

Master Lesson Plan
for
Polynomials

Board	Standard	Subject	Chapter	Language
NCERT	STD IX	Mathematics	Polynomials	English

CURRICULAR GOALS	CG-3: Discovers and proves algebraic identities and models real-life situations in the form of equations to solve the
COMPETENCIES AS PER NCF/NIPUN BHARAT	C-3.1: States and motivates/proves remainder theorem, factor theorem, and division algorithm
SDG(Sustainable Development Goals)	SDG4: Quality Education
Technology aids	PPTs
Learning Teaching Materials (LTM)	Placards/A4 sheet paper cut into half, horizontally, with each term of the given polynomials
Cross-Cutting Themes	Technology in Education - PPTs for every component Values and Beliefs - VC_Heroes and Zeroes

MS_Objective Polynomials

Objective: Teacher will be able to get an overview of the asset objectives in the MLP for the chapter Polynomials.

Notes to the teacher: This asset lays down the proposed plan for transacting this chapter. It states the asset objectives of the MLP. This asset is for the teacher's reference and need not be taught to the students.

The students will be able to –

- distinguish between an algebraic expression and a polynomial
- analyse a polynomial in one variable
- apply the knowledge of polynomials in one variable in answering the questions
- find zero of a polynomial
- internalise that decisions made day-to-day could have a significant impact on her/his future
- solve questions on zeroes and value of a polynomial
- solve questions based on the division of polynomials by a polynomial
- appreciate the use of Pascal's triangle in writing the expansion of binomials with any power
- relate polynomials to real life
- prove the Factor theorem
- apply Factor theorem in factorisation of polynomials
- apply identities in expansion and factorisation of quadratic polynomials
- verify experimentally, the identity involving the square of a trinomial
- apply identities involving cubic polynomials in expansion and factorisation
- apply identities for expansion and factorisation of polynomials

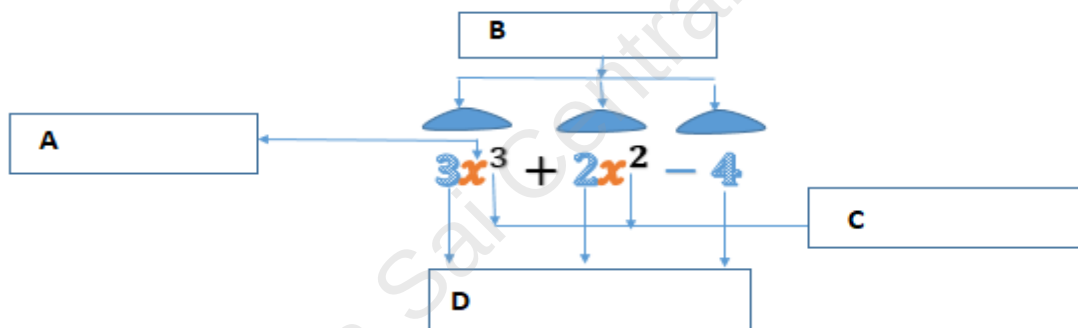
IQ_Algebraic Expressions

Objective: The student will be able to distinguish between an algebraic expression and a polynomial.

Teaching Aids: PPT

Notes to the teacher: The teacher may pose the following questions to the students, allow them to think and come out with as many answers as possible before arriving at a conclusion. If there is any difficulty in getting the answers, the teacher can ask leading questions.

1. An algebraic expression is a mathematical expression consisting of numbers , variables and signs of mathematical operations.
Fill the correct option in the space provided to mark the parts of an algebraic expression..



- a. Coefficients of the terms
- b. Variable
- c. Powers of the variable
- d. Terms

Answer: a) Box D b) Box A c) Box C d) Box B

2. Form an algebraic expression:
 - a. With four terms and using -2 and $\frac{4}{5}$ as the coefficients of any of the two terms.
 - b. For the statement: Half of a number subtracted from the cube of the number.

Suggested Answer:

a) $-2y^7 + 3y^3 + \frac{4}{5}y + 4$

[Notes to the teacher: The teacher may mention here that the power of the variable should be different in all the terms, otherwise the like terms will have to be combined.

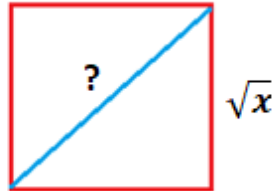
b) $x^3 - \frac{1}{2}x$

3. Give two algebraic expressions related to measurements which are used in everyday

life.

Suggested Answer: Examples - To find the area and perimeter of shapes e.g. xy representing the area of the rectangle with sides x and y . $2(x + y)$ represents its perimeter.

4. Represent the length of the diagonal as an algebraic expression.



Answer: $(\text{Diagonal})^2 = (\sqrt{x})^2 + (\sqrt{x})^2$ (Using Pythagoras Theorem)

$$(\text{Diagonal})^2 = 2x$$

$$\text{Diagonal} = \sqrt{2}\sqrt{x} = \sqrt{2}x^{\frac{1}{2}}$$

5. Give a statement which can be expressed as an algebraic expression with a negative exponent.

Examples:

- Sum of a number and its reciprocal gives algebraic expression $x + x^{-1}$
- 4 divided by the square of a number gives algebraic expression $\frac{4}{x^2} = 4x^{-2}$

6. Some of the answers of the above questions are arranged in the tabular form. What difference do you observe in the algebraic expressions of the two columns?

[Note to the teacher: The teacher may arrange the answers in tabular form as follows.]

$-2y^7 + 3y^3 + \frac{4}{5}y + 4$	$\sqrt{2}x^{\frac{1}{2}}$
$2x + 2y$	$x + x^{-1}$
$x^3 - \frac{1}{2}x$	$4x^{-2}$

Answer: The powers of the variable x in the first column are non-negative integers.
The powers of the variables in the second column are negative or fractions.

The algebraic expressions in the first column are called **polynomials**.

A polynomial is an algebraic expression with the non-negative integral exponents.

So, $3x^0 = 3$ is also a polynomial but $x^{-2} + 3$ and $x^{\frac{3}{4}}$ are not, they are only algebraic expressions.

You will learn more about polynomials in this chapter.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Inquisitive Questions	Polynomials	Polynomials

MS_Know the polynomials

Objective: Students will be able to analyse a polynomial in one variable.

Teaching Learning Material (TLM 1): Teacher shall make use of placards/A4 sheet paper cut into half, horizontally, with each term of the respective polynomial as mentioned in 'Notes to teacher' in this asset and the same shall be displayed by the students.

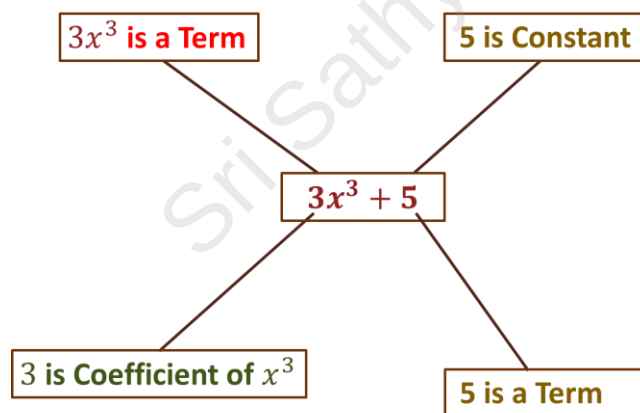
Get ready: Teacher will keep box board cards with half the size of A4 (quantity as per the number of students and 15 extra) or A4 plain white paper cut into two parts and distribute one to each student and also retain around 15 pieces with him/ her. A black/blue marker to be carried by the teacher. Sketch pens to be carried by students. Box board is referred to as a placard in the asset.

Notes to teacher: Teacher will recapitulate the difference between algebraic expression and polynomials and continue with polynomials in one variable.

Polynomials are algebraic expressions with only whole numbers as the exponent of the variable. The word polynomial is derived from a Greek term , 'poly' which means many and Latin term 'nomial' which means terms or names.

Let's take a polynomial $3x^3 + 5$. This is a polynomial in one variable x . ■

[Note to teacher: Teacher may call three students and ask any two students to write each term ($3x^3$ and 5) of the polynomial and the third student to write the sign on the box board or A4 sheet given by the teacher and they will display it as a polynomial $3x^3 + 5$ in front of the class and the teacher may explain the following terms.]



Variable: In the above polynomial, x is a variable, value of a variable can change in the given polynomial..in general, letters like x , y , z , etc. are used to represent variables.

Coefficient: Number with which the variable is multiplied in a term, is called a coefficient. In polynomial $3x^3 + 5$, 3 is the coefficient of x . Sometimes the coefficient can also be a letter. Letters like a , c , b , etc. are used to represent the coefficient. For example in polynomial ax , a is the coefficient of x .

Constant: A constant is a standalone number in a given polynomial. The value of the constant remains the same, or does not change. In the polynomial $3x^3 + 5$, 5 is a constant.

Term: Expressions which are connected to each other by + or - sign are terms of a polynomial. For example, in the polynomial $3x^3 + 5$, $3x^3$ and 5 are the terms. In $x^2 - x + 2$, the terms are x^2 , $-x$ and 2

[**Notes to the teacher:** Teacher may call three students and give each one a placard or A4 size cut paper and ask each one to write one of the terms from $2x^3, y^3, 5$. The teacher may call two more students to write the sign '+'. All five students may hold the placard/A4 paper sheet in front of the class to make a polynomial.]

$2x^3 + y^3 + 3$ is a polynomial in two variables x and y , 2 is the coefficient of x^3 and 3 is a constant term.

[**Notes to the teacher:** Teacher may call three students and give a placard / A4 paper sheet wherein two of them will write the terms x^{-3} and y^{-3} and the third student will write '-' sign. These three students will hold the placards/A4 paper sheet in front of the class. The teacher may have a question and answer session on the questions like, "Is the given expression a polynomial or an algebraic expression? Give reasons." Teacher may give the second question as $\sqrt{x}$ and have a similar discussion]

$x^{-3} + y^{-3}, \sqrt{x}$ are not polynomials as the exponent of $x^{-3} + y^{-3}$ are negative and exponent of $\sqrt{x}$ ($x^{\frac{1}{2}}$) is a fraction, which is not a whole number.

(i) $3x^3, 5x, 4y, \frac{x^2}{2} + 5, 5y + 6$ are all polynomials in one variable.

(ii) $u^2 + uv + v^2 + 3uvw$, is a polynomial in 3 variables.

(ii) $x^2 + z^2 + y^2 + w^2$ is a polynomial in 4 variables.

[**Notes to the teacher:** Teacher can ask all the students to write a polynomial with one term and specify the variable to be used by the students sitting in different rows/columns. The teacher may call a student who has written a polynomial with variable z to display that in front of the class. The student will hold the placard/A4 paper sheet in front of the class.]

Polynomials in one variable are denoted based on the variable used in that polynomial.

Example: (i) $q(z) = 5z^4$

$q(z)$ is a polynomial in variable ' z '.

Polynomial has only one term.

Polynomials with one term are called **monomials** (mono means one).

More examples of monomial are: $p(x) = x^8, f(y) = y^{17}, g(z) = 5z, r(x) = -7x$, etc.

Teacher may call two students who have written the polynomial with variable x .

Students will hold the playcards/A4 paper sheets in front of the class to make a polynomial with two terms connected by +/- sign.

(ii) $p(x) = x^3 + x$

$p(x)$ is a polynomial in variable x

$p(x)$ has two terms and it is called binomial. (bi means two)

$y + 2, z^2 + \frac{1}{5}, x^{10} - \sqrt{5}$ are some more examples of binomial

(iii) $r(y) = 3y^2 + 4y + 5$

$r(y)$ is polynomial in variable ' y '.

The polynomial $r(y) = 3y^2 + 4y + 5$ has three terms.

Polynomials with three terms are called **trinomials** (tri means three).

$3y^8 + y^2 + 9y, 3x^5 + 4x - 5$ are more examples of trinomial

[Notes to the teacher: Teacher may ask questions like, "Can real numbers be a polynomial?" and hence introduce the word non-zero constant]

(iv) $t(x) = 9$

9 is a constant. It can be written as $9x^0$ or $9y^0$ as $x^0 = 1$. Similarly $y^0 = 1$.

So every non-zero constant is also a polynomial. It is called a constant **polynomial**.

