  $y + 1$

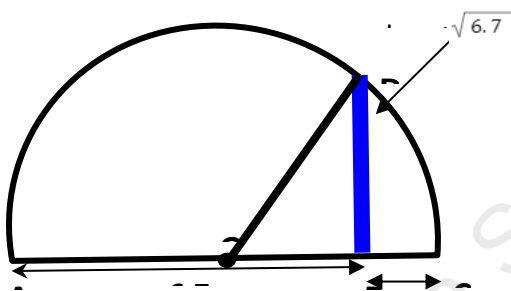
$$\begin{aligned}
 OX \text{ (radius)} &= \frac{(y+1)}{2} \text{ and,} \\
 OY \text{ (radius - 1)} &= \frac{(y+1)}{2} - 1 = \frac{(y+1) - 2}{2} = \frac{(y-1)}{2} \\
 (XY)^2 &= (OX)^2 - (OY)^2 = \left(\frac{(y+1)}{2}\right)^2 - \left(\frac{(y-1)}{2}\right)^2 = \left(\frac{(y+1)^2}{4} - \frac{(y-1)^2}{4}\right) \\
 &= \frac{(y^2 + 2y + 1) - (y^2 - 2y + 1)}{4} = \frac{y^2 + 2y + 1 - y^2 + 2y - 1}{4} = \frac{4y}{4} = y \\
 (XY)^2 &= y \text{ and thus, } XY = \sqrt{y}
 \end{aligned}$$

### **Follow-up activity:**

Using the above result, we can find  $\sqrt{x}$  for any positive real number  $x$  as follows:

E.g. to find  $\sqrt{6.7}$

- 1) Draw a line segment AB = 6.7 cm.
- 2) Extend AB to C such that BC = 1 cm.
- 3) Mark the midpoint of AC as O.
- 4) With O as centre, AO as radius, draw a semicircle.
- 5) Draw a perpendicular at B which intersects the semicircle at D.



(6) Now length of BD =  $\sqrt{6.7}$

| Time to teach | Asset type         | Theme        | Sub Theme |
|---------------|--------------------|--------------|-----------|
| 15 minutes    | Suggested Activity | Real Numbers | Surds     |

### **QA\_Apply Laws of Exponents**

**Observation:** Students will be able to solve questions on laws of exponents.

**Teaching Aids:** PPT/Worksheet

**Notes to the teacher:** The teacher may present the questions and allow the students to write or say the answers before revealing them to the students.

**I) Choose the correct answer**

- 1)  $4\sqrt{12} \times 7\sqrt{6}$  is  
a)  $24\sqrt{2}$  b)  $28\sqrt{2}$  c)  $168\sqrt{2}$  d)  $64\sqrt{2}$
- 2) If  $3^n = 27$  then  $3^{n-2}$  is  
a) 3 b) 1 c) 9 d) 27
- 3) If m, n are natural numbers such that  $m^n = 25$  then  $n^{mn}$  is  
a)  $2^5$  b)  $2^{10}$  c)  $5^2$  d)  $5^{10}$
- 4) The value of n for which  $\sqrt{n}$  is an irrational number is  
a) 3 b) 4 c) 25 d)  $\frac{16}{25}$
- 5) If a, b are natural numbers such that  $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a + b\sqrt{15}$  then the values of a, b are respectively  
a) 1, 4 b) 4, 2 c) 2, 4 d) 4, 1

**Solutions:**

- 1) c)  $168\sqrt{2}$  2) a) 3 3) b)  $2^{10}$  4) a) 3 5) d) 4, 1

**II) State True or False**

- 1) The product of a non-zero rational number and an irrational number is an irrational number.  
2) Between two rational numbers there are infinitely many integers.  
3) The sum of two irrational numbers need not be an irrational number.

