

Master Lesson Plan for

Triangles

Board	Standard	Subject	Chapter	Language
NCERT	STD IX	Mathematics	Triangles	English

Sri Sathya Sai Central Trust

CURRICULAR GOALS	CG-4: Analyses characteristics and properties of two-dimensional geometric shapes and develops mathematical arguments to explain geometric relationships
COMPETENCIES AS PER NCF/NIPUN BHARAT	C-4.1: Describes relationships including congruence of two-dimensional geometric shapes (such as lines, angles, triangles) to make and test conjectures and solve problems
SDG(Sustainable Development Goals)	SDG4 – Quality Education
Technology aids	PPTs
Learning Teaching Materials (LTM)	Cut-outs of congruent triangles
Learning Aids	Worksheets, Project Report Template
NEP recommended Strategies	Art Integration: SA_project Congruent triangles, Case Study: VC_Equality leads to harmony
Cross-Cutting Themes	Values and Dispositions – Integrated the value of Equality Inclusion – Gender sensitisation through Value content Technology in education – Powerpoint presentations

MS_Objective Triangles

Objective: Teacher will be able to get an overview of the asset objectives in the MLP for the chapter

Notes to the teacher: This asset lays down the proposed plan for transacting this chapter. It states the asset objectives of the MLP. This asset is for the teacher's reference and need not be taught to the students.

Students will be able to –

- develop the concept of congruence of plane figures
- interpret the correspondence between parts of congruent triangles in symbolic form
- identify congruent triangles using the SAS congruence rule
- relate the concept of congruent triangles with real life
- establish the ASA criteria for congruence of triangles
- apply the SAS and ASA congruence rules in geometrical problems
- appreciate the concept of congruence of triangles
- establish that angles opposite to equal sides of an isosceles triangle are equal and also the converse
- apply the properties of the isosceles triangle in solving questions
- internalise the need for treating everyone equal, in society
- identify congruent triangles using the SSS criterion for congruence
- identify congruent triangles using the RHS rule for congruence of triangles
- build a 3-D model or an art design based on the concept of congruence of triangles
- solve problems using the SSS and RHS congruence rules
- verify experimentally that the sum of any two sides of a triangle is greater than the third side
- summarise the key points covered in the chapter 'Triangles'.

Time to teach	Asset type	Theme	Sub Theme
5 minutes	Main Script	Triangle	Congruence among Right-Angled Triangle Congruent triangles Criteria for ASA congruence Criterion for RHS congruence Criterion for SAS congruence Criterion for SSS congruence Inequalities in a Triangle Properties of triangles

IQ_Congruent Figures

Objective: The student will be able to develop the concept of congruence of plane figures.

Teaching Aids: PPT

Notes to the teacher: Teacher may pose the following questions to the students, allow them to think and come out with as many answers as possible before arriving at a conclusion. If there is any difficulty in getting the answers, the teacher can ask leading questions.

1. There are three pairs of figures given below. Watch what happens when we try to superimpose the cutouts of the copies of these figures on one another. You will notice that the figures in only the last pair coincide exactly with each other.

[Notes to the teacher: The teacher may use PPT to show superimposition of the figures.]

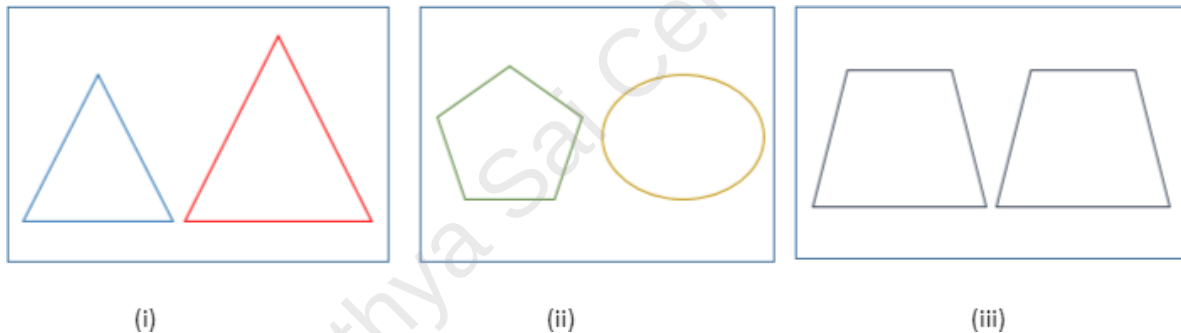


Image Source: PPT Shapes

1. Why do the figures in the first two pairs not cover each other completely when superimposed like the third pair? What features of the third pair make the coinciding of figures possible? What are such figures called?

Expected Answer:

The pair in figure (i) have different sizes. The pair in figure (ii) have different shapes.

The pair in figure (iii) have the same shape and size.

The figures whose shapes and sizes are the same are called congruent figures. 'Congruent' means equal in all respects. They coincide exactly with each other on superimposing.

2. Can you think of some identical objects in real life which can be called congruent?

Suggested Answer:

Two lenses of a spectacle, tyres of a cycle, badminton racquets, etc. are some examples of congruent objects.



Image Source:

Spectacle: pixabay.com/vectors/glasses-spectacles-eyeglasses-geek-312005/ - Attribution: Clker-Free-Vector-Images

Badminton Racket: pixabay.com/illustrations/badminton-bat-sports-badminton-game-5424103/ - Attribution: nts01

Bicycle: pixabay.com/illustrations/bicycle-basket-with-flowers-3758313/ - Attribution: pixabay

3. The following pair of figures are congruent. What is the requirement for pairs of line segments, angles and circles to be congruent?

A ————— B

C ————— D

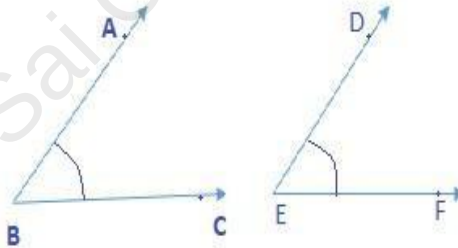


Image Source: PPT Shapes

Expected Answer:

The requirement for the above pairs to be congruent are: Line segments - equal length.

Angles - equal measure.

Circles - equal radius.

4. Given a pair each of square and rhombus, with sides length 3 cm, which figures are congruent?
Why are the figures of the other pair not congruent in spite of having equal sides?



Image Source: PPT Shapes

Expected Answer:

Two squares are congruent.

The rhombuses are not congruent because the angles of the given rhombuses are not respectively equal.

5. Following pair of triangles are congruent. So, the copies of these triangles coincide exactly with each other when superimposed.

Side **PQ** of Δ **PQR** matches with the side **CB** of Δ **CBA** and $\angle$ **P** with $\angle$ **C**.

That is written as **PQ** $\leftrightarrow$ **CB** and $\angle$ **P** $\leftrightarrow$ $\angle$ **C**.