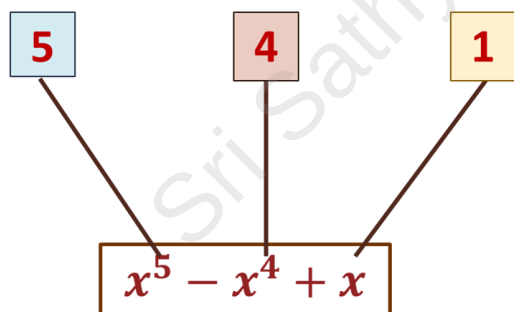
But the number zero is called the zero polynomial, for example $t(x) = 0$ is called the zero polynomial

(v) $u(s) = x^{50} + x^{48} + x^{47} + \dots + x + 1..$

$u(s)$ is a polynomial with 51 terms.

Degree of the polynomial is the highest power of the variable in the polynomial.

For example, degree of the polynomial $x^5 - x^4 + x$ is 5. The powers of variable in each term is as follows:



Terms of this polynomial are x^5, x^4 and x

Power of variables in each term is 5, 4 and 1 respectively. Here the highest power of the variable in the polynomial $x^5 - x^4 + x$ is 5. So, the degree of the polynomial is the highest power of the polynomial which is 5.

[Notes to the teacher: Teacher may call three students and specify the terms of the polynomial to the students to be written on placard/ A4 paper sheet to explain example 1, and make them stand in a row along with two students with sign - and + written on the placard/A4 paper sheet,

to show the polynomial to the class. Teacher may ask the questions listed below to the class and elicit answers from the students. Teacher can have an interactive class by asking the following or similar questions]

Example 1:

$$f(x)=3x^{10} - 2x^{11} + 5x$$

$3x^{10}$	$-$	$2x^{11}$	$+$	$5x$
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1. Degree of the polynomial $f(x)=3x^{10} - 2x^{11} + 5x$ is 11
2. Coefficient of the term with highest degree: $f(x)= -2$
3. How many terms are in this polynomial: 3
4. How can we categorise this polynomial: Suggested answer: Trinomial, as there are three terms.

[Notes to the teacher: Teacher may ask three students to display polynomial $3x - 11 + 5x^2$ by writing each term on a placard/ A4 paper sheet and two students can show the signs - and +. Students will arrange themselves in proper order to form a polynomial and show it to the class. Teacher may ask the students to arrange themselves in a row such that the polynomial is in standard form. Teacher may ask the below mentioned questions to the class and elicit answers from the students]

Example 2: $g(x)= 3x - 11 + 5x^2$

$5x^2$	$+$	$3x$	$+$	-11
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A polynomial is generally written in standard form. A polynomial is said to be in standard form if its terms are arranged in decreasing or increasing order of exponents of the variable in the polynomial.

Notes to the teacher: Teacher may ask the students following questions or any other similar question.]

Degree of the polynomial $3x - 11 + 5x^2$ is 2

Which is the constant term in the polynomial $3x - 11 + 5x^2$? Answer: -11

Example 3: [Notes to the teacher: The teacher may use the TLM to explain the example 3].]

$p(x) = 7$

Here 7 can be written as $7 \times x^0$ or $7 \times y^0$. So, the degree of a **non-zero constant polynomial** is always **zero**.

Zero Polynomial

'0' is called the zero polynomial.

Degree of zero polynomial is **not defined**.

$f(x) = 0 = 0 \times 5 = 0 \times x = 0 \times x^2 = 0$ (here all are zero polynomial, but the degree of each varies). Hence degree of zero polynomial is undefined.

Classification of polynomials based on the degree.

(i) $p(x) = 3x^3 - 5x^2 + 4x + 5$

Degree of the above polynomial is 3. A polynomial of degree 3 is called **cubic polynomial**

A cubic polynomial can have a maximum of 4 terms and minimum 1 term.

Any cubic polynomial in x is of the form $ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, d are constants. A cubic polynomial can be in any variable like y, z , etc.

Example: $16x^3, -12y^3 + 5y^2 + 6y + 2$

(ii) $q(y) = -10y^2 + 5y - 23$

Degree of polynomial $q(y)$ is 2. A polynomial of degree 2 is called a **quadratic polynomial**.

A quadratic polynomial can have a maximum of 3 terms and minimum 1 term.

Any quadratic polynomial in x is of the form $ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constants. A quadratic polynomial can be in any variable like y, z , etc.

Example: $2z^2 - z, x^2 + 2$

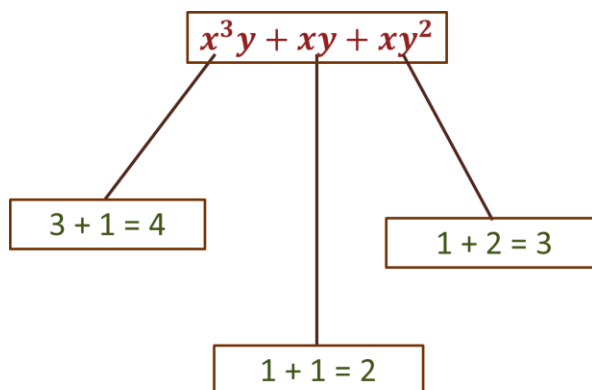
(iii) $s(t) = 6t + 5$

Degree of the polynomial $s(t)$ is 1

A polynomial of degree 1 is called a **linear polynomial**.

A linear polynomial can have a maximum of two terms.

Let us take a polynomial $x^3y + xy + xy^2$ in two variables. To find its degree we add the power of all the variables in each term. Highest sum of the power of variables in any term is the degree.



Here 4 is the highest sum of the powers of the variables in the given polynomial. Therefore 4 is the degree of this polynomial. This polynomial is written as $p(x,y)$.

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Main Script	Polynomials	Coefficients of a polynomial Degree of polynomial Linear, quadratic and cubic polynomials Monomial, Binomial and Trinomial Polynomial in one variable

QA_About Polynomials

Objective: Students will be able to apply the knowledge of polynomials in one variable, in answering the questions.

Teaching Aids: PPT, Worksheet

Notes to the teacher: The teacher may make use of the following questions and the attached worksheet to enhance the ability of students to apply the knowledge of polynomials in one variable in answering the questions.

Q I. Identify whether each one of the following algebraic expressions is a polynomial in one variable or not. Give reasons for your answer.

a. $6x^2 - 5x + 1$

Answer: It is a polynomial in one variable.

Reason: There is only one variable, x , and the powers of x are non-negative integers.

b. $x^3 + \sqrt{5}$

Answer: It is a polynomial in one variable.

Reason: There is only one variable, x and the power of x is a non-negative integer.

c. $\sqrt{x} + 3$

Answer: No. It is not a polynomial but an algebraic expression in one variable.

Reason: There is only one variable, x .

But the term containing the variable x , is $\sqrt{x} = x^{1/2}$. Here, the power of x is not an integer.

d. $3x^3 + 2x + \frac{1}{x}$

Answer: No. It is not a polynomial but only an algebraic expression in one variable.

Reason: There is only one variable, x .

But the term $\frac{1}{x} = x^{-1}$. The power of the variable here is a negative integer.

e. $4y^2 + \sqrt{3}y + 2$

Answer: Yes. It is a polynomial in one variable.

Reason: There is only one variable, y , and the power of y , in all the terms is, a non-negative integer.

f. $4x^2 - 2y + 1$

Answer: No. It is a polynomial in two variables.

Reason: There are two variables, x and y .

Q II. Classify the following polynomials as monomials, binomials or trinomials.

- a. $5x^2 - 3$
- b. $2x^2 + x + 7$
- c. $8x$
- d. $3y$
- e. $3x^{10} + 2x^8 - 5$
- f. $15x^2 - 13y$

Answer:

Monomials: $8x$ and $3y$

Binomials: $5x^2 - 3$ and $15x^2 - 13y$

Trinomials: $2x^2 + x + 7$ and $3x^{10} + 2x^8 - 5$

Q III. Classify the following as linear, quadratic or cubic polynomials.

- a. $6x^2 + 2x + 7$
- b. $21x - 3y + 7$
- c. $2x^3 + 5x^2 - 7x + 6$
- d. $9x^2 + 14$
- e. $2x + 5$
- f. $35x^3$

Answer:

Linear polynomials: $21x - 3y + 7$ and $2x + 5$ as the degree of the polynomials is 1.

Quadratic polynomials: $6x^2 + 2x + 7$ and $9x^2 + 14$, as the degree of the polynomials is 2.

Cubic polynomials: $2x^3 + 5x^2 - 7x + 6$ and $35x^3$, as the degree of the polynomials is 3.

Q IV. Choose the correct answer:

- a. What is the degree of the polynomial $6x^2 + x^5 - 2$?
 - 1. 2
 - 2. 6
 - 3. -2
 - 4. 5

Answer: 4) 5

- b) 5 is a constant polynomial. What is its degree?

1. 0
2. 5
3. 1
4. Not defined

Answer: 1) 0

(Notes to the teacher: The teacher may remind the students that $5 = 5 \times 1 = 5 \times x^0$. This will be true for any non-zero constant polynomial. So the degree of a non-zero constant polynomial is always 0)

c) What is the degree of a zero polynomial?

1. 0
2. 1
3. Not defined
4. None of the above

Answer: 3) Not defined

(Notes to the teacher: The teacher may remind the students that $0 = 0 \times x^{100} = 0 \times x^{1000} = 0 \times x$ any number.

Thus the degree of the zero polynomial is not fixed. Hence it is said to be 'not defined')

d) Which of the following is a polynomial of degree 5?

1. $x^7 + x^5 + x + 5$
2. $5x+5$
3. $18x^3 + x^2 + 7x^5 + 13$
4. 5

Answer: 3) $18x^3 + x^2 + 7x^5 + 13$

e) Which of the following is a monomial of degree 8?

1. $8x$
2. 2^8
3. 8
4. $7x^8$

Answer: 4) $7x^8$

f) Which of the following is a binomial of degree 6?

1. $8x^{16} + 5y^6$
2. $6x^3 + 6y^3$
3. $8x^6 + 5x$
4. 6

Answer: 3) $8x^6 + 5x$

Q.V. Write the required polynomial for each of the following:

- a. A trinomial of degree 25
- b. A binomial of degree 15
- c. A monomial of degree 60.
- d. A quadratic polynomial
- e. A cubic polynomial
- f. A polynomial with degree 0

Answers:

- a. Trinomial of degree 25: $15x^{25} - 7x + 6$
- b. Binomial of degree 15: $9y^{15} + 1$
- c. Monomial of degree 60: $51x^{60}$
- d. A quadratic polynomial: $5x^2 - 7x + 6$
- e. A cubic polynomial: $8y^3 + 1$
- f. A polynomial with degree 0: 25

(Notes to the teacher: The students may come up with different answers to the above questions. The teacher may determine the correctness of the answer based on the required criteria)

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Questions to Assess	Polynomials	Polynomial in one variable

MS_Zero of a Polynomial

Objective: Students will be able to calculate the zero of a polynomial.

Teaching Aids: PPT

Notes to the teacher: Teacher may begin the class by a brief recapitulation of type of polynomials.

Consider a polynomial $p(x) = x^2$

Here x is a variable, it can take any value in this polynomial.

Taking $x = 2$, i.e. substituting the value of x as 2 in the polynomial, we get:

$$p(2) = 2^2 = 4$$

The value of $p(x)$ at $x = 2$ is 4

$$\text{Taking } x = -3, p(-3) = (-3)^2 = 9$$

So, the value of $p(x)$ at $x = -3$ is 9.

In general, the value of a polynomial $p(x)$ at $x = a$, denoted as $p(a)$, is obtained by substituting x by a , where a is any real number.

Examples 1: Find the value of $p(x) = 7x^2 + 8x - 2$ at:

i) $x = 1$ and ii) at $x = -1$ (iii) $x = 0$

$$p(1) = 7 \times 1^2 + 8 \times 1 - 2 = 7 + 8 - 2 = 13$$

$$p(-1) = 7 \times (-1)^2 + (8 \times -1) - 2 = 7 - 8 - 2 = -3$$

$$p(0) = 7 \times 0 + 8 \times 0 - 2 = 0 - 2 = -2$$

Example 2: $p(x) = x^3 + 4x^2 + 4x + 1$

Find the value of $p(x)$ at i) $x = a$ ii) $x = -1$

Substituting $x = a$ in $p(x)$,

$$p(a) = a^3 + 4a^2 + 4a + 1$$

Substituting $x = -1$

$$p(-1) = (-1)^3 + 4(-1)^2 + 4(-1) + 1$$

$$= -1 + 4 - 4 + 1 = 0$$

Here we observe that $x = -1$ makes the value of polynomial zero or $p(-1) = 0$.