**Solutions:**

- 1) True 2) False 3) True

**III) Fill in the blanks**

- 1)  $2\sqrt{3} + \sqrt{27} = \underline{\hspace{2cm}}$   
2)  $\sqrt{10} \times \sqrt{15} = \underline{\hspace{2cm}}$   
3)  $\frac{\sqrt{3}}{\sqrt{6}} = \underline{\hspace{2cm}}$   
4)  $\frac{1}{\sqrt{3}+\sqrt{2}} = \underline{\hspace{2cm}}$

**Solutions:**

- 1)  $5\sqrt{3}$  2)  $\sqrt{150}$  3)  $\frac{1}{\sqrt{2}}$  4)  $\sqrt{3} - \sqrt{2}$

**IV) Answer the following questions**

- 1) What is the rationalising factor of  $\sqrt[5]{x^2y^3z^4}$  where x, y and z are rational numbers?  
2) Find two irrational numbers a, b such that  $\sqrt{13 + 2\sqrt{42}} = a + b$ .

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**Solutions:**

1) The rationalising factor of  $\sqrt[5]{x^2y^3z^4}$  is  $\sqrt[5]{x^3y^2z}$

Explanation: The product of two factors  $\sqrt[5]{x^2y^3z^4} \times \sqrt[5]{x^3y^2z} = \sqrt[5]{x^2y^3z^4 \times x^3y^2z} = \sqrt[5]{x^5y^5z^5} = \sqrt[5]{(xyz)^5} = xyz$ , a rational number.

2) Let  $\sqrt{13 + 2\sqrt{42}} = \sqrt{x} + \sqrt{y}$  \_\_\_\_\_ (1) where  $x, y$  are rational numbers.

Squaring both sides,

$$(\sqrt{x} + \sqrt{y})^2 = 13 + 2\sqrt{42}$$

$$x + y + 2\sqrt{xy} = 13 + 2\sqrt{42}$$

Equating rational parts and surds,  $x + y = 13$ ,  $xy = 42$

The values of  $x, y$  satisfying these equations,  $x = 6$ ,  $y = 7$

Substituting  $x = 6$ ,  $y = 7$  in equation (1),

$$\sqrt{13 + 2\sqrt{42}} = \sqrt{6} + \sqrt{7}$$

Given that  $\sqrt{13 + 2\sqrt{42}} = a + b$

$$a = \sqrt{6} \text{ and } b = \sqrt{7}$$

| Time to teach | Asset type          | Theme        | Sub Theme                                                                                                                                        |
|---------------|---------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| 15 minutes    | Questions to Assess | Real Numbers | Laws of exponents for real numbers<br>Operation on real numbers<br>Properties of surds<br>Radical sign<br>Rationalising the denominator<br>Surds |

**MS\_Summary\_Number Systems**

**Objective:** Students will be able to summarise the chapter 'Number Systems'.

**Teaching Aids:** PPT

**Notes to the teacher:** The teacher can draw the mind map or write the main points of the summary on the black/white board.

The teacher can introduce the asset by saying, "Let us summarise what we have learnt from this topic".

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- The need to extend the number system beyond the rational numbers is explored.



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### **Annexure to Master Lesson Plan**