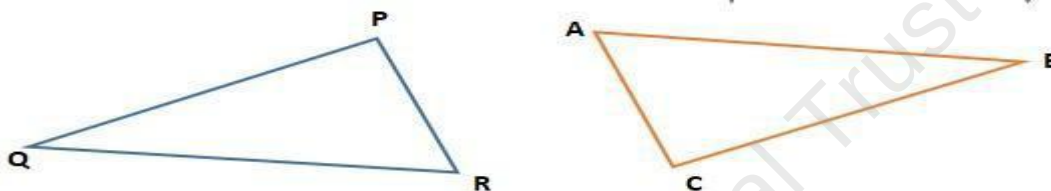


Image Source: PPT Shapes

Select the other correct parts of Δ **CBA** corresponding to the parts of Δ **PQR**.

PR $\leftrightarrow$ **Select**

QR $\leftrightarrow$ **Select**

$\angle$ **Q** $\leftrightarrow$ **Select**

$\angle$ **R** $\leftrightarrow$ **Select**

Answer:

PR $\leftrightarrow$ **CA**

QR $\leftrightarrow$ **BA**

$\angle$ **Q** $\leftrightarrow$ $\angle$ **B**

$\angle$ **R** $\leftrightarrow$ $\angle$ **A**

To check whether the figures are congruent, we need some specific information about the measures describing these figures. You will learn about such conditions for congruent triangles in this chapter.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Inquisitive Questions	Triangle	Congruent triangles

MS_Congruence of Triangles

Objective: The student will be able to interpret the correspondence between parts of congruent triangles in symbolic form.

Teaching Aids: No

Teaching Learning Material (TLM): Cutouts of congruent triangles.

What do you need? Two A4 size sheets of thick paper, a pair of scissors, a ruler and pencil, coloured markers.

Get ready: Prepare cutouts of two congruent triangles from the sheets of paper. Mark the corresponding equal parts of the two cutouts using coloured markers. Label the vertices in the interior of the cutouts.



How to use: The teacher may use the TLM to explain the one-one correspondence between the parts of congruent triangles and in writing the congruence of triangles in symbolic form.

Notes to the teacher: The teacher may introduce the topic by briefly explaining congruent figures: 'Congruent' means equal in all respects or figures whose shapes and sizes are both the same. They cover each other completely when superimposed on one another.

Let's learn about congruence of triangles.

If two triangles are congruent then the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

So, in congruent triangles, corresponding parts are equal and written in short as 'CPCT' (corresponding parts of congruent triangles).

[Notes to the teacher: The teacher may use TLM here.]

Here are cutouts of two congruent triangles ABC and PQR. The equal sides and angles in the two triangles are marked in the same colours.

If we superimpose one cutout on another, the two should exactly cover each other.

[Notes to the teacher: The teacher may superimpose the two cutouts so that the corresponding equal parts are not matched.]

However, you can notice that if the corresponding equal parts are not placed on one another the triangles do not cover each other exactly on superimposing. So, it is important to match the corresponding equal parts of the two triangles for exact covering to take place on superimposition.

[Notes to the teacher: The teacher may superimpose the two cutouts so that the corresponding equal parts are exactly matched.]

So, the cutouts of congruent triangles ABC and PQR are to be turned around and placed on one another such that side QP covers AB, PR covers BC and RQ covers CA; $\angle P$ covers $\angle B$, $\angle Q$ covers $\angle A$ and $\angle R$ covers $\angle C$.

This one-one correspondence between the sides and vertices is written as: $AB \leftrightarrow QP$, $BC \leftrightarrow PR$, $AC \leftrightarrow QR$, $A \leftrightarrow Q$, $B \leftrightarrow P$ and $C \leftrightarrow R$

So, under this correspondence the triangles are congruent, and their congruence is expressed symbolically as:

$$\triangle ABC \cong \triangle QPR.$$

But it is not correct to write $\triangle ABC \cong \triangle PQR$.

Let's consider an example.

[**Notes to the teacher:** The teacher may use the black/white board to draw the following figures to explain. It may be mentioned here that the corresponding sides and angles of the congruent triangles are marked with equal number of dashes and arcs respectively.]

Example: In the following figure, side DE corresponds to side LM, side DF to LN and side EF to MN.

Also angle $D \leftrightarrow L$, $E \leftrightarrow M$ and $F \leftrightarrow N$



So, we can express the congruence of the triangles DEF and LMN symbolically as:

$$\triangle DEF \cong \triangle LMN$$

$$\triangle EDF \cong \triangle MLN$$

$$\triangle DFE \cong \triangle LNM$$

and so on.

However, it is not correct to write $\triangle DEF \cong \triangle MLN$ because the one-one correspondence between the vertices is not reflected in the name of the triangles.

So, it is necessary to write the correspondence of vertices correctly for the congruence of triangles in symbolic form.

Time to teach	Asset type	Theme	Sub Theme
5 minutes	Main Script	Triangle	Congruent triangles

MS_SAS Congruence Rule

Objective: The student will be able to identify congruent triangles using the SAS rule of congruence.

Teaching Aids Provided: PPT

Preparation for the lesson: The teacher may divide the class into four groups on the day prior to teaching the topic.

Each member of every group works at home to construct a triangle with the given measure (the other measures may vary for different members) on a thick sheet of paper and obtains a cutout of the triangle.

Group 1: A triangle with one of its sides 6 cm.

Group 2: A triangle with two of its sides as 5 cm and 6 cm

Group 3: $\triangle ABC$ where $AB = 6$ cm, $BC = 8$ cm, $\angle C = 50^\circ$

Group 4: $\triangle ABC$ where $AB = 6$ cm, $BC = 8$ cm, $\angle B = 50^\circ$

Notes to the teacher: The teacher may do a brief recap of how to construct the triangle, if required.]

During the lesson:

Each group compares the triangles cutouts made by the members by superimposing to check if the triangles within the group are congruent.]

Discussion Time:

[Notes to the teacher: The teacher may pose the following question to the groups and encourage the members to discuss and give their observations on the triangles cutouts prepared by them.]

Did all the groups get congruent triangles cutouts?

[Notes to the teacher: After the response from each group the teacher may compare the result of Group

C and Group D and pose the following question to the students, followed by students' response and conclusion]

In both the groups C and D, the triangles were constructed with two sides and one angle. Why is it that the triangles in group D are congruent and not in group C?

After the discussion...

Conclusion:

Thus given two sides and one angle of a triangle, the order of given measurements to construct a triangle also plays a vital role in constructing a unique triangle.

The above gives us a rule for checking the congruence of triangles.

This is taken as the first criterion for congruence of triangles and we can prove the other criteria through this.