We call this value of x , the zeroes of the polynomial $p(x)$.

Hence we say $x = -1$ is a zero of $p(x)$

A zero of a polynomial $p(x)$ is a real number m such that $p(m) = 0$.

Let us take the linear polynomial

$$p(x) = x + 2$$

Here x is a variable and '2' is a constant.

Now, we can look for the value of x for which $x+2 = 0$.

If we substitute $x = -2$, we get $-2 + 2 = 0$.

How can we obtain this value from the given polynomial?

[Notes to the teacher: Teacher can use questions to elucidate method (s) for finding the value of x that makes the polynomial zero]

Yes, in order to find the zero of a polynomial, we can equate the polynomial to zero.

So, $x + 2 = 0$ will give $x = -2$. Hence $x = -2$ is the zero of the polynomial.

Zero of a polynomial $p(x)$ is also called the root of the polynomial equation $p(x) = 0$.

Consider another example, $p(x) = x - 5$.

In order to find the value of x which makes $p(x)$ zero, we equate $p(x)$ to 0.

$$p(x) = 0, \text{ i.e. } x - 5 = 0$$

Hence $x = 5$ i.e. $x = 5$ makes the polynomial zero.

So, $x = 5$ is called zero of the polynomial.

For any linear polynomial $p(x) = ax + b$, $a \neq 0$, the zero can be obtained by solving the equation $p(x) = 0$. Hence $x = -\frac{b}{a}$ is the only zero of $p(x)$.

Note that a linear polynomial has exactly one zero.

Example 3: Verify if $x = 2$ is zero of the polynomial $p(x) = x^3 - 2x^2 + 2$.

Expected solution: Substituting $x = 2$ in $p(x)$, we get:

$$p(2) = 2^3 - 2 \times 2^2 + 2 = 8 - 8 + 2 = 0 + 2 = 2.$$

Here $p(2) \neq 0$. So $x = 2$ is not the zero of $p(x)$.

Example 4: What is the zero of the polynomial $p(x) = 9$?

Expected solution: 9 is a constant polynomial.

It can be expressed as:

$$p(x) = 9 \times x^0$$

Here for any value of x , x^0 is 1.

x can take any non zero real value, say 3. $3^0 = 1$. $p(x)$ will still be equal to 9 and will not be equal to zero.

Therefore, there is no zero for a constant polynomial.

Thus a non-zero constant polynomial has no zero.

Example 5: What is the zero of a zero polynomial?

Expected solution: $p(x) = 0$ is a zero polynomial.

Zero of the polynomial is the value of the variable in that polynomial which makes the

polynomial zero.

x can take any real value and still $p(x)$ will be equal to 0.
Hence any real value of x is the zero of a zero polynomial.

Example 6: Find the zero of the polynomial $p(x) = 3x^2 + x$

Solution: Equate $p(x) = 0$

$$3x^2 + x = 0$$

$$x(3x + 1) = 0 \text{ which will give } x = 0 \text{ OR } 3x + 1 = 0$$

$$\text{If } 3x + 1 = 0, x = \frac{-1}{3}$$

Here there are two zeroes $x = 0$ and $x = \frac{-1}{3}$.

From the above examples we note that,

- A polynomial can have more than one zero.
- '0' may be zero of a polynomial.
- Zero of a polynomial need not be zero.
- Linear polynomial will have exactly one zero.
- Any real value of x is zero of a zero polynomial.
- There is no zero for a constant polynomial.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Polynomials	Zeros and roots of polynomials

VC_Hero Or Zero?

Objective:

Concept- Value Connection:

Zero of a polynomial is that value of the variables, at which the value of the polynomial becomes zero. Also the zero of a polynomial can be obtained by equating the polynomial to a zero.

Let's understand how such zeroes exist in our lives and reduce our value as a human being. Bad habits and bad company act like zeroes that bring down your worth. Some friends try to pull you into bad habits. In the beginning they are fun to be with, but the wrong habits that you developed by their association, diminish your worth.

Some children make decisions on impulse or to look 'cool' to their peers without regard to how that could define their future. When we are young, it is but natural that we want to look and act like our heroes. If some of those acts prove detrimental to our health and general well being, we could instead become zeroes, meaning, failures in life. You need to decide what you would like to be, a Hero or a Zero?

Notes to the teacher:

1. Please narrate the following situation to the class.
2. Allow them a minute's time to ponder over it before asking the questions for discussion.
3. Please ensure that every student gets a chance to express her/his views.

Situational analysis :

Jatin's classmates Raju and Rohan asked him to join them to watch a movie. Initially he refused as examinations were just weeks away. However, they persuaded him saying that the movie would be a nice change and that he could study in the evening. Jatin fell for it and told a lie to his mother that he was going to study with his friends.

After the movie, his friends stopped to smoke and tried to convince Jatin that it was cool to smoke and that there was nothing wrong. They gave examples of great and famous personalities who were known to smoke. Smoking now and then, as they put it, was a stress buster!

Now, Jatin was in a dilemma. He knew that it was wrong to smoke and that his parents would be shocked if he did. He had read enough about the harm caused by cigarette smoke! However, he was afraid that his friends would ridicule and brand him as a coward and a weakling and that he may not be able to face them again. How could he do something which he knew to be clearly wrong?

Discussion questions:

- If you were Jatin, how would you deal with such friends?
- What are the possible activities that could affect a student adversely?
- Can friends really influence you one way or the other?
- Student life may at times seem to be monotonous and tiring. What options do we have?

Points to ponder:

- It is good to have fun through sports, games or travel without affecting the studies, as these enhance a student's life skills.
- Excessive TV watching, indiscriminate use of the social media, getting involved in political activism, etc. affect student life adversely.
- A good friend is one who tells you when you are wrong and stands by you at all times.
- Your future is defined by the company you keep. Good friends help you become a hero while the contrary could reduce you to a zero!



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Time to teach	Asset type	Theme	Sub Theme
12 minutes	Value Content	Truth	Discrimination

QA_ Value and Zero of Polynomial

Objective: Students will be able to solve questions on zeroes and value of a polynomial.

Teaching Aids: Worksheet,PPT

Notes to the teacher: <<Teacher may present the questions and allow the students to write or say the answers before revealing them.>>

I. Find the zero of the polynomial

a. $p(y) = y - 3$

Answer: $y - 3 = 0$

. Therefore $y = 3$ is the zero of $p(y)$

b. $p(y) = my + n$

Answer : $my + n = 0$, $y = -\frac{n}{m}$ is the zero of $p(y)$

c. $q(x) = 10x - 100$

Answer: $10x - 100 = 0$, $x = \frac{100}{10} = 10$ is the zero of $q(x)$

II a) Find the value of k if $x = 1$ is the zero of $p(x) = 5x^2 + k$

Answer: $x = 1$ is the zero of $p(x)$

$$\therefore p(1) = 0$$

$$\Rightarrow p(1) = 5 \times 1 + k = 0 .$$

$$\Rightarrow k = -5$$

b) Find the value of a if $y = 2$ is the zero of $p(y) = 2y^2 - \frac{3y}{7} + a$

Answer: $y = 2$ is the zero of $p(y)$

$$\therefore p(2) = 2 \times 2^2 - \frac{3 \times 2}{7} + a = 0$$

$$\Rightarrow 8 - \frac{6}{7} + a = 0$$

$$\Rightarrow \frac{56-6}{7} + a = 0$$

$$a = \frac{-50}{7}$$

III Find the value of the polynomial $p(x) = x^3 - x^2 + x - 1$ at

a. $x = 0$

Answer: $p(0) = 0 - 0 + 0 - 1 = -1$

b) $x = -1$

Answer: $(-1)^3 - (-1)^2 + (-1) + (-1) = -1 - 1 - 1 - 1 = -4$

c) $x = 3$

Answer: $(3)^3 - (3)^2 + (3) + (-1) = 27 - 9 + 3 - 1 = 30 - 10 = 20$

IV a) Verify whether $x = -2$ is zero of $p(x) = 2x + 2$

Answer: $p(-2) = -4 + 2 = -2$. Therefore, $x = -2$ is not zero of the polynomial.

b) Check if $x = 3, -3$ are zeroes of $p(x) = (x + 3)(x - 3)$

Answer: $p(3) = (3 + 3)(3 - 3) = 6 \times 0 = 0$

So, 3 is zero of $p(x)$

$p(-3) = (-3 + 3)(-3 - 3) = 0 \times -6 = 0$

Therefore, -3 is zero of $p(x)$

V) Can 0 be zero of a polynomial? If yes, give an example.

Answer : Yes, 0 can be zero of a polynomial. Any polynomial with terms containing variables and with no constant term will have 0 as zero of the polynomial.

Example: $p(x) = x^3 + x^2 + x$

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Questions to Assess	Polynomials	Zeros and roots of polynomials

MS_Division of Polynomials

Objective: The students will be able to perform the division of a polynomial by a polynomial.

Teaching Aids: PPT

Notes to the teacher: The teacher may help the students to recollect the division of whole numbers and then explain the division algorithm of polynomials.

Let us recall some facts about division of numbers.

When we divide 17 by 5, 17 is called the dividend and 5 is the divisor.

$$\begin{array}{r} 3 \\ 5 \overline{) 17} \\ \underline{-15} \\ 2 \end{array}$$

We get 3 as the quotient and 2 is the remainder.

This can also be written as follows: $17 = (5 \times 3) + 2$

That is, Dividend = (Divisor $\times$ Quotient) + Remainder

If the remainder is zero, then Dividend = Divisor $\times$ Quotient.

For example when 15 is divided by 5, the quotient is 3 and the remainder is zero.

In such a case, we say that 5 and 3 are factors of 15.

15 is a multiple of 5 and 3.

For division of numbers, (with positive divisors) the following facts need to be remembered:

Dividend = (Divisor $\times$ Quotient) + Remainder

The remainder is always less than the divisor.

If the remainder is zero, then the divisor and quotient are factors of the dividend; in such a case the dividend is said to be a multiple of the divisor and the quotient.

Let us now see how to divide a polynomial $p(x)$ by a monomial $g(x)$.

Example 1:

Let $p(x) = 5x^3 + 3x^2 + 2x$ and $g(x) = x$

$$\begin{aligned} p(x) \div g(x) &= (5x^3 + 3x^2 + 2x) \div x \\ &= (5x^3 \div x) + (3x^2 \div x) + (2x \div x) \\ &= 5x^2 + 3x + 2 \end{aligned}$$

So, the quotient = $5x^2 + 3x + 2$ and the remainder = 0.

In example 1, we observe that $g(x)$ that is x , is common to each term of $p(x)$.

So, $p(x) = 5x^3 + 3x^2 + 2x$

$$= x(5x^2 + 3x + 2)$$

We say that x and $5x^2 + 3x + 2$ are factors of $5x^3 + 3x^2 + 2x$ and $5x^3 + 3x^2 + 2x$ is a multiple of x and $5x^2 + 3x + 2$.

Example 2:

Divide $p(x)$ by $g(x)$ if $p(x) = 2x^2 + 3x + 1$ and $g(x) = x$

$$\begin{aligned} p(x) \div g(x) &= (2x^2 + 3x + 1) \div x \\ &= (2x^2 \div x) + (3x \div x) + (1 \div x) \\ &= 2x + 3 + 1/x \end{aligned}$$

Here, $1/x$ is not an accepted term in a polynomial as $\frac{1}{x} = x^{-1}$

So we write, $2x^2 + 3x + 1 = x(2x + 3) + 1$

We know that Dividend = (Divisor $\times$ Quotient) + Remainder

In the above example, quotient = $2x + 3$ and remainder = 1

Example 3:

Let us now learn how to divide a polynomial by another polynomial

$$p(x) = 3x + 5x^2 + 2 \text{ and } g(x) = 1 + x$$

Divide $p(x)$ by $g(x)$.

Let us see how to do this step by step.

