



## QA Applying Identities

**QI. Choose the correct answer:**

1. If  $a + \frac{1}{a} = 4$ , evaluate  $a^2 + \frac{1}{a^2}$ 
  - a) 16
  - b) 14
  - c) 8
  - d) 18
2. If  $p - \left(\frac{1}{p}\right) = 3$ , evaluate  $p^2 + \left(\frac{1}{p^2}\right)$ 
  - a) 9
  - b) 6
  - c) 7
  - d) 11
3. If  $a + b + c = 1$  and  $ab + bc + ca = -1$ , evaluate  $a^2 + b^2 + c^2$ 
  - a) 1
  - b) -1
  - c) 2
  - d) 3
4. If  $a, b$  and  $c$  are natural numbers such that  $a^2 + b^2 + c^2 = 14$  and  $ab + bc + ca = 11$ , evaluate  $a + b + c$ 
  - a) 36
  - b) 6
  - c) 14
  - d) 11
5. If  $a - b = 1$  and  $ab = 2$ , evaluate  $a^3 - b^3$ 
  - a) 1
  - b) 2
  - c) 7
  - d) -5

**QII. Answer the following using identities.**

1. If  $a^{1/3} + b^{1/3} + c^{1/3} = 0$ , prove that  $(a + b + c)^3 = 27abc$
2. If  $a^2 + b^2 + c^2 - ab - bc - ca = 0$ , prove that  $a^3 + b^3 + c^3 = 3abc$
3. Factorise:  $8a^3 + 27b^3 + 343c^3 - 126abc$
4. Factorise:  $8a^3 - 27b^3 - 36a^2b + 54ab^2$
5. Evaluate:  $526^2$
6. If  $a$  is a natural number and  $a - \frac{1}{a} = \frac{8}{3}$ , evaluate  $a + \frac{1}{a}$
7. Verify that  $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$



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## ANSWERS

QI.

1. b) 14
2. d) 11
3. d) 3 (Use the identity for  $(a + b + c)^2$ )
4. b) 6 (Use the identity for  $(a + b + c)^2$ )
5. c) 7 (Use the identity for  $(a - b)^3$ )

QII.

1. If  $a^{1/3} + b^{1/3} + c^{1/3} = 0$ , prove that  $(a + b + c)^3 = 27abc$

Solution

$$a^{1/3} + b^{1/3} + c^{1/3} = 0$$

$$\therefore a^{1/3} + b^{1/3} = -c^{1/3} \text{ ---(1)}$$

Cubing both sides,

$$a + b + 3a^{1/3}b^{1/3}(a^{1/3} + b^{1/3}) = -c$$

$$a + b + 3a^{1/3}b^{1/3}(-c^{1/3}) = -c \text{ [using (1)]}$$

$$a + b + c = 3a^{1/3}b^{1/3}c^{1/3}$$

Cubing both sides,

$$(a + b + c)^3 = 27abc$$

Hence proved.

2. If  $a^2 + b^2 + c^2 - ab - bc - ca = 0$ , prove that  $a^3 + b^3 + c^3 = 3abc$

Consider the identity

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$$

$$(a + b + c) \times 0 = a^3 + b^3 + c^3 - 3abc$$

$$0 = a^3 + b^3 + c^3 - 3abc$$

$$3abc = a^3 + b^3 + c^3$$

$$a^3 + b^3 + c^3 = 3abc$$

Hence proved.

3. Factorise using identities:  $8a^3 + 27b^3 + 343c^3 - 126abc$

$$8a^3 + 27b^3 + 343c^3 - 126abc = (2a)^3 + (3b)^3 + (7c)^3 - 3(2a)(3b)(7c)$$

$$(2a)^3 + (3b)^3 + (7c)^3 - 3(2a)(3b)(7c) \text{ is comparable to the identity,}$$