Axiom (SAS congruence rule): Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle. (Side-Angle-Side)

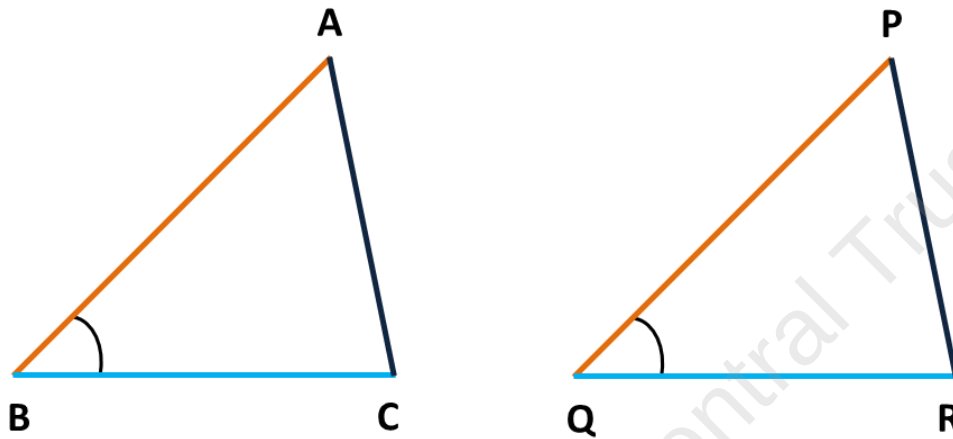


Image Source: PPT Shapes

$$\triangle ABC \cong \triangle PQR$$

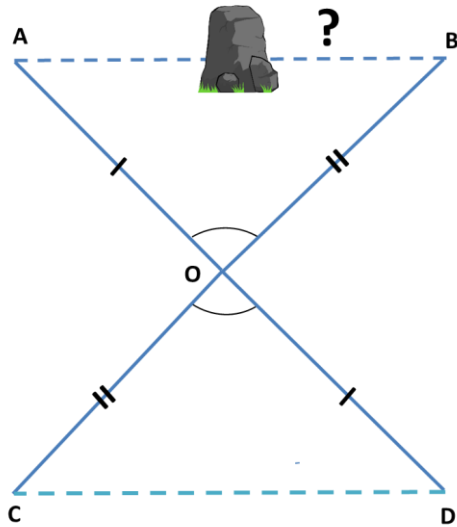
This result cannot be proved with the help of previously known results and so it is accepted as an axiom.

We can say that the SAS congruence rule holds, but not SSA or ASS, as shown by the activity of Group C.

Let's consider a situation where the SAS criterion can be used.

A surveyor wants to find the distance between two objects A and B but there is an obstacle between them which prevents direct measurement. He thinks of an indirect way to measure it.

He marks a point O at some distance, from where both A and B are visible. Then he marks a point C such that points B, O and C are collinear and $OB = OC$. Similarly he marks a point D and gets $OA = OD$.



Rock: pixabay.com/vectors/svg-rock-vector-gray-vector-2346025/ - Attribution: ktos

Surveyor: pixabay.com/illustrations/workers-work-project-builder-6152007 - Attribution: Elf-Moondance

How can he use this to find the length of AB indirectly?

[Note to The Teacher: The teacher may encourage the students to come up with the answer.]

In $\triangle AOB$ and $\triangle DOC$

$OA = OD$ and $OB = OC$ (By construction)

$\angle AOB = \angle DOC$ (Vertically opposite angles)

$\therefore \triangle AOB \cong \triangle DOC$ (SAS rule)

So, $AB = CD$ (CPCT)

The surveyor measured the length of the segment CD and found the distance between A and B.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Triangle	Criterion for SAS congruence

DD_Need for Congruent Triangles

Objective: *The student will be able to relate the concept of congruent triangles with real life.*

Teaching Aids Provided: PPT

The concept of congruent triangle is of great importance not only in the field of mathematics and sciences, but also in real life. Let's know some of the ways congruent triangles influence our lives.

Construction and Architecture

Congruent triangles are used in construction of buildings to reinforce structures so that they are strong and stable. They bring balance to the structure by distributing the forces on the building evenly so that it can sustain a large amount of force without collapsing or getting deformed.

You can observe the use of congruent triangles in supporting the roof of village houses, in designs and structures of old temples and churches. They not only give strength and stability but also bring beauty and symmetry in the structure. Some world famous buildings in which congruent triangles are used in construction of structures are the Eiffel tower, pyramids of Egypt, Louvre museum in Paris.

They are also used in the towers supporting overhead high tension power lines.

Egyptian Pyramids



Louvre museum in Paris



Towers for High tension



Wire Village House



Pyramids: pixabay.com/photos/pyramid-pyramids-egypt-pharaoh-66361/ - Attribution: PixelAnarchy

Louvre Museum: pixabay.com/photos/building-architecture-landmark-102840/ - Attribution: EdiNugraha

Tower: pixabay.com/photos/electricity-electric-tower-electric-262467/ - Attribution: venturaartist

House: pixabay.com/photos/chalet-swiss-wood-alps-switzerland-403976/ - Attribution: Albatrossie

Truss Bridge

A truss bridge is a bridge whose load bearing superstructure is composed of a truss, a structure of connecting parts, usually forming congruent triangles. Congruent triangles allow forces or weight to spread evenly across the bridge. This makes them strong enough to bear heavy weight without collapsing.



Truss Bridge: pixabay.com/photos/bridge-shore-dusk-architecture-5481229/ - Attribution: Kanenori

Geodesic Dome

Geodesic domes are hemispherical or spherical structures consisting of a network of congruent triangles, which help in distributing the stress equally along the entire structure. It is because of this strength and balance that they can withstand very heavy loads. Among their many design applications are sports and military equipment and arenas, exhibition halls, green houses, etc.



Planetarium

Planetarium: commons.wikimedia.org/wiki/File:Planetarium_Puttaparthi.jpg - Attribution: J929j929



Museum

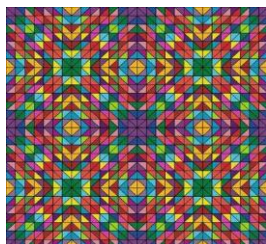
Museum: pixabay.com/photos/figuerres-spain-museum-dali-4682966/ - Attribution: falco

Art and Design

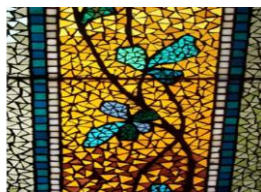
Congruent triangles are used in art and design to add beauty and harmony. They can be seen on floor tiles and window panes, in wall paintings, carpet and textile designs and so on.