|                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| What is the purpose of this Master lesson Plan? | This Master Lesson Plan (MLP) prepared by Sri Sathya Sai Vidya Vahini (SSSVV), covers the entire chapter for the given board. The MLP for the chapter given in the textbook is broken down into smaller modules known as assets. Any chapter can be taught using the 7 asset types in SSSVV methodology. Several different assets together complete the entire chapter. Depending on the requirement, each asset is supported by teaching aids such as animated presentation, audio, video, and learning aids such as worksheets. The MLP is for the teacher's preparation and the teaching aids are for use in the classroom. |
| How to use it?                                  | You can prepare for your class with this MLP by following 3 simple steps: 1. Read your textbook; 2. Go through the information given in the assets (document & multimedia); 3. Prepare your teaching notes. Great! Now you are ready to transact the chapter.                                                                                                                                                                                                                                                                                                                                                                  |
| Duration                                        | Kindly note the duration given for each asset. Due care has been taken in planning the assets, to ensure the chapter is completed within the time specified by the Board. Note: Preparation time is not included in the asset duration. The teaching time depends on the duration of the teaching aids and is not impacted by the length of the MLP.                                                                                                                                                                                                                                                                           |
| What is a 'Main Script' (MS) Asset?             | The Main Script asset is there for explaining the main concepts of the chapter clearly. The information given in the textbook is simplified, organised and structured to give more clarity. Additionally, you may find a video or mnemonics or a graphic organiser to deepen the understanding of the concept.                                                                                                                                                                                                                                                                                                                 |
| How to teach using the MS asset?                | Please use the blackboard, slides and interaction to develop the concept.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| What is an 'Inquisitive Questions' (IQ) asset?  | The Inquisitive Questions asset uses questions to promote higher order thinking like the n <sup>th</sup> why, what-if, new perspectives, cause-effect, and others, creating curiosity.                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| How to teach using the IQ asset?                | You may use the questions to connect with the students, encourage exploration to engage them in the learning process. You may allow multiple responses and instead of rejecting any of the responses, ask why, before revealing the suggested answer or hints.                                                                                                                                                                                                                                                                                                                                                                 |
| What is a 'Day to day Relevance' (DD) Asset?    | The Day to Day Relevance asset helps students understand the practical relevance of every topic, making them eager participants in the classroom. If students connect the concepts to their environment, they would learn meaningfully without dislike or simply memorizing for exams.                                                                                                                                                                                                                                                                                                                                         |
| How to teach using the DD                       | You may use the asset by asking questions about their experience and use it to establish why they are learning the topic. If necessary you may substitute with a                                                                                                                                                                                                                                                                                                                                                                                                                                                               |

|                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|---------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| asset?                                      | recent or local example.                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| What is an 'Interesting Aside' (IA) asset?  | The Interesting Aside asset uses attention grabbing titbits or anecdotes to bring joy and satisfy the child's hunger to know more. This is a quick/short asset intended to bring attention back to the main concept being taught.                                                                                                                                                                                                                                             |
| How to teach using the IA asset?            | You may use this asset to give a piece of interesting information relevant to the topic, without prolonged explanation/discussion. You may suggest appropriate books for further reading.                                                                                                                                                                                                                                                                                     |
| What is a 'Suggested Activity' (SA) Asset?  | The Suggested Activity asset provides a detailed step by step procedure for the teacher to conduct a hands-on activity. This promotes Activity Based Learning.                                                                                                                                                                                                                                                                                                                |
| How to teach using the SA asset?            | You may use this asset to energise your students to learn by doing simple, fun-filled activities based on the topic. The asset includes - preparation, activity, and follow-up. You may engage the students in the preparation and follow-up stages to develop collaboration and responsibility. You could try it once, yourself, before the actual delivery to be more comfortable with it. Note: The duration mentioned in the asset is usually for the activity part only. |
| What is a 'Value Content' (VC) Asset?       | The Value Content asset integrates Universal Human Values (such as empathy, inclusivity, respect for nature that translates into responsible behaviour) seamlessly into the curriculum, as the "End of education is character".                                                                                                                                                                                                                                               |
| How to teach using the VC asset?            | You may use this asset to engage with the students to bring out their innate values and connect with the intrinsic values in the topic/concept. The asset includes pointers on the specific pedagogical technique followed.                                                                                                                                                                                                                                                   |
| What is a 'Questions to Assess' (QA) asset? | The Questions to Assess asset uses an interactive approach to check learning of different types of learners, and provides feedback to the teacher for appropriate action. The questions cater to all the levels of Bloom's Taxonomy. Questions from 'Apply' level and above enable students to reflect on their learning.                                                                                                                                                     |
| How to teach using the QA asset?            | You may go through the slides to understand its flow and know when to click for the answer (slides include suggested answers). While presenting in the class, you may display the question and encourage multiple responses, before revealing the answer. In some cases, this asset may include extra questions/worksheet that can be given as homework.                                                                                                                      |