Step 1: Write the dividend and divisor in the standard form (in decreasing powers of the variable).

$$p(x) = 5x^2 + 3x + 2 \text{ and } g(x) = x + 1$$

$$\begin{array}{r} \\ x+1 \overline{) 5x^2 + 3x + 2} \end{array}$$

Step 2: Divide the first term of the dividend ($5x^2$) by the first term of the divisor (x).

$$5x^2 \div x = 5x$$

Write this $5x$ as the first term of the quotient.

$$\begin{array}{r} 5x \\ x+1 \overline{) 5x^2 + 3x + 2} \end{array}$$

Step 3: Multiply the divisor by the answer obtained in the previous step, that is, $5x$.

$$5x(x + 1) = 5x^2 + 5x$$

Write this product below the dividend in such a way that the like terms are one below the other.

Then subtract $5x^2 + 5x$ from the dividend.

$$\begin{array}{r}
 5x \\
 \hline
 x+1 \overline{) 5x^2 + 3x + 2} \\
 \underline{5x^2 + 5x} \\
 (-) (-) \\
 \hline
 -2x + 2
 \end{array}$$

Step 4: Look at the remainder. If the degree of the remainder is:

- greater than or equal to that of the divisor, treat the remainder as the new dividend and repeat steps 2 and 3 .
- less than that of the divisor, the division is over. You can write down the quotient and remainder.

Here the degrees of the remainder and the divisor are both 1. So we have to treat $-2x + 2$ as the new

dividend and repeat steps 2 and 3

So, we divide $-2x$ by x to get the second term of the quotient.

$$-2x \div x = -2.$$

$$\begin{array}{r}
 5x - 2 \\
 \hline
 x+1 \overline{) 5x^2 + 3x + 2} \\
 \underline{5x^2 + 5x} \\
 (-) (-) \\
 \hline
 -2x + 2
 \end{array}$$

Multiply the divisor by -2 .

$$-2(x + 1) = -2x - 2$$

Then subtract this product from the remaining terms of the dividend.

$$\begin{array}{r}
 5x - 2 \\
 \hline
 x+1 \overline{) 5x^2 + 3x + 2} \\
 \underline{5x^2 + 5x} \\
 (-) (-) \\
 \hline
 -2x + 2 \\
 \underline{-2x - 2} \\
 (+) (+) \\
 \hline
 4
 \end{array}$$

We get a remainder 4 which is a constant polynomial of degree 0.

The degree of the remainder is now less than that of the divisor. That concludes our division.

Step 5:

The answer is written as follows:

Quotient = $5x - 2$ and Remainder = 4.

We observe that Dividend = (Divisor $\times$ Quotient) + Remainder

$$5x^2 + 3x + 2 = (x + 1)(5x - 2) + 4$$

Example 4:

$$p(x) = 7x^4 - 4x^3 - 7x + 2 \text{ and } g(x) = x - 1.$$

Divide $p(x)$ by $g(x)$.

Solution:

In this example, there is no term containing x^2 in the dividend. So we have to insert $0x^2$ in the place of x^2 . Let us now follow the steps of long division sequentially.

(Notes to the teacher: The teacher may remind the students frequently to write the dividend in the standard form and check whether there is any missing power of the variable in the dividend. In case there is a missing power of the variable, it should be included with zero as its coefficient)

$$\begin{array}{r}
 \overline{7x^3 + 3x^2 + 3x - 4} \\
 x-1 \overline{) 7x^4 - 4x^3 + 0x^2 - 7x + 2} \\
 \underline{7x^4 - 7x^3} \\
 (-) (+) \\
 3x^3 + 0x^2 - 7x + 2 \\
 \underline{3x^3 - 3x^2} \\
 (-) (+) \\
 3x^2 - 7x + 2 \\
 \underline{3x^2 - 3x} \\
 (-) (+) \\
 -4x + 2 \\
 \underline{-4x + 4} \\
 (+) (-) \\
 -2
 \end{array}$$

Answer:

$$\text{Quotient} = 7x^3 + 3x^2 + 3x - 4$$

$$\text{Remainder} = -2$$

$$7x^4 - 4x^3 - 7x + 2 = (x - 1)(7x^3 + 3x^2 + 3x - 4) - 2$$

We observe that Dividend = (Divisor $\times$ Quotient) + Remainder

We can conclude that for two polynomials, $p(x)$ and $g(x)$ fulfilling the condition that the degree of $p(x) \geq$ the degree of $g(x)$, we can find two polynomials, $q(x)$ and $r(x)$ such that $p(x) = g(x)q(x) + r(x)$ where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$. Here $p(x)$ is the dividend, $g(x)$ is the divisor, $q(x)$ is the quotient and $r(x)$ is the remainder. This is the Division Algorithm for Polynomials.

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Main Script	Polynomials	Division algorithm for polynomials Division of polynomial by a polynomial

DD_Uses of Polynomials

Objective: The student will be able to relate polynomials to real life.

Teaching Aids: PPT

Notes to the teacher: The teacher may introduce the topic as follows:

Polynomials are not only required in mathematics class and in all higher level mathematics courses but also have great applications in all spheres of life.

Almost everyone from engineers to businessmen use polynomials in their work.

For the rest of us, they are less evident but we use them without even realising it.

Suppose you buy 2 pencils, 4 ballpoint pens and 3 notebooks. When you check the cost of each item, you form a polynomial in your head unknowingly, to calculate the cost.

If we denote the cost of each pencil by letter p , ballpoint pen by b and notebook by n , we get a polynomial $2p + 4b + 3n$



Pencils ₹ 5 each



Ballpoint Pens ₹ 25 each



Notebooks ₹ 30 each

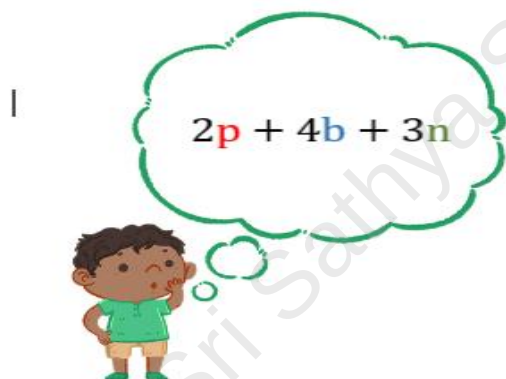


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Whatever be the cost of each item, we calculate the total cost using the polynomial, mentally.

Insert the cost in the polynomial and we get the total cost.

$$\begin{aligned} &2(5) + 4(25) + 3(30) \\ &= 10 + 100 + 90 \end{aligned}$$

Total cost = ₹ 200

Similarly, for paying a taxi fare or while deciding the proportion of ingredients of recipes to cook a dish, we actually visualise polynomials to help us calculate.

Let's explore some more applications of polynomials.

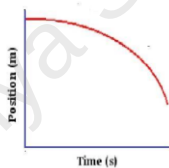
- **Construction and material planning:** Architects use polynomials to design buildings, roads and other structures and to draw the blueprint which is used as a guide for the actual construction. They are used for materials planning by civil engineers, that is to calculate the amount of material to be used in construction. They are also used for designing bridges and checking their strength.

$$2(l+b) \quad 2(lb + bh + lh)$$

- **Business:** People deal with money matters like paying wages, savings, loan installments and rent, which involves paying and receiving money at different intervals of time. Polynomials prove useful in such situations. Polynomials are also used in financial planning, marketing, budgeting and price or demand predictions. They are used to analyse how a rise in the price of goods affects its sales. Economists use them for economic planning and analysing growth patterns.

$$px^n + px^{n-1} + px^{n-2} \dots \dots p, \text{ where } x = \left(1 + \frac{r}{100}\right), n \text{ is a positive integer}$$

- **Scientific research:** Researchers use polynomials in their experiments to analyse and interpret data and draw conclusions. For example, medical researchers use them as a tool to describe the pattern of bacterial growth. The graphs representing the polynomials are used in analysing results in areas like sports and space research.



$$a(x - h)^2 + k$$

- **Polynomials are applicable in almost all the fields of science.**

Polynomials are used in physics, chemistry, geography, astrophysics, statistics, economics, medical research and population studies, to name a few.

For example, polynomials are used in **designing and building many devices** like microphones, satellite dishes and car headlights.

Astronomers use polynomials to estimate the number of stars in the sky and to calculate distance between stars and galaxies.

Space scientists use them to calculate the velocity of the spacecraft and acceleration needed to launch them into space, and so on.

The power of polynomials helps in **meteorology** to make accurate weather forecasts about extreme weather conditions and thus prevent destruction and loss of life by timely planning and action.

In the present times the greatest application of polynomials has been made in the **field of Electronics**. Know that when you use a calculator, you are using a polynomial.

Scientific calculators and computers perform accurate calculations using a polynomial called Taylor Approximation or other polynomial approximation.

Computer scientists use coordinate systems and a technique called cryptography which involves the use of polynomials.

So, polynomials are a great mathematical tool, though not used evidently in everyday life. They are the power behind all the scientific and technological advancements in the world, which greatly influence our day to day life.

Time to teach	Asset type	Theme	Sub Theme
5 minutes	Day-to-day Relevance	Polynomials	Polynomials

MS_Factor Theorem

Objective: The students will be able to prove the factor theorem.

Teaching Aids: PPT

Notes to the teacher: The teacher may help the students to recollect the concepts of a factor and the remainder theorem, before introducing the factor theorem.

Consider the following multiplication fact:

$$12 = 3 \times 4$$

Here 3 and 4 are called factors of 12.

When is a number ' a ' said to be a factor of another number ' b '?

When we divide b by a and get zero as the remainder.

The same is true in the case of polynomials too.

We know that when a polynomial, $p(x)$ is divided by a linear polynomial $x - a$, then the remainder is $p(a)$.

Suppose $p(a) = 0$, what can we conclude about $x - a$ and $p(x)$?

Yes, $x - a$ is a factor of $p(x)$.

The factor theorem states this fact as follows:

Factor Theorem:

If $p(x)$ is a polynomial of degree $n \geq 1$ and a is any real number, then

- (i) $x - a$ is a factor of $p(x)$, if $p(a) = 0$ and
- (ii) $p(a) = 0$, if $x - a$ is a factor of $p(x)$.

Proof:

According to the remainder theorem,

$$p(x) = (x - a)q(x) + p(a)$$

(i) If $p(a) = 0$,

$$\text{then } p(x) = (x - a)q(x)$$

$$\Rightarrow x - a \text{ is a factor of } p(x).$$

(ii) If $x - a$ is a factor of $p(x)$,

$$\text{then } p(x) = (x - a)g(x) \text{ for some polynomial } g(x).$$

$$\text{So } p(a) = (a - a)g(a)$$

$$p(a) = 0 \times g(a)$$

$$\Rightarrow p(a) = 0.$$

Let us now see how the factor theorem is used in solving problems.

Example 1: Examine whether $x + 2$ is a factor of $x^3 + 4x^2 + 2x - 4$.

Let $p(x) = x^3 + 4x^2 + 2x - 4$.

By factor theorem $x + 2$ is a factor of $p(x)$, if $p(-2) = 0$

$$p(x) = x^3 + 4x^2 + 2x - 4$$

$$\begin{aligned} p(-2) &= (-2)^3 + 4(-2)^2 + 2(-2) - 4 \\ &= -8 + 16 - 4 - 4 \\ &= 0. \end{aligned}$$

So $x + 2$ is a factor of $x^3 + 4x^2 + 2x - 4$.

Example 2: Find the value of k , if $x - 1$ is a factor of $x^3 + kx^2 + 2x - 4$.

$$\text{Let } p(x) = x^3 + kx^2 + 2x - 4.$$

By factor theorem if $x - 1$ is a factor of $p(x)$, $p(1) = 0$

$$p(x) = x^3 + kx^2 + 2x - 4$$

$$p(1) = 1^3 + k(1)^2 + 2(1) - 4$$

$$0 = 1 + k + 2 - 4$$

$$0 = -1 + k$$

$$\Rightarrow k = 1.$$

Let us learn to factorise a polynomial of type $ax^2 + bx + c$ by splitting the middle term

Let the factors of $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$

$$\text{So } ax^2 + bx + c = (px + q)(rx + s)$$

$$ax^2 + bx + c = prx^2 + (ps + qr)x + qs$$

Equating the coefficients of the like terms, we get

$$a = pr \quad \text{_____ (i)}$$

$$b = ps + qr$$

$$c = qs \quad \text{_____ (ii)}$$

So, $ac = (pr)(qs)$ [Multiplying step (i) and (ii)]

This shows that b is the sum of two numbers ps and qr and ac is the product of the same two numbers ps and qr .