Triangular Tessellation (Tiling) Stained Glass Window

Indian Mystical Design



Warli Tribal Art Design



Stained Glass Window



Sri Yantra

Painting: pixabay.com/illustrations/multicolored-colors-colorful-2703532/ - Attribution: DavidZydd

Stained glass painting: flickr.com/photos/9756990@N07/1342409676 - Attribution: Lyle

Warli Painting: flickr.com/photos/72746018@N00/4489190461/ - Attribution: Jean-Pierre Dalbéra

Indian Mystical Design: pixabay.com/vectors/sri-yantra-object-devotion-hindu-33588 - Attribution: Clker-Free-Vector Images

Time to teach	Asset type	Theme	Sub Theme
5 minutes	Day-to-day Relevance	Triangle	Congruent triangles

MS_ASA Congruence Rule

Objective: The student will be able to establish the ASA criteria for congruence of triangles.

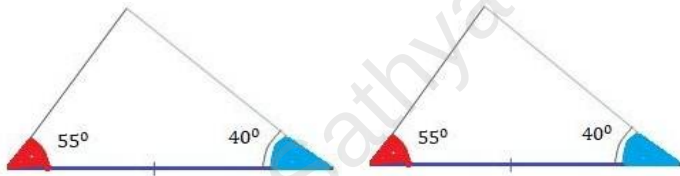
Teaching Aids Provided: PPT

Teaching Learning Material (TLM): Cutouts of congruent triangles.

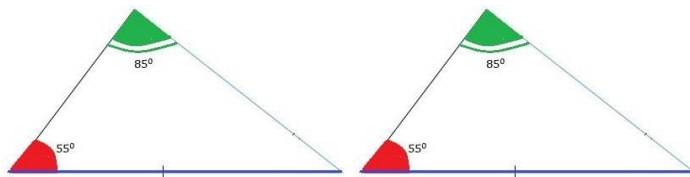
What do you need?: Four sheets of A4 size thick paper, a pair of scissors, coloured markers and a geometry box.

Get ready: Make cutouts of four congruent triangles from the sheets of paper. (With two angles, say 55° and 40° and the side included between these angles is, say 20 cm.

Set I: In two of the cutouts mark the two pairs of corresponding equal angles (55° and 40°) and the pair of equal sides (20 cm) with a marker.



Set II: In the remaining two cutouts mark the two pairs of corresponding equal angles such that the pair of corresponding equal sides of 20 cm is not included between the marked angles.



When to use: The teacher may use the TLM in the classroom to introduce the ASA and AAS congruence rules.

Notes to the teacher: The teacher may start with a brief recap of the SAS congruence rule.

In the SAS congruence rule, two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Now let us consider two triangles in which two angles (55° and 40°) and the side (20 cm) included between these angles (20 cm) are equal.

[**Notes to the teacher:** The teacher may use the **set I** of TLM to show by superimposing one triangle upon another, that they perfectly cover each other.]

Observe that when the cutouts of these triangles are placed upon one another, one triangle covers the other completely. So we can say that the two triangles are congruent.

If this activity is repeated with more pairs of triangles with different measures of the two pairs of angles and the included side, the result will be the same.

This result is called the **Angle-Side-Angle** criterion for congruence and is written as **ASA** criterion.

ASA result is called a theorem because it can be proved using the SAS axiom. Let's state and prove this theorem.

Theorem - ASA congruence rule:

Statement: Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of the other triangle.

[**Notes to the teacher:** The teacher may involve the students in the following steps required in proving a theorem:

- Figure: Draw a labelled figure to represent the information given in the statement of the theorem.
- Given: Write the given information as shown in the figure.
- To prove: Write what is to be proved as labelled in the figure.
- Construction: Write about any appropriate addition to the figure, if required, to prove the theorem.
- Proof: Use the above information step by step to write the proof.]

Given: $\triangle ABC$ and $\triangle DEF$ with $\angle B = \angle E$, $\angle C = \angle F$ and $BC = EF$

To prove: $\triangle ABC \cong \triangle DEF$

Proof: There are three cases possible for sides AB and DE, when we prove the congruence of the triangles.

Case I: Assume that $AB = DE$

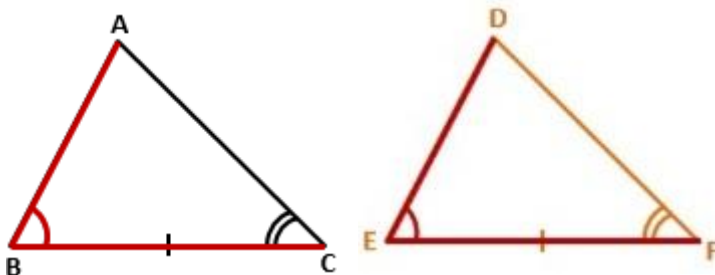


Figure 1

Let's see what it means for $\triangle ABC$ and $\triangle DEF$ (Figure 1) $AB = DE$ (Assumed)

$\angle B = \angle E$ (Given) $BC = EF$ (Given)

$\therefore \triangle ABC \cong \triangle DEF$ (By SAS axiom for congruence)

Case II: The other possibility is $AB > DE$.

Construction: Let's take a point X on AB such that $XB = DE$. (Figure 2)

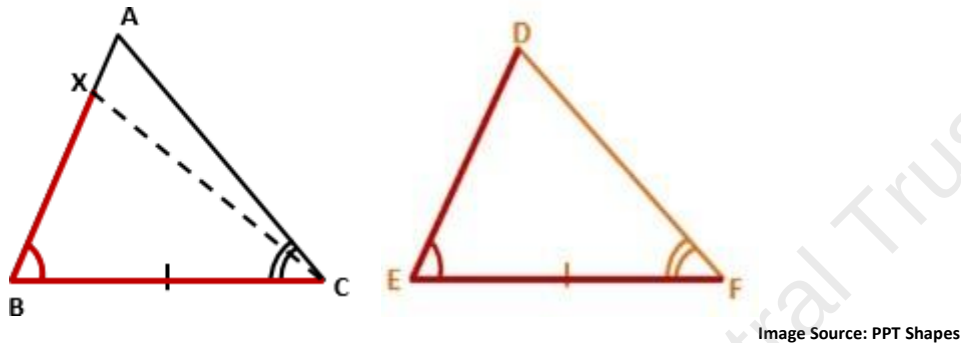


Figure 2 Observe the $\triangle XBC$ and $\triangle DEF$ $XB = DE$ (By construction)

$\angle B = \angle E$ (Given) $BC = EF$ (Given)]

$\therefore \triangle XBC \cong \triangle DEF$ (SAS rule)

We get, $\angle XCB = \angle DFE$ (CPCT)

[**Notes to the teacher:** The teacher may give the full form of CPCT, corresponding parts of congruent triangles.)