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$(2a)^3 + (3b)^3 + (7c)^3 - 3(2a)(3b)(7c)$$

$$= (2a + 3b + 7c)[(2a)^2 + (3b)^2 + (7c)^2 - (2a)(3b) - (3b)(7c) - (7c)(2a)]$$

$$= (2a + 3b + 7c)(4a^2 + 9b^2 + 49c^2 - 6ab - 21bc - 14ca)$$

$$\text{Answer: } 8a^3 + 27b^3 + 343c^3 - 126abc$$

$$= (2a + 3b + 7c)(4a^2 + 9b^2 + 49c^2 - 6ab - 21bc - 14ca)$$

4. Factorise using identities:  $8a^3 - 27b^3 - 36a^2b + 54ab^2$

$$8a^3 - 27b^3 - 36a^2b + 54ab^2 = (2a)^3 - (3b)^3 - 18ab[2a - 3b]$$

$$= (2a)^3 - (3b)^3 - 3(2a)(3b)[2a - 3b]$$

$$(2a)^3 - (3b)^3 - 3(2a)(3b)[2a - 3b] \text{ is comparable to the identity}$$

$$x^3 - y^3 - 3xy(x - y) = (x - y)^3$$

$$(2a)^3 - (3b)^3 - 3(2a)(3b)[2a - 3b] = (2a - 3b)^3$$



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**Answer:**

$$8a^3 - 27b^3 - 36a^2b + 54ab^2 = (2a - 3b)(2a - 3b)(2a - 3b)$$

### 5. Solution

Expanded form of 526 = 500 + 20 + 6

$$526^2 = (500 + 20 + 6)^2$$

$(500 + 20 + 6)^2$  is comparable to  $(x + y + z)^2$

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$$

$$\begin{aligned}(500 + 20 + 6)^2 &= 500^2 + 20^2 + 6^2 + 2 \times 500 \times 20 + 2 \times 20 \times 6 + 2 \times 6 \times 500 \\ &= 250000 + 400 + 36 + 20000 + 240 + 6000 \\ &= 276676.\end{aligned}$$

$$\text{Answer: } 526^2 = 276676.$$

### 6. Given: $a$ is a natural number and $a - \frac{1}{a} = \frac{8}{3}$ , evaluate $a + \frac{1}{a}$

To evaluate:  $a + \frac{1}{a}$

To find  $a + \frac{1}{a}$ , we need the value of  $a^2 + \frac{1}{a^2}$

$a - \frac{1}{a}$  and  $a^2 + \frac{1}{a^2}$  are comparable to the terms in the following identity.

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$\left(a - \frac{1}{a}\right)^2 = a^2 - 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$$

$$\left(\frac{8}{3}\right)^2 = a^2 - 2 + \frac{1}{a^2}$$

$$\frac{64}{9} + 2 = a^2 + \frac{1}{a^2}$$

$$\frac{64+18}{9} = a^2 + \frac{1}{a^2}$$

$$\frac{82}{9} = a^2 + \frac{1}{a^2}$$

$$a^2 + \frac{1}{a^2} = \frac{82}{9} \text{----(1)}$$

Now  $a^2 + \frac{1}{a^2}$  and  $a + \frac{1}{a}$  are comparable to the terms in the following identity.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + 2a\left(\frac{1}{a}\right) + \left(\frac{1}{a}\right)^2$$

$$= a^2 + 2 + \frac{1}{a^2}$$

$$= \frac{82}{9} + 2 \text{ [Substituting from (1)]}$$



## QA Applying Identities



$$\left(a + \frac{1}{a}\right)^2 = \frac{100}{9}$$

$$a + \frac{1}{a} = \pm \frac{10}{3}$$

But it is given that  $a$  is a natural number.

$$\text{So } a + \frac{1}{a} \neq -\frac{10}{3}$$

$$\therefore a + \frac{1}{a} = \frac{10}{3}$$

$$\text{Answer: } a + \frac{1}{a} = \frac{10}{3}$$

### 7. Solution

Consider the RHS of the given equation

$$\begin{aligned} & \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] \\ &= \frac{1}{2}(a+b+c)[a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2] \\ &= \frac{1}{2}(a+b+c)[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca] \\ &= \frac{1}{2}(a+b+c) \times 2(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= \frac{1}{2} \times 2(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \end{aligned}$$

This is comparable to the LHS of the following identity

$$\begin{aligned} (x+y+z)(x^2+y^2+z^2-xy-yz-zx) &= x^3+y^3+z^3-3xyz \\ \therefore (a+b+c)(a^2+b^2+c^2-ab-bc-ca) &= a^3+b^3+c^3-3abc \\ &= \text{LHS of the given statement.} \end{aligned}$$

RHS = LHS.

Hence verified.