Let us use this fact to factorise a quadratic polynomial.

Example 3: Factorise $2x^2 + 7x + 6$

First identify two numbers whose sum is 7 and product is 2×6 , that is 12.

We know that $3 + 4 = 7$ and $3 \times 4 = 12$.

So writing $7x$ as $3x + 4x$

$$\begin{aligned} 2x^2 + 7x + 6 &= 2x^2 + 3x + 4x + 6 \\ &= x(2x + 3) + 2(2x + 3) \\ &= (x + 2)(2x + 3) \end{aligned}$$

Alternate Method: We can solve the above example using the factor theorem as follows.

$$2x^2 + 7x + 6 = 2\left(x^2 + \frac{7}{2}x + 3\right) = 2p(x), \text{ (say).}$$

If a and b are zeros of $p(x)$, then

$$2x^2 + 7x + 6 = 2(x - a)(x - b).$$

$$2x^2 + 7x + 6 = 2x^2 - 2x(a + b) + 2ab \quad (1)$$

Here, $ab = 3$ and $a + b = \frac{-7}{2}$ (By comparing the coefficients of like terms on both sides of (1))

Some possibilities for a and b may be $\pm 2, \pm \frac{3}{2}$

By trial, we find that $p(-2) = 0$ and $p(\frac{-3}{2}) = 0$

So, $(x + 2)$ and $(x + \frac{3}{2})$ are factors of $p(x)$.

$$\begin{aligned} 2x^2 + 7x + 6 &= 2(x + 2)(x + \frac{3}{2}) \\ &= 2(x + 2)(\frac{2x + 3}{2}) \\ &= (x + 2)(2x + 3) \end{aligned}$$

In this example the splitting method is more convenient.

But the splitting method cannot be used directly to factorise a cubic polynomial. We need to use the factor theorem to find at least one factor first.

Example 4: Factorise $2x^3 + 3x^2 - 11x - 6$

$$\text{Let } p(x) = 2x^3 + 3x^2 - 11x - 6$$

Now as $p(x)$ is a cubic polynomial, let us assume that

$$2x^3 + 3x^2 - 11x - 6 = (2x - a)(x - b)(x - c) \text{ for some real numbers } a, b \text{ and } c.$$

[Notes to the teacher: The teacher may remind the students that the coefficient of x^3 is 2 which is a prime number. So there will be at least one factor of the type $(2x - a)$]

In the above equation, the constant term in the RHS will be $-abc$

$$\text{So we get } -6 = -abc$$

The factors of the constant term -6 are $\pm 1, \pm 2, \pm 3$ and ± 6

So the values of a, b and c may be $\pm 1, \pm 2, \pm 3$ and ± 6

So we find the value of $p(x)$ by substituting the above values for x one by one.

By trial, we find that $p(2) = 0$

So $x - 2$ is a factor of $p(x)$.

Now $2x^3 + 3x^2 - 11x - 6 = 2x^3 - 4x^2 + 7x^2 - 14x + 3x - 6$ (Splitting the terms in such a way so as to get $(x - 2)$ as a common factor)

$$\begin{aligned} 2x^3 + 3x^2 - 11x - 6 &= 2x^3 - 4x^2 + 7x^2 - 14x + 3x - 6 \\ &= 2x^2(x - 2) - 7x(x - 2) + 3(x - 2) \\ &= (x - 2)(2x^2 - 7x + 3) \text{ [This can also be obtained by dividing } p(x) \text{ by } (x - 2)] \end{aligned}$$

Now, factorising $(2x^2 - 7x + 3)$ by splitting the middle term (or using factor theorem)

$$\begin{aligned} 2x^2 - 7x + 3 &= 2x^2 - 6x - 1x + 3 \\ &= 2x(x - 3) - 1(x - 3) \\ &= (x - 3)(2x - 1) \end{aligned}$$

$$\begin{aligned}\therefore 2x^3 + 3x^2 - 11x - 6 &= (x - 2)(2x^2 - 7x + 3) \\ &= (x - 2)(x - 3)(2x - 1)\end{aligned}$$

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Polynomials	Factor theorem(Factorisation of polynomial)

Sri Sathya Sai Central Trust

QA_Applying Factor Theorem

Objective: Students will be able to apply the Factor theorem to factorise a polynomial.

Teaching Aids: PPT, Worksheet.

Notes to the teacher: The teacher may make use of the following questions to enhance the ability of students to apply the knowledge of the factor theorem to solve problems.

Q I. Choose the correct answer:

1. When a polynomial $p(x)$ is divided by the binomial $2x - 3$, the remainder is given by
- $p(3)$
 - $p(-3)$
 - $p(\frac{-3}{2})$
 - $p(\frac{3}{2})$

Answer: d. $p(\frac{3}{2})$

2. If $r(x)$ is the remainder when $p(x)$ is divided by $g(x)$, then which of the following statements is true?
- Degree of $r(x) \leq$ degree of $g(x)$
 - Degree of $r(x) <$ degree of $g(x)$
 - Degree of $r(x) \geq$ degree of $g(x)$
 - Degree of $r(x) >$ degree of $g(x)$

Answer: b. Degree of $r(x) <$ degree of $g(x)$

3. Given that $g(x)$ is a factor of $p(x)$, which of the following statements is true?
- When $p(x)$ is divided by $g(x)$, the remainder is zero.
 - When $g(x)$ is divided by $p(x)$, the remainder is zero.
 - When $p(x)$ is divided by $g(x)$, the remainder is not defined.
 - None of the above.

Answer: a. When $p(x)$ is divided by $g(x)$, the remainder is zero.

4. $(x - 1)$ is a factor of which of the following polynomials?
- $x^3 + 2x^2 + 2x + 1$
 - $x^3 - 2x^2 + 2x - 1$
 - $3x^3 + 2x^2 + x + 1$
 - $3x^3 - 2x^2 + x - 1$

Answer: b. $x^3 - 2x^2 + 2x - 1$

5. To factorise $x^2 + 2x - 15$, which of the following is the correct way to split the middle

term, $2x$?

- a. $3x - x$
- b. $-x - x$
- c. $4x - 2x$
- d. $5x - 3x$

Answer: d. $5x - 3x$

Q II. If $p(x) = 9x^2 - 3x - 20$ and $g(x) = 3x - 5$, check whether $p(x)$ is a multiple of $g(x)$.

Solution:

When $p(x)$ is divided by $3x - 5$, the remainder is $p(\frac{5}{3})$

$$p(x) = 9x^2 - 3x - 20$$

$$\begin{aligned} p\left(\frac{5}{3}\right) &= 9\left(\frac{5}{3}\right)^2 - 3\left(\frac{5}{3}\right) - 20 \\ &= 9\left(\frac{25}{9}\right) - 3\left(\frac{5}{3}\right) - 20 \\ &= 25 - 5 - 20 \\ &= 0. \end{aligned}$$

When $p(x)$ is divided by $g(x)$, the remainder is zero. So $p(x)$ is a multiple of $g(x)$.

Q III. If $p(x) = 6x^3 + 7x^2 - 18x + 6$ and $g(x) = 3x - 1$, use the factor theorem to find whether $g(x)$ is a factor of $p(x)$.

Solution:

When $p(x)$ is divided by $3x - 1$, the remainder is $p(\frac{1}{3})$

$$\begin{aligned} p(x) &= 6x^3 + 7x^2 - 18x + 6 \\ p\left(\frac{1}{3}\right) &= 6\left(\frac{1}{3}\right)^3 + 7\left(\frac{1}{3}\right)^2 - 18\left(\frac{1}{3}\right) + 6 \\ &= 6\left(\frac{1}{27}\right) + 7\left(\frac{1}{9}\right) - 18\left(\frac{1}{3}\right) + 6 \\ &= \frac{6}{27} + \frac{7}{9} - \frac{18}{3} + 6 \\ &= \frac{2}{9} + \frac{7}{9} - 6 + 6 \\ &= \frac{9}{9} + 0 \\ &= 1 \end{aligned}$$

When $p(x)$ is divided by $g(x)$, the remainder is 1. So, $g(x)$ is not a factor of $p(x)$.

Q IV. Find the value of k , if $x + 1$ is a factor of $5x^3 - 3x^2 + kx + 10$.

Solution:

$$\text{Let } p(x) = 5x^3 - 3x^2 + kx + 10.$$

$x + 1$ is a factor of $p(x)$. So, $p(-1) = 0$.

$$p(x) = 5x^3 - 3x^2 + kx + 10$$

$$p(-1) = 5(-1)^3 - 3(-1)^2 + k(-1) + 10$$

$$0 = -5 - 3 - k + 10$$

$$0 = 2 - k$$

$$k = 2.$$

Answer: k = 2

Q V. Factorise $12x^3 - 19x^2 + 8x - 1$

Solution: Let $p(x) = 12x^3 - 19x^2 + 8x - 1$

The factors of the constant term -1 are ± 1

By trial, we find that $p(1) = 0$

So $x - 1$ is a factor of $p(x)$.

$12x^3 - 19x^2 + 8x - 1 = 12x^3 - 12x^2 - 7x^2 + 7x + x - 1$ (Splitting the terms to get $(x - 1)$ as a common factor)

$$12x^3 - 19x^2 + 8x - 1 = 12x^3 - 12x^2 - 7x^2 + 7x + x - 1$$

$$= 12x^2(x - 1) - 7x(x - 1) + 1(x - 1)$$

$$= (x - 1)(12x^2 - 7x + 1) \text{ [This can also be obtained by dividing } p(x) \text{ by } (x -$$

1)]

$$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$$

$$= 3x(4x - 1) - 1(4x - 1)$$

$$= (3x - 1)(4x - 1)$$

$$\therefore 12x^3 - 19x^2 + 8x - 1 = (x - 1)(12x^2 - 7x + 1)$$

$$= (x - 1)(3x - 1)(4x - 1)$$

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Questions to Assess	Polynomials	Factor theorem(Factorisation of polynomial)

Objective: Students will be able to apply identities in expansion and factorisation of quadratic polynomials.

Teaching Aids: PPT

Notes to the teacher: The teacher may encourage the students to recollect the identities learnt in the earlier classes before enabling them to derive and verify the square of a trinomial.

Consider the equation, $2x + 3 = 5$

This equation is true only when x takes the value 1.

Now consider the equation $(x + y)^2 = x^2 + 2xy + y^2$

Let us substitute $x = 2$ and $y = 3$ in the above equation.

$$\text{LHS} = (2 + 3)^2 = 5^2 = 25$$

$$\text{RHS} = 2^2 + 2(2)(3) + 3^2$$

$$= 4 + 12 + 9$$

$$= 25.$$

We find that LHS = RHS.

(Notes to the teacher: The teacher may ask the students to substitute some more values for x and y and verify the identity)

The above equation is true for all real values of x and y .

Such an equation is called an algebraic identity.

An algebraic identity is an equation which is true for all values of the variables contained in it.

Let us recall the four identities studied in the earlier classes.

$$\text{Identity I: } (x + y)^2 = x^2 + 2xy + y^2$$

$$\text{Identity II: } (x - y)^2 = x^2 - 2xy + y^2$$

$$\text{Identity III: } x^2 - y^2 = (x + y)(x - y)$$

$$\text{Identity IV: } (x + a)(x + b) = x^2 + (a + b)x + ab.$$

Now let us see how these identities are used in computations and factorisations.