But, $\angle ACB = \angle DFE$ (Given)

$\Rightarrow \angle ACB = \angle DFE = \angle XCB$

This is possible only if point X coincides with point A or $AB = DE$

So, $\triangle ABC \cong \triangle DEF$ (By SAS rule)

Case III: Let $AB < DE$

Construction: We can choose a point Y on DE such that $YE = AB$. (Figure 3)

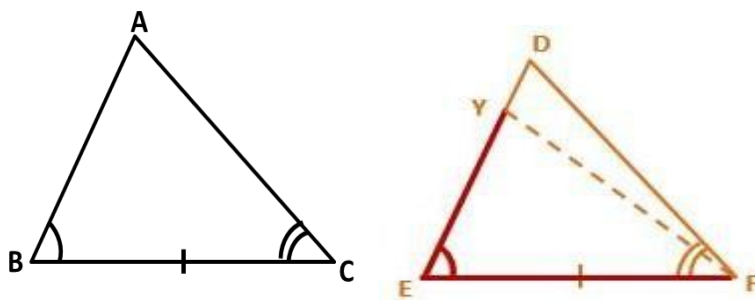


Figure 3

Image Source: PPT Shapes

[Notes to the teacher: The teacher may use logical steps similar to case II to prove $AB = DE$ on the black/white board as follows:

In $\triangle ABC$ and $\triangle YEF$

$AB = YE$ (By construction)

$\angle B = \angle E$ (Given) $BC = EF$ (Given)

$\therefore \triangle ABC \cong \triangle YEF$ (SAS rule)

We get, $\angle ACB = \angle YFE$ (CPCT)

But, $\angle ACB = \angle DFE$ (Given)

$\Rightarrow \angle ACB = \angle DFE = \angle YFE$

This is possible only if point Y coincides with point D or $AB = DE$.

$\therefore \triangle ABC \cong \triangle DEF$ (SAS)]

In this case also $\triangle ABC \cong \triangle DEF$ (By SAS axiom) Thus the theorem ASA congruence rule is proved.

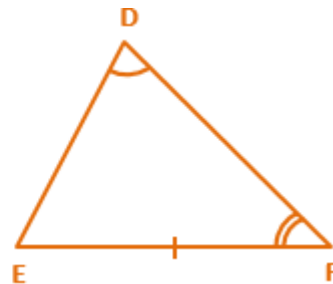
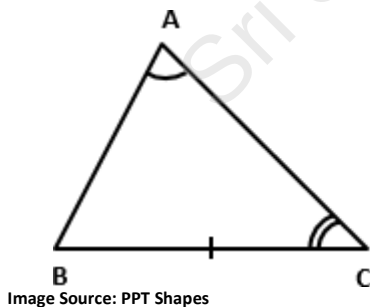
Now let's consider two triangles with two pairs of angles and one pair of corresponding sides equal but the side is not included between the corresponding equal pairs of angles and check if the triangles are still congruent.

[Notes to the teacher: The teacher may use the **set II** of TLM to explain the AAS rule]

Observe that they exactly overlap each other on superimposing. So, they are congruent.

[Notes to the teacher: The teacher may ask the students why this is so and explain with example the reason, using black/white board. The sum of the three angles of a triangle = 180° . So if two pairs of angles are equal, the third pair is = $180^\circ - \text{sum of equal angles}$, making it also equal. Then by ASA rule of congruence, the triangles are congruent.]

Thus, we get the AAS congruence rule:



AAS Congruence Rule (Angle-Angle-Side): Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

Let's consider some examples where these rules can be used to prove the congruence of triangles.

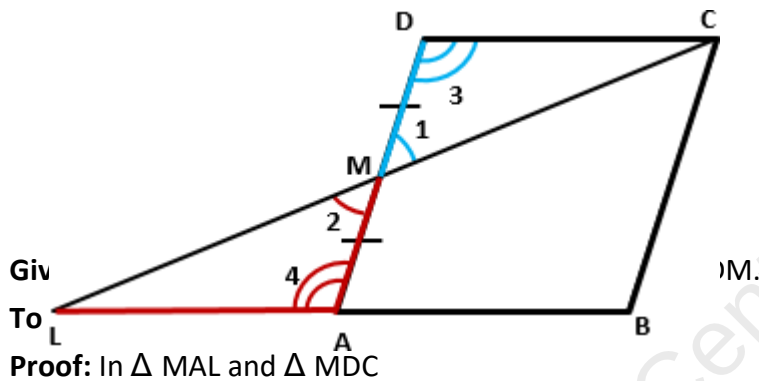
[Notes to the teacher: While solving problems, the teacher may encourage the students to:

- read the question and draw a labelled figure to represent the information given in the question.
- use the figure and break down the question into what is given (Given) and what is to be proved (To Prove).
- observe the triangles and check for existing equalities based on what is given and what can be derived from the given information to identify the congruence rule that can be applied.
- write them all step by step and get the 'Proof'.]

Examples:

1. In figure 4, $BL \parallel CD$, M is the midpoint of AD . Prove that $\triangle MDC \cong \triangle MAL$ and $DC = AL$.

Suggested Answer:



$\angle 1 = \angle 2$ (Vertically opposite angles) $DM = AM$ (Given)

$\angle 3 = \angle 4$ (Alternate interior angles, $BL \parallel CD$)

$\therefore \triangle MDC \cong \triangle MAL$ (ASA rule)

$\Rightarrow DC = AL$ (CPCT)

Hence proved.

2. PR and QS are equal perpendiculars to a line segment PQ . Show that RS and PQ bisect each other at Z .

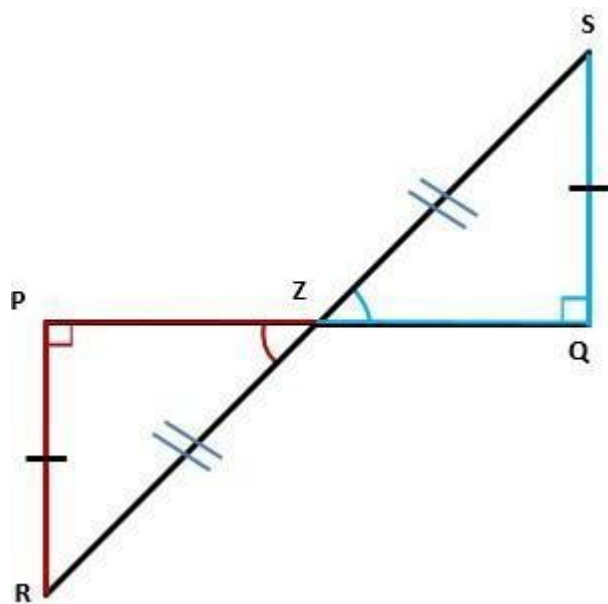


Image Source: OC

Suggested Answer:

Given: $PR \perp PQ$, $QS \perp PQ$ and $PR = QS$. PQ and RS intersect at Z . **To Show:** RS and PQ bisect each other at Z or $PZ = QZ$ and $RZ = SZ$.

Proof: In $\triangle PZR$ and $\triangle QZS$

$\angle ZPR = \angle ZQS = 90^\circ$ ($PR \perp PQ$, $QS \perp PQ$)

$\angle PZR = \angle QZS$ (Vertically opposite angles) $PR = QS$ (Given)

$\therefore \triangle PZR \cong \triangle QZS$ (AAS Rule)

$\Rightarrow PZ = QZ$ and $RZ = SZ$ (CPCT)

So, RS and PQ bisect each other at Z . Hence proved

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Main Script	Triangle	Criteria for ASA congruence

QA_Applying SAS and ASA Rules

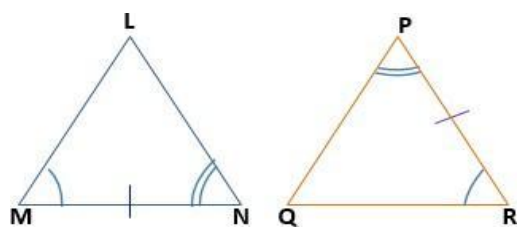
Objective: The student will be able to apply the SAS and ASA congruence rules in geometrical problems.

Teaching Aids: PPT and Worksheet

Notes to the teacher: Teacher may present the question and allow the students to say or write the answers before revealing them to the students.

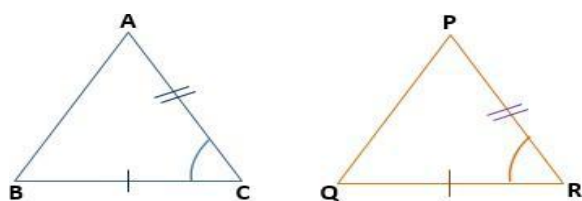
1. Given two triangles with $MN = PR$, $\angle N = \angle P$ and $\angle M = \angle R$.
 - a. Which criterion of congruence can be used to prove the two triangles congruent?
 - b. Choose the correct option to write their congruence in symbolic form.
 $\triangle LMN \cong \triangle$ Select ▾

Answer: a) ASA b) $\triangle QRP$



2. In $\triangle ABC$ and $\triangle PQR$, $\angle C = \angle R$, and $BC = QR$. Choose the third pair of corresponding equal parts, so that

- a. $AB = PQ$
- b. $BC = QR$
- c. $AC = PR$
- d. $\angle B = \angle Q$ Answer: (c) $AC = PR$



$\triangle ABC \cong \triangle PQR$ by SAS rule of congruence

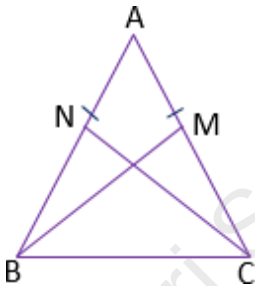
3. Which congruence criteria can be used to prove $\triangle ABC \cong \triangle FGH$, given that $\angle B = \angle G$, $\angle A = \angle F$ and $AB = FG$ and $BC = GH$. Answer: (c) ASA, AAS and SAS



In $\triangle ABC$ and $\triangle FGH$, $\angle A = \angle F$, $AB = FG$ and $\angle B = \angle G$
 $\therefore \triangle ABC \cong \triangle FGH$ (ASA)

[Notes to the teacher: The teacher may mention here that the congruence of triangles can be proved by using three appropriate corresponding equal parts of the two given triangles.] Similarly by using appropriate corresponding equal parts of the two triangles, rules AAS and SAS can also be used to prove the congruence of the given triangles.

4. In the following figure, M and N are two points on the equal sides AB and AC of $\triangle ABC$ such that $AN = AM$. Fill in the following blanks to prove that $\triangle ANC \cong \triangle AMB$ and $NC = MB$.



In $\triangle s$ ANC and AMB, we have

= [Given]

= (Given)

= (Common)

$\therefore \triangle ANC \cong \triangle AMB$ (Rule)

$NC = MB$ ()

Suggested Answer:

In $\triangle s$ ANC and AMB, we have $AN = AM$ [Given]

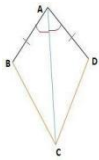
$$AC = AB \quad [\text{Given}]$$

$$\angle A = \angle A \quad [\text{Common angle}]$$

$$\therefore \triangle ANC \cong \triangle AMB \quad (\text{SAS Rule})$$

5. Dhanu likes to fly kites. He is happy to see that the quadrilateral ABCD with $AB = AD$ and AC bisecting $\angle A$, resembles a kite.

Is Dhanu right in thinking that ABCD is a kite? Give reasons.

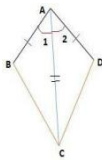


Boy: pixabay.com/vectors/kite-boy-kid-children-play-flying-2521627/ - Attribution: jambulboy

Kite: pixabay.com/vectors/kite-green-flying-autumn-fall-303238 - Attribution: Clker-Free-Vector-Images

Suggested Answer:

By definition in geometry, a kite has two pairs of adjacent sides equal. So, to prove that quadrilateral ABCD is a kite, we must prove $BC = DC$.

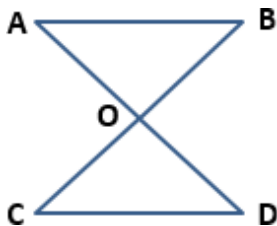


The three pairs of corresponding equal parts marked in the figure make $\triangle ABC \cong \triangle ADC$ (SAS Rule)

$$\therefore BC = DC \quad (\text{CPCT})$$

So, ABCD is a kite.

6. In the given figure, $AB \parallel CD$ and AD and BC bisect each other at O. Prove that $\triangle AOB \cong \triangle DOC$.



Suggested Answer:

In $\triangle AOB \cong \triangle DOC$

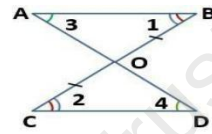
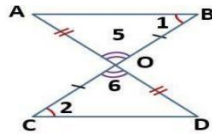
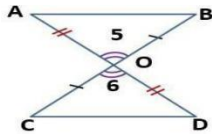
$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4 \quad (\text{Alternate interior angles})$$

AD and AC bisect each other at O. So, $AO = DO$ and $BO = CO$

$\angle 5 = \angle 6$ (Vertically opposite angles)

[**Notes to the teacher:** The teacher may mention here that the congruence of triangles can be proved by using three appropriate corresponding equal parts of the two given triangles.]

So, the triangles $\triangle AOB$ and $\triangle DOC$ can be proved congruent using the congruence rule SAS, ASA or AAS.



$\triangle AOB \cong \triangle DOC$. (SAS rule))

$\triangle AOB \cong \triangle DOC$ (ASA rule)

$\triangle AOB \cong \triangle DOC$ (AAS rule)

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Questions to Assess	Triangle	Criteria for ASA congruence Criterion for SAS congruence

IA_Story Time

Objective: The student will be able to appreciate the concept of congruence of triangles.