Example 1

Find the product using the appropriate identity: $(x + 5)(x + 5)$

Solution:

$$(x + 5)(x + 5) = (x + 5)^2$$

Using identity $(x + y)^2 = x^2 + 2xy + y^2$ we get:

$$(x + 5)^2 = x^2 + 2x(5) + 5^2$$

$$= x^2 + 10x + 25$$

Answer: $(x + 5)(x + 5) = x^2 + 10x + 25$

Example 2

Find the product using the appropriate identity: $(2x + 3)(2x + 5)$

Solution:

The given product is comparable to the LHS of Identity $(x + a)(x + b) = x^2 + (a + b)x + ab$.

$$(2x + 3)(2x + 5) = (2x)^2 + (3 + 5)(2x) + (3)(5).$$

$$= 4x^2 + (8)(2x) + 15$$

$$= 4x^2 + 16x + 15$$

Answer:

$$(2x + 3)(2x + 5) = 4x^2 + 16x + 15$$

Example 3

Find the value of 103×97 using a suitable algebraic identity

$$103 = 100 + 3 \text{ and } 97 = 100 - 3$$

$$\text{So } 103 \times 97 = (100 + 3)(100 - 3)$$

$$(100 + 3)(100 - 3) = 100^2 - 3^2 \text{ [using identity } x^2 - y^2 = (x + y)(x - y)]$$

$$= 10000 - 9$$

$$= 9991$$

Answer:

$$103 \times 97 = 9991$$

Example 4

Find the value of 98^2 using a suitable algebraic identity

$$98 = 100 - 2$$

$$98^2 = (100 - 2)^2$$

$$(100 - 2)^2 \text{ is comparable to the LHS of Identity } (x - y)^2 = x^2 - 2xy + y^2$$

$$(100 - 2)^2 = 100^2 - 2(100)(2) + 2^2$$

$$= 10000 - 400 + 4$$

$$= 9604$$

Example 5

Factorise: $64a^2 - 48ab + 9b^2$

Solution

One of the first things to do before attempting to factorise a polynomial is to see if there is any common factor for all the terms in the given polynomial. In this case we observe that there is no common factor for all the terms. So we use the suitable identity to factorise.

We observe that $64a^2 = (8a)^2$, $9b^2 = (3b)^2$ and $48ab = 2(8a)(3b)$

$$64a^2 - 48ab + 9b^2 = (8a)^2 - 2(8a)(3b) + (3b)^2$$

$$= (8a - 3b)^2 \text{ [by Identity } (x - y)^2 = x^2 - 2xy + y^2 \text{]}$$

Answer:

$$64a^2 - 48ab + 9b^2 = (8a - 3b)(8a - 3b)$$

Example 6

Factorise: $27a^2 - 12b^2$

Solution:

We observe that 3 is a common factor for both the terms. So we proceed as follows.

$$27a^2 - 12b^2 = 3(9a^2 - 4b^2)$$

Now we factorise $(9a^2 - 4b^2)$ using identity III

$$(9a^2 - 4b^2) = (3a)^2 - (2b)^2$$

$$= (3a + 2b)(3a - 2b)$$

$$27a^2 - 12b^2 = 3(9a^2 - 4b^2)$$

$$= 3(3a + 2b)(3a - 2b)$$

Answer:

$$27a^2 - 12b^2 = 3(3a + 2b)(3a - 2b)$$

Now we shall derive an identity to find the square of a trinomial using identity $(x + y)^2 = x^2 + 2xy + y^2$.

Find $(x + y + z)^2$

Let $x + y = t$

$$(x + y + z)^2 = (t + z)^2$$

$$(t + z)^2 = t^2 + 2tz + z^2$$

$$(x + y + z)^2 = (x + y)^2 + 2(x + y)z + z^2$$

$$= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$$

$$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx \text{ (Rearranging the terms)}$$

$$\text{Identity V: } (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

In the above identity, the RHS is called the expanded form of the identity while the LHS is the factorisation of the RHS.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Polynomials	Standard identities and their applications

SA_Verify Square of Trinomial

Objective: The student will be able to verify experimentally, the identity involving the square of a trinomial, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$.

Teaching Aids: PPT

Notes to the Teacher: The teacher may introduce the activity by a brief recap of the identities involving squares of a binomial.

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

Let's perform an experiment to verify this algebraic identity.

Aim: To verify experimentally, the identity for expansion of $(a + b + c)^2$

Resources Required: Every student must have a pencil, a ruler, graph paper A4 size, a pair of scissors, an adhesive, a sheet of A4 size white paper (or the notebook used by the students for recording math lab activities) and coloured pencils).

Setting for the Activity: Indoor (classroom)

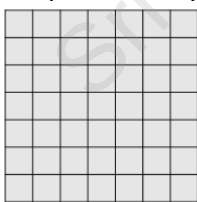
Type of Activity: Individual, math lab activity

Preparation of Activity: There need not be any change in the seating arrangement of the students in the classroom. To ensure that the students have the necessary materials for conducting the activity, they may be informed about it in advance.

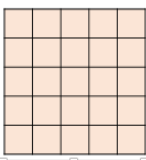
Role of the Teacher: Demonstrator/ Facilitator/ Observer

Procedure: The teacher may demonstrate the following steps of the activity using the paper cutting and pasting or black/white board or the PPT provided with this asset.

- A. From the graph paper let's cut out the following:
1. A square of any side length ' a ' units (say $a = 7$ cm). Colour it using a coloured pencil.



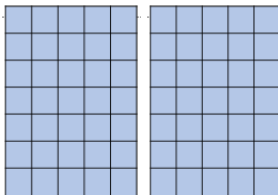
2. A square of any side length ' b ' units, such that $b < a$ (say $b = 5$ cm). Colour it with a different colour.



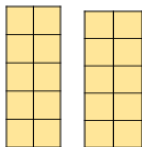
3. A square of side length ' c ' units, such that $c < b$ (say $c = 2$ cm). Colour it with a third colour.



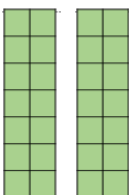
4. Two rectangles of dimension $a \times b$. Colour them using another coloured pencil.



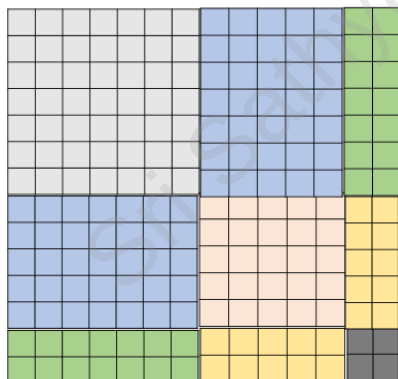
5. Two rectangles of dimension $b \times c$. Colour them using a pencil colour different from the ones used in the above steps.



6. Two rectangles of dimension $a \times c$. Now use the sixth colour for colouring these two cutouts.



- B. Assemble the cutouts obtained in step A on a sheet of paper so as to form a square. Paste the cutouts in the form of a square.



[Notes to the teacher: The teacher may encourage the students and give them some time to assemble the cutouts to form a square, before disclosing the method to them.]

Observation:

[Notes to the teacher: The teacher may write the following questions on the black/white board (or use the PPT attached with this asset) and students may record the observations individually in their notebook, based on the value of a , b and c used by them for the cutouts. This may be

followed by a discussion in the class about the observations of the students and the outcome of the experiment. Then the teacher may sum up their observations using the PPT.]

Using the values of a , b and c taken for the experiment, note down the following observations:

The area of square with side ' a ' units = $a^2 = 7^2 = 49$ sq units

The area of square with side ' b ' units = $b^2 = 5^2 = 25$ sq units

The area of square with side ' c ' units = $c^2 = 2^2 = 4$ sq units

The area of each rectangle with sides ' a ' and ' b ' units = $ab = 7 \times 5 = 35$ sq units

The area of each rectangle with sides ' b ' and ' c ' units = $bc = 5 \times 2 = 10$ sq units

The area of each rectangle with sides ' a ' and ' c ' units = $ac = 7 \times 2 = 14$ sq units

Side of square obtained in step B = $a + b + c = 7 + 5 + 2 = 14$ units

Its area = $(a + b + c)^2 = (7 + 5 + 2)^2 = 14^2 = 196$ sq units -----(1)

Total area of all the cutouts = area of the square obtained in step B

$$= a^2 + b^2 + c^2 + ab + ab + bc + bc + ac + ac$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

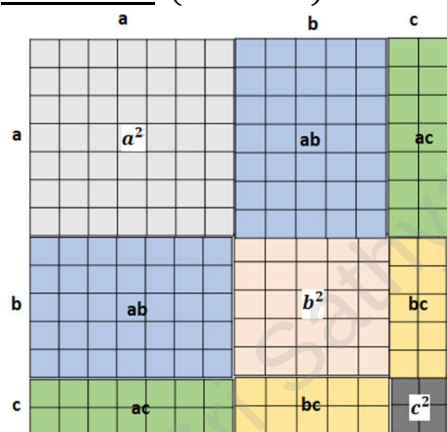
$$= 49 + 25 + 4 + 2(7)(5) + 2(5)(2) + 2(7)(2)$$

$$= 78 + 70 + 20 + 28$$

$$= 196 \text{ sq units} \text{ -----(2)}$$

So, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$ [From steps (1) and(2)]

Conclusion: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$



Hence, the identity for expansion of $(a + b + c)^2$ is verified experimentally.

Follow-up activity: The students will take up questions and apply the above identity to write the expansion of the square of trinomials and factorisation of polynomials.

The following question may be discussed and later answered by the students in their notebook.

Example 1

Write the expansion of:

a. $(x + 2y + z)^2$

b. $(3p - q - 2r)^2$

Suggested answer:

Using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$, we get the expansion of:

- a. $(x + 2y + z)^2$
 $= x^2 + (2y)^2 + z^2 + 2(x)(2y) + 2(2y)(z) + 2xz$
 $= x^2 + 4y^2 + z^2 + 4xy + 4yz + 2xz$
- b. $(3p - q - 2r)^2$
 $= (3p)^2 + (-q)^2 + (-2r)^2 + 2(3p)(-q) + 2(-q)(-2r) + 2(3p)(-2r)$
 $= 9p^2 + q^2 + 4r^2 - 6pq + 4qr - 12pr$

Example 2

Factorise the following:

- a. $4x^2 + y^2 + 4z^2 - 4xy - 4yz + 8xz$
- b. $16a^2 + 4b^2 + 9c^2 - 16ab + 12bc - 24ac$

Suggested answer:

- a. $4x^2 + y^2 + 4z^2 - 4xy - 4yz + 8xz$
 $= (2x)^2 + (-y)^2 + (2z)^2 + 2(2x)(-y) + 2(-y)(2z) + 2(2x)(2z)$
 $= (2x - y + 2z)^2$ (using the identity verified above)

[Notes to the teacher: The teacher may point out here that the – sign of the fourth and the fifth terms of the question is either because of the – sign with y or the – sign with both x and z . So, this answer can also be written as

$$(-2x)^2 + (y)^2 + (-2z)^2 + 2(-2x)(y) + 2(y)(-2z) + 2(-2x)(-2z) = (-2x + y - 2z)^2]$$

- b. $16a^2 + 4b^2 + 9c^2 - 16ab + 12bc - 24ac$
 $= (-4a)^2 + (2b)^2 + (3c)^2 + 2(-4a)(2b) + 2(2b)(3c) + 2(-4a)(3c)$
 $= (-4a + 2b + 3c)^2$

Precautions:

1. The square and rectangles cutouts should be measured and cut accurately. Inaccurate measurement may not give the desired result.

Troubleshooting:

The teacher may keep some extra materials required for the activity, in case of need.

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Suggested Activity	Polynomials	Standard identities and their applications

Sri Sathya Sai Central Trust

MS_Cubic Polynomial Identities

Objective: Students will be able to apply the identities involving cubic polynomials in expansion and factorisation.

Teaching Aids: PPT

Notes to the teacher: The teacher may encourage the students to verify the identities involving cubic polynomials and explain their application in solving problems.

Let us quickly recall the first five algebraic identities involving quadratic polynomials.

$$\text{Identity I: } (x + y)^2 = x^2 + 2xy + y^2$$

$$\text{Identity II: } (x - y)^2 = x^2 - 2xy + y^2$$

$$\text{Identity III: } x^2 - y^2 = (x + y)(x - y)$$

$$\text{Identity IV: } (x + a)(x + b) = x^2 + (a + b)x + ab.$$

$$\text{Identity V: } (x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

We shall now use identity I to obtain $(x + y)^3$

$$\begin{aligned}(x + y)^3 &= (x + y)(x + y)^2 \\&= (x + y)(x^2 + 2xy + y^2) \\&= x(x^2 + 2xy + y^2) + y(x^2 + 2xy + y^2) \\&= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3 \\&= x^3 + 3x^2y + 3xy^2 + y^3 \\&= x^3 + y^3 + 3xy(x + y)\end{aligned}$$

So we get

$$\text{Identity VI: } (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

Replacing y with $-y$ in the above equation, we get,

$$\text{Identity VII: } (x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

$$\text{Identity VI: } (x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\text{Identity VII: } (x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

Let us verify these identities.