Teaching Aids Provided: PPT

Notes to the teacher: The teacher may introduce the story by saying, “Here’s a story of how the knowledge and application of ASA rule of congruence of triangles helped an officer get a promotion.”

There is a story about Napoleon’s officer. Napoleon I was a French military general and statesman. Once passing through a forest, there was a river to be crossed.



Napoleon I

The Officer

Napoleon: pixabay.com/illustrations/napoleon-bonaparte-leader-military-5688620/ - Attribution: ArtsyBeeKids

River: pixabay.com/illustrations/illustration-vector-forest-nature-3878415 - Attribution: pixabay

Officer: pixabay.com/vectors/hairy-verdun-soldier-war-tribute-3207343/ - Attribution: gregroose

It was required to know the width of the river and there were no means available for doing that. One intelligent officer estimated the width of the river without crossing it.



Napoleon’s Officers discussing the solution to the problem

The Smart Officer

Officer: pixabay.com/vectors/hairy-verdun-soldier-war-tribute-3207343/ - Attribution: gregroose

Officers: pixabay.com/vectors/soldier-war-14-first-world-war-3208335/ - Attribution: gregroose

Bulb: pixabay.com/vectors/light-light-bulb-on-bulb-148483/ - Attribution: OpenClipart-Vectors

Now let's see how the officer accomplished this task.

The officer stood erect on the bank of the river, facing the river and lowered the visor(eyeshade or shield) of his cap until he could see only the edge of the opposite bank. (Figure 1)



VISOR

Visor: pixabay.com/vectors/hat-baseball-cap-cap-cap-cap-295184/ - Attribution: Clker-Free-Vector-Images
River Width: OC

Figure 1

He then turned towards his right (or left) to the bank of the river on his side keeping his head and eye still. There he located a spot that was in line with his eye and the tip of his visor.

[Notes to the teacher: The teacher may use a cap to show to the students how the officer lowered the visor of his cap to set it at a correct angle by looking at a student sitting in the last row.]



Boy face: pixabay.com/vectors/boy-wink-teen-teenager-face-7024401/ - Attribution: BeezeeStock
River with man: OC

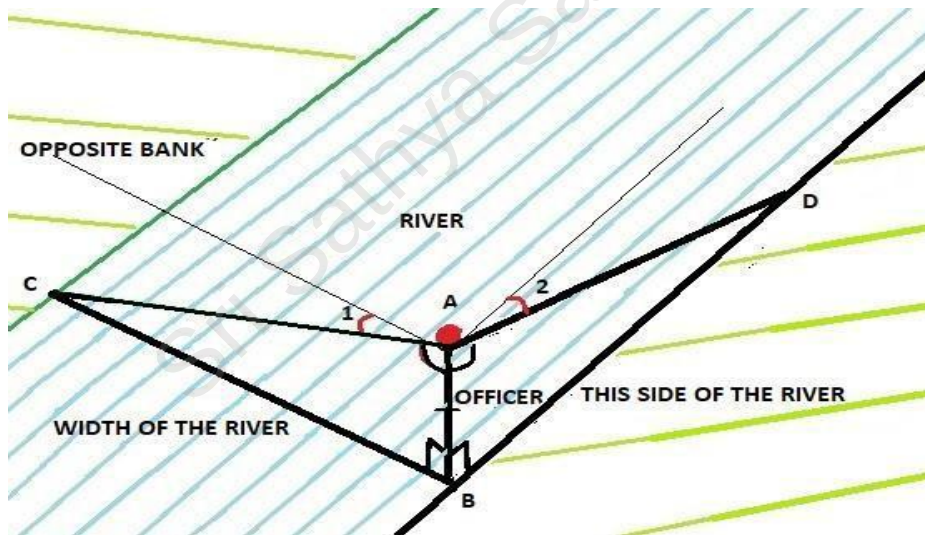
Figure 2

The officer then measured the distance to this spot from where he stood and declared that distance to be the width of the river!

Now watch why his estimate is correct.

In figure 3, segment AB represents the officer who stood erect, making an angle of 90° with the ground.

BC is for the width of the river and BD is the estimated width of the river.



River width: OC:

Figure 3

Observe the three equal corresponding parts marked in $\triangle ABC$ and $\triangle ABD$. So, the two triangles are congruent by ASA criteria. Thus $BC = BD$, as they are the corresponding parts of the congruent triangles (CPCT).

[Notes to the teacher: The teacher may explain to the students that $\angle 1$ is the angle through which the officer moved the visor as he looked at the opposite bank of the river. He kept the angle of the visor fixed to find point D on his side of the river. So, $\angle 2 = \angle 1$, this made $\angle BAC = 90^\circ - \angle 1$ and $\angle$

$\angle BAD = 90^\circ - \angle 2$, both equal. So, $\triangle ABC \cong \triangle ABD$ by the ASA criteria.]

So, the officer's estimation of the width of the river is correct and is equal to the distance BD on the bank of the river.

No wonder, the officer got a promotion!

Time to teach	Asset type	Theme	Sub Theme
5 minutes	Interesting Asides	Triangle	Construction of triangles by ASA

MS_Isosceles Triangle Theorem

Objective: The students will be able to establish that angles opposite to equal sides of an isosceles triangle are equal, and also the converse.

Teaching Aids: NA.

Teaching Learning Material (TLM): Triangle Cutouts

What do you need? A4 size sheet of paper and a paper cutter.

Get ready:

1. Take an A4 size sheet of paper or any other paper. Fold as shown in figure 1 to make a square.



Figure 1

Paper Folding: Original Creation

2. Cut off the extra paper to obtain a cutout of a square. (Figure 2)

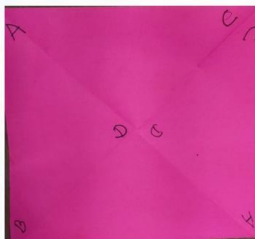


Figure 2

Paper cutting: Original Creation

3. Cut the square along the diagonal to obtain cutouts of two isosceles triangular regions. (Figure 3)

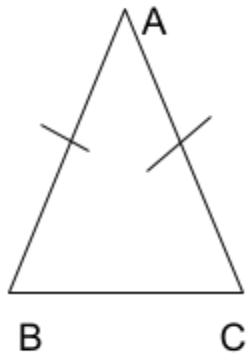


Figure 3

Isosceles Triangle: Original Creation

How to use: The teacher may use the TLM while introducing Theorem 1 and Theorem 2, through paper folding.

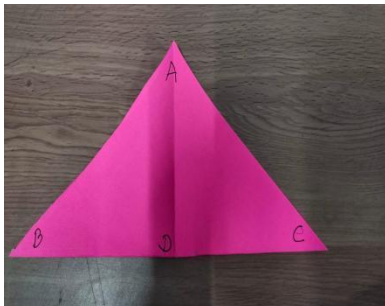
Notes to the teacher: The teacher may begin the class by categorising triangles based on the sides. Triangle with any two sides equal is called an isosceles triangle.