Let's substitute $x = 3$ and $y = 2$ in Identity VI.

$$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$$

$$\begin{aligned}\text{LHS} &= (x + y)^3 \\&= (3 + 2)^3 \\&= 5^3\end{aligned}$$

$$\begin{aligned}
 &= 125 \\
 \text{RHS} &= x^3 + y^3 + 3xy(x + y) \\
 &= 3^3 + 2^3 + 3(3)(2)(3 + 2) \\
 &= 27 + 8 + 18(5) \\
 &= 27 + 8 + 90 \\
 &= 125
 \end{aligned}$$

LHS = RHS.

So the Identity is verified to be true.

[Notes to the teacher: The teacher may encourage the students to verify Identity VII for some more values of x and y])

Example 1

Write in the expanded form: $(2a + 3b)^3$

The given expression is comparable to the LHS of Identity VI $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$ where $x = 2a$ and $y = 3b$

$$\begin{aligned}
 (2a + 3b)^3 &= (2a)^3 + (3b)^3 + 3(2a)(3b)(2a + 3b) \\
 &= 8a^3 + 27b^3 + 18ab(2a + 3b) \\
 &= 8a^3 + 27b^3 + 36a^2b + 54ab^2
 \end{aligned}$$

Answer:

$$(2a + 3b)^3 = 8a^3 + 27b^3 + 36a^2b + 54ab^2$$

(Note to the teacher: The teacher may encourage the students to do example 2 on their own.)

Example 2

Write in the expanded form: $(3a - 5b)^3$

The given expression is comparable to the LHS of Identity VII where $x = 3a$ and $y = 5b$

$$\begin{aligned}
 (x - y)^3 &= x^3 - y^3 - 3xy(x - y) \\
 (3a - 5b)^3 &= (3a)^3 - (5b)^3 - 3(3a)(5b)(3a - 5b) \\
 &= 27a^3 - 125b^3 - 45ab(3a - 5b) \\
 &= 27a^3 - 125b^3 - 135a^2b + 225ab^2
 \end{aligned}$$

Answer:

$$(3a - 5b)^3 = 27a^3 - 125b^3 - 135a^2b + 225ab^2$$

Example 3

Evaluate using a suitable identity: 201^3

$$201^3 = (200 + 1)^3$$

This is comparable to the LHS of Identity VI where $x = 200$ and $y = 1$

$$\begin{aligned}
 (x + y)^3 &= x^3 + y^3 + 3xy(x + y) \\
 (200 + 1)^3 &= 200^3 + 1^3 + 3(200)(1)(200 + 1) \\
 &= 8000000 + 1 + 600(200 + 1)
 \end{aligned}$$

$$= 8000001 + 120000 + 600$$

$$= 8120601$$

Answer:

$$201^3 = 8120601$$

(Note to the teacher: The teacher may encourage the students to do Example 4 by themselves)

Example 4

Evaluate using a suitable Identity: 399^3

$$399^3 = (400 - 1)^3$$

This is comparable to the LHS of Identity VII where $x = 400$ and $y = 1$

$$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$$

$$(400 - 1)^3 = 400^3 - 1^3 - 3(400)(1)(400 - 1)$$

$$= 64000000 - 1 - 1200(400 - 1)$$

$$= 64000000 - 1 - 480000 + 1200$$

$$= 64001200 - 480001$$

$$= 63521199$$

Answer:

$$399^3 = 63521199$$

Example 5

Factorise: $8a^3 + 125b^3 + 60a^2b + 150ab^2$

$$8a^3 + 125b^3 + 60a^2b + 150ab^2 = (2a)^3 + (5b)^3 + 3(2a)^2(5b) + 3(2a)(5b)^2$$

$$= (2a)^3 + (5b)^3 + 3(2a)^2(5b) + 3(2a)(5b)^2$$

$$= (2a + 5b)^3 \text{ [using Identity VI]}$$

$$= (2a + 5b)(2a + 5b)(2a + 5b)$$

Answer:

$$8a^3 + 125b^3 + 60a^2b + 150ab^2 = (2a + 5b)(2a + 5b)(2a + 5b)$$

Now let us arrive at one more Identity by finding the following product

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Expanding the above, we get,

$$x(x^2 + y^2 + z^2 - xy - yz - zx) + y(x^2 + y^2 + z^2 - xy - yz - zx) + z(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + xy^2 + xz^2 - x^2y - xyz - x^2z + x^2y + y^3 + yz^2 - xy^2 - y^2z - xyz + x^2z + y^2z + z^3 - xyz - yz^2 - xz^2$$

$$= x^3 + y^3 + z^3 - 3xyz \text{ (Combining the like terms)}$$

$$\text{Hence, } (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$$

$$\text{Identity VIII: } (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$$

Let us verify Identity VIII substituting $x = 1$ and $y = -1$ and $z = 2$

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$$

$$\begin{aligned} \text{LHS} &= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \\ &= [1 + (-1) + 2][1^2 + (-1)^2 + 2^2 - 1(-1) - (-1)(2) - (2)(1)] \\ &= (2)(1 + 1 + 4 + 1 + 2 - 2) \\ &= 2 \times 7 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= x^3 + y^3 + z^3 - 3xyz \\ &= 1^3 + (-1)^3 + 2^3 - 3(1)(-1)(2) \\ &= 1 - 1 + 8 + 6 \\ &= 14. \end{aligned}$$

LHS = RHS.

So the identity is verified to be true.

Now, if $x + y + z = 0$, let's see what happens to Identity VIII.

$$(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) = x^3 + y^3 + z^3 - 3xyz$$

$$\begin{aligned} 0 \times (x^2 + y^2 + z^2 - xy - yz - zx) &= x^3 + y^3 + z^3 - 3xyz \\ 0 &= x^3 + y^3 + z^3 - 3xyz \\ 3xyz &= x^3 + y^3 + z^3 \\ x^3 + y^3 + z^3 &= 3xyz \end{aligned}$$

Remember:

$$\text{If } x + y + z = 0, \text{ then } x^3 + y^3 + z^3 = 3xyz$$

Example 6

Factorise: $27a^3 + 125b^3 + c^3 - 45abc$

We observe that $45abc$ can be written as $3(3a)(5b)c$. Further, $27a^3 = (3a)^3$ and $125b^3 = (5b)^3$

$$\text{So, } 27a^3 + 125b^3 + c^3 - 45abc = (3a)^3 + (5b)^3 + c^3 - 3(3a)(5b)c$$

We now factorise the given expression using Identity VIII.

$$\begin{aligned} 27a^3 + 125b^3 + c^3 - 45abc &= (3a)^3 + (5b)^3 + c^3 - 3(3a)(5b)c \\ &= (3a + 5b + c)[(3a)^2 + (5b)^2 + c^2 - (3a)(5b) - (5b)(c) - (3a)(c)] \\ &= (3a + 5b + c)(9a^2 + 25b^2 + c^2 - 15ab - 5bc - 3ac) \end{aligned}$$

Example 7

Without actually calculating the cubes find the value of $(-20)^3 + 3^3 + 17^3$

Solution:

Here $-20 + 3 + 17 = 0$

In Identity VIII, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

$$-20 + 3 + 17 = 0$$

$$\begin{aligned}\text{So, } (-20)^3 + 3^3 + 17^3 &= 3(-20)(3)(17) \\ &= -3060\end{aligned}$$

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Main Script	Polynomials	Standard identities and their applications

IA_Binomial expansion shortcut

Objective: The student will be able to appreciate the use of the Pascal triangle in writing the expansion of binomials with non negative integral power.

Teaching Aids: PPT

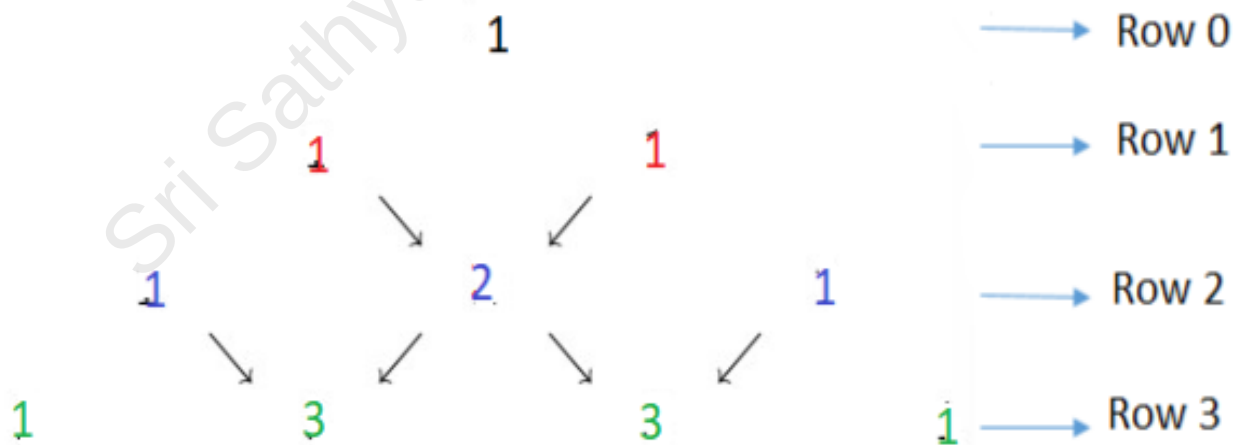
Notes to the teacher: The teacher may start with a brief recap of the polynomial identities.

The identities for expansion of $(x + y)^2$, $(x - y)^2$, $(x + y)^3$ and $(x - y)^3$ involve the square and cube of binomials.

Here's a shortcut to easily write the expansion of not only the square and the cube of a binomial but also for other powers of the binomial, it's the Pascal's triangle!

Pascal's triangle is a triangular arrangement of numbers, named after a French mathematician Blaise Pascal. It is used widely in algebra, probability theory and many other mathematical concepts.

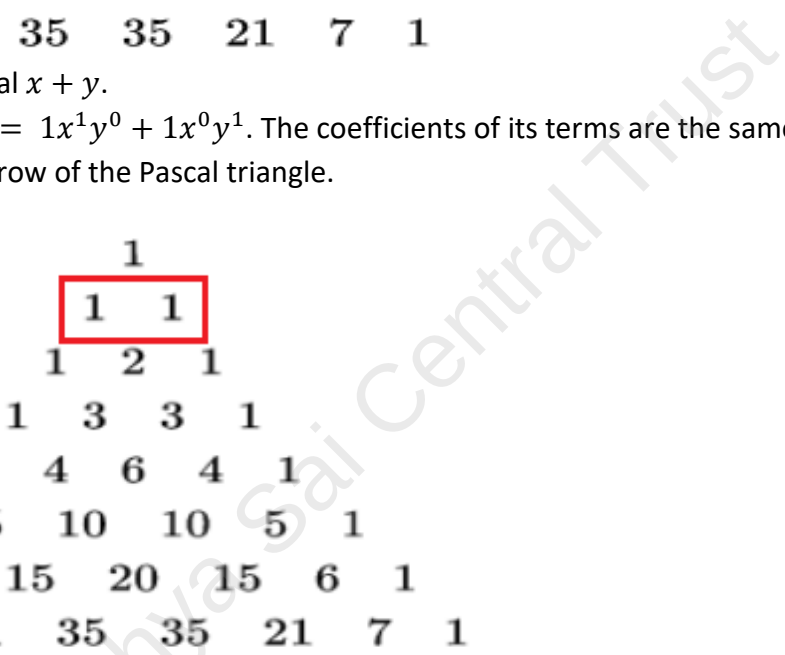
The Pascal's triangle is constructed by writing 1 in the topmost row which is counted as the row number 0. The rows after this are obtained by writing 1 at each end and the remaining numbers by adding the two terms of the row above lying on the right and left, as shown in the figure.



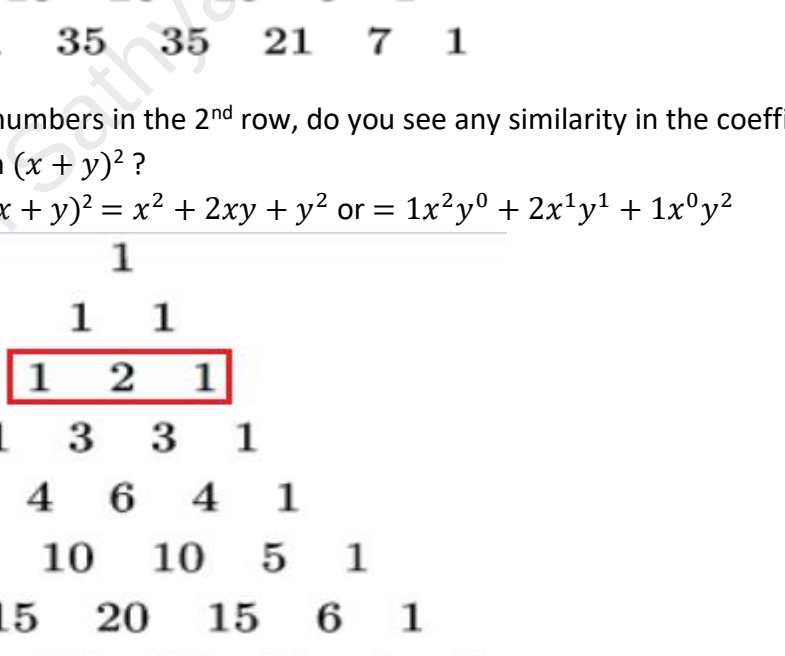
Continuing like this, we can generate infinite rows in this triangle.

Now let's know how to use the Pascal triangle to write the expansion of a binomial expression.

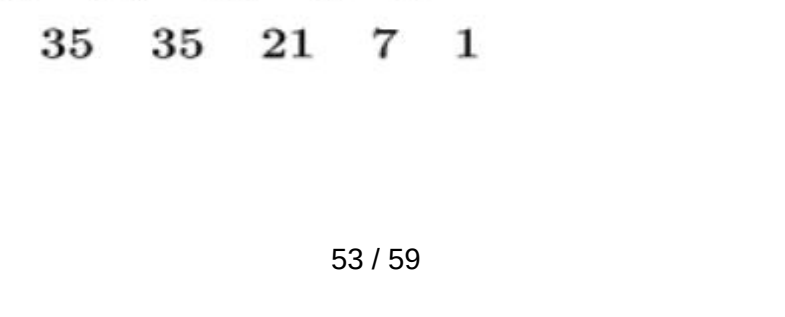
The Pascal triangle given below is constructed till the 7th row.



$(x + y)^1 = x + y = 1x^1y^0 + 1x^0y^1$. The coefficients of its terms are the same as the numbers in the 1st row of the Pascal triangle.



The expansion of $(x + y)^2 = x^2 + 2xy + y^2$ or $= 1x^2y^0 + 2x^1y^1 + 1x^0y^2$

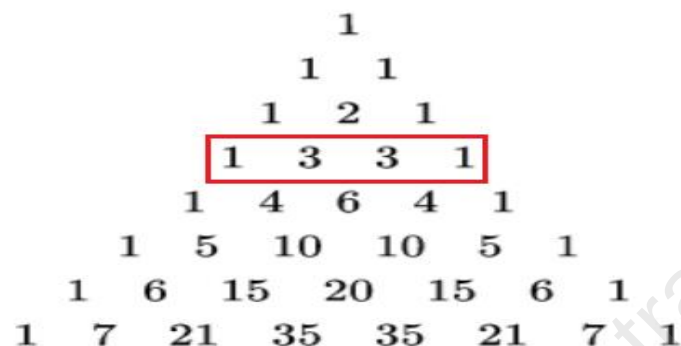


Yes, they are the same.

Notice that the powers of x (the first term of the binomial) decrease from 2 to 0 and the powers of the second term y increase from 0 to 2 as we move from the first to the last term of the expansion.

So, to write the expansion of $(x + 2y)^3$

1. Select the third row of the Pascal triangle beginning with 1, 3.



2. Write down the coefficients as the numbers in this row i.e. 1 3 3 1
3. Insert + sign between the coefficients.
4. Along with the coefficients, write the first and the second term of the binomial with their powers, such that the powers of x decrease from 3 to 0 and the powers of $2y$ increase from 0 to 3, as we move from left to right.

So, we write: $1x^3(2y)^0 + 3x^2(2y)^1 + 3x^1(2y)^2 + 1x^0(2y)^3$

The expansion is complete $(x + 2y)^3 = 1x^32^0y^0 + 3x^22^1y^1 + 3x^12^2y^2 + 1x^02^3y^3$

$$= x^3 + 6x^2y + 12xy^2 + 8y^3$$

Remember that the number of terms is always one more than the power of the binomial.

Here, the number of terms is $3 + 1 = 4$

So, now it's easy to write the expansion of $(x + y)^6$

Remember to select the row of the Pascal triangle starting with 1, 6 and get the expansion using the coefficients as 1 6 15 20 15 6 1.

$$\begin{aligned}(x + y)^6 &= 1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6 \\ &= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6\end{aligned}$$

While writing the expansion of the binomial $x - y$ with any power, the + and - signs are inserted between the terms alternately, starting from the first term.

$$\text{So, } (x - y)^3 = +x^3 - 3x^2y + 3xy^2 - y^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$\begin{aligned}\text{And } (x - y)^6 &= +x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6 \\ &= x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6\end{aligned}$$

To write the expansion of $(x \pm y)^n$:

-
- ★ Find the coefficients of the terms of its expansion from the row of the Pascal triangle beginning with 1, n .
 - ★ Insert + sign between the coefficients. For the binomial $(x - y)^n$, the – and + signs are inserted between the coefficients alternatingly, starting from the left.
 - ★ Along with each coefficient, write x and y with their powers, such that the power of x (first term of the binomial) decreases from n to 0 and the power of y (the second term of the binomial) increases from 0 to n as we move from the first to the last term of the expansion.

Remember that for expanding a binomial expression with very large powers, say 25, the use of Pascal triangle is not practical as the generation of so many rows becomes inconvenient. This difficulty is solved by a mathematical formula called the Binomial Theorem, about which you will learn in the higher classes.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Interesting Asides	Polynomials	Standard identities and their applications

QA_Applying Identities

Objective: Students will be able to apply identities for expansion and factorisation of polynomials.

Teaching Aids: PPT, Worksheet.

Notes to the teacher: The teacher may present the questions and allow the students to write or say the answers before revealing them.

QI. Fill in the blanks with the correct answer.

1. The factors of $\frac{25}{4}p^2 - \frac{1}{49}q^2$ are $(\frac{5}{2}p + \frac{1}{7}q)$ and _____.

Answer: $(\frac{5}{2}p - \frac{1}{7}q)$

2. The product $(m + 3)(m + 7)$ is _____.

Answer: $m^2 + 10m + 21$

3. The expanded form of $(\frac{1}{4}a - \frac{1}{2}b + 1)^2$ is _____.

Answer: $\frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$

4. $(2a + b + 3)(\text{_____}) = 8a^3 + b^3 + 27 - 18ab$.

Answer: $4a^2 + b^2 + 9 - 2ab - 3b - 6a$

QII. Evaluate the following using a suitable identity.

1. **105×106**

$$105 \times 106 = (100 + 5)(100 + 6)$$

$(100 + 5)(100 + 6)$ is comparable to the LHS of the following identity

$$(x + a)(x + b) = x^2 + (a + b)x + ab.$$

$$(100 + 5)(100 + 6) = 100^2 + (5 + 6)(100) + (5 \times 6)$$

$$= 10000 + 11 \times 100 + 30$$

$$= 10000 + 1100 + 30$$

$$= 11130$$

Answer: $105 \times 106 = 11130$

2. **$a^3 - b^3$ if $a - b = (-8)$ and $ab = (-12)$**

Solution

$a^3 - b^3$, $a - b$ and ab are comparable to the terms in the identity,

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(-8)^3 = a^3 - b^3 - 3 \times (-12) \times (-8)$$

$$\Rightarrow -512 = a^3 - b^3 - 288$$

$$\Rightarrow -512 + 288 = a^3 - b^3$$

$$\Rightarrow -224 = a^3 - b^3$$



Sri Sathya Sai
VIDYA VAHINI
श्री सत्यसाई विद्या वाहिनी

Sri Sathya Sai Central Trust

Annexure to Master Lesson Plan

What is the purpose of this Master lesson Plan?	This Master Lesson Plan (MLP) prepared by Sri Sathya Sai Vidya Vahini (SSSVV), covers the entire chapter for the given board. The MLP for the chapter given in the textbook is broken down into smaller modules known as assets. Any chapter can be taught using the 7 asset types in SSSVV methodology. Several different assets together complete the entire chapter. Depending on the requirement, each asset is supported by teaching aids such as animated presentation, audio, video, and learning aids such as worksheets. The MLP is for the teacher's preparation and the teaching aids are for use in the classroom.
How to use it?	You can prepare for your class with this MLP by following 3 simple steps: 1. Read your textbook; 2. Go through the information given in the assets (document & multimedia); 3. Prepare your teaching notes. Great! Now you are ready to transact the chapter.
Duration	Kindly note the duration given for each asset. Due care has been taken in planning the assets, to ensure the chapter is completed within the time specified by the Board. Note: Preparation time is not included in the asset duration. The teaching time depends on the duration of the teaching aids and is not impacted by the length of the MLP.
What is a 'Main Script' (MS) Asset?	The Main Script asset is there for explaining the main concepts of the chapter clearly. The information given in the textbook is simplified, organised and structured to give more clarity. Additionally, you may find a video or mnemonics or a graphic organiser to deepen the understanding of the concept.
How to teach using the MS asset?	Please use the blackboard, slides and interaction to develop the concept.
What is an 'Inquisitive Questions' (IQ) asset?	The Inquisitive Questions asset uses questions to promote higher order thinking like the n th why, what-if, new perspectives, cause-effect, and others, creating curiosity.
How to teach using the IQ asset?	You may use the questions to connect with the students, encourage exploration to engage them in the learning process. You may allow multiple responses and instead of rejecting any of the responses, ask why, before revealing the suggested answer or hints.
What is a 'Day to day Relevance' (DD) Asset?	The Day to Day Relevance asset helps students understand the practical relevance of every topic, making them eager participants in the classroom. If students connect the concepts to their environment, they would learn meaningfully without dislike or simply memorizing for exams.
How to teach using the DD	You may use the asset by asking questions about their experience and use it to establish why they are learning the topic. If necessary you may substitute with a

asset?	recent or local example.
What is an 'Interesting Aside' (IA) asset?	The Interesting Aside asset uses attention grabbing tidbits or anecdotes to bring joy and satisfy the child's hunger to know more. This is a quick/short asset intended to bring attention back to the main concept being taught.
How to teach using the IA asset?	You may use this asset to give a piece of interesting information relevant to the topic, without prolonged explanation/discussion. You may suggest appropriate books for further reading.
What is a 'Suggested Activity' (SA) Asset?	The Suggested Activity asset provides a detailed step by step procedure for the teacher to conduct a hands-on activity. This promotes Activity Based Learning.
How to teach using the SA asset?	You may use this asset to energise your students to learn by doing simple, fun-filled activities based on the topic. The asset includes - preparation, activity, and follow-up. You may engage the students in the preparation and follow-up stages to develop collaboration and responsibility. You could try it once, yourself, before the actual delivery to be more comfortable with it. Note: The duration mentioned in the asset is usually for the activity part only.
What is a 'Value Content' (VC) Asset?	The Value Content asset integrates Universal Human Values (such as empathy, inclusivity, respect for nature that translates into responsible behaviour) seamlessly into the curriculum, as the "End of education is character".
How to teach using the VC asset?	You may use this asset to engage with the students to bring out their innate values and connect with the intrinsic values in the topic/concept. The asset includes pointers on the specific pedagogical technique followed.
What is a 'Questions to Assess' (QA) asset?	The Questions to Assess asset uses an interactive approach to check learning of different types of learners, and provides feedback to the teacher for appropriate action. The questions cater to all the levels of Bloom's Taxonomy. Questions from 'Apply' level and above enable students to reflect on their learning.
How to teach using the QA asset?	You may go through the slides to understand its flow and know when to click for the answer (slides include suggested answers). While presenting in the class, you may display the question and encourage multiple responses, before revealing the answer. In some cases, this asset may include extra questions/worksheet that can be given as homework.