Here $\triangle ABC$ is an isosceles triangle, as $AB = AC$

[**Notes to teacher:** The teacher may use the TLM here. Cutout of one of the triangles may be used to show that when sides overlap each other (as they are equal), the angles opposite to equal sides also overlap each other showing that they are equal]

1. Let's fold the triangular region from vertex A to make the sides overlap each other. (Figure 4)



Paper Activity: Original Creation

Figure 4

2. We find that $\angle B$ overlaps $\angle C$. Hence $\angle B = \angle C$. (Figure 5)

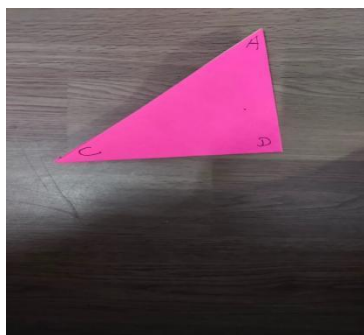


Figure 5

Angles Equal: Original Creation

You can observe that the angles opposite to equal sides exactly cover each other. So, they are equal. If you repeat this activity for any other isosceles triangle, it will give the same result. This important result can be proved as a theorem using rules of congruence of triangles.

[Note: Teacher can show the proof of the theorem as explained below on the black board]

Theorem I: Angles opposite to equal sides of an isosceles triangle are equal.

[Notes to the teacher: The teacher may involve the students in the following steps required in proving a theorem:

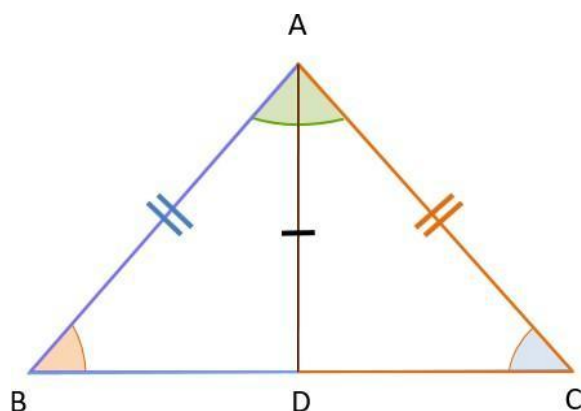
- **Figure:** Draw a labelled figure to represent information given in the statement of the theorem.
- **Given:** Write the given information as shown in the figure.
- **To prove:** Write what is to be proved.
- **Construction:** Write about any appropriate addition to the figure, if required, to prove the theorem.
- **Proof:** Use the above information step by step to prove the theorem.]

Given: $\triangle ABC$ with $AB = AC$

To prove: $\angle B = \angle C$

Construction: Draw bisector of $\angle A$ intersecting BC at D .

[Notes to teacher: The teacher may give a small memory tip to the students to remember that the folding of this triangle along vertex A , is actually bisecting the angle A .]



Proof: In $\triangle ADB$ and $\triangle ADC$, $AB = AC$ (Given)

$AD = AD$ (Common)

$\angle BAD = \angle CAD$ (by construction)

$\triangle ADB \cong \triangle ADC$ (SAS rule)

$\angle B = \angle C$ (CPCT)

Hence, proved.

[Notes to the teacher: The above theorem can be proved if AD is a median or AD is an altitude. The rules of congruence will vary for each].

Examples:

[Notes to the teacher: While solving problems, the teacher may encourage the students to

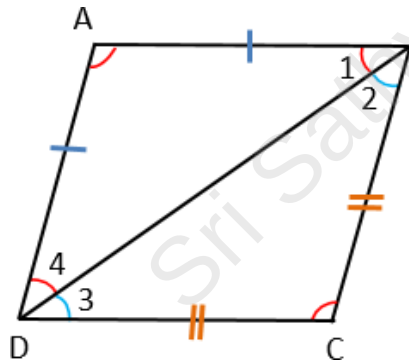
- read the question and draw a labelled figure to represent the information given in the question.
- use the figure and break down the question into what is given (Given) and what is to be proved (To Prove).
- observe the triangles and check for existing equalities based on what is given and what can be derived from the given information to identify the rules that can be applied.
- write them all step by step and get the 'Proof'.]

1. Show that opposite angles of a rhombus are equal, using isosceles triangle theorem.

Suggested Answer:

Given: Rhombus ABCD

To Prove: $\angle A = \angle C$ and $\angle B = \angle D$ Construction: Join diagonal BD



Proof: In $\triangle ADB$

$AD = AB$ (Sides of rhombus ABCD)

$\therefore \angle 1 = \angle 4$ (Angles opposite to equal sides of triangle ADB) (i)

Similarly in $\triangle CDB$

$\angle 2 = \angle 3$ (ii)

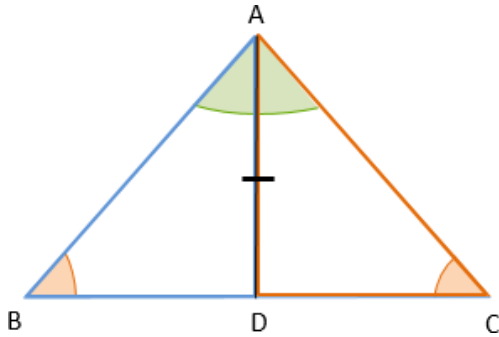
$\angle 1 + \angle 2 = \angle 4 + \angle 3$ [Adding steps (i) and (ii)]

$\Rightarrow \angle B = \angle D$ Similarly $\angle A = \angle C$

[Notes to the teacher: The teacher may use TLM again here. Cutout of the other triangle may be used to show that when the base angles of a triangle overlap (if angles are equal), then the sides will

also overlap and are thus equal.(sides opposite to equal angles of a triangle are equal.)]

Theorem 2: Side opposite to equal angles of a triangle are equal. (Converse of theorem I)



Given: $\triangle ABC$ with $\angle B = \angle C$

To prove: $AB = AC$

Construction: Draw AD , bisector of $\angle A$. AD intersects BC at point D .

Proof: In $\triangle ADB$ and $\triangle ADC$,

$\angle B = \angle C$ (Given) $AD = AD$ (Common)

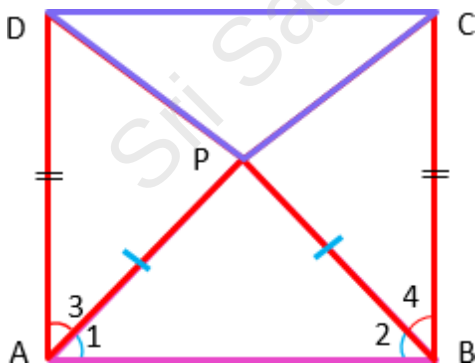
$\angle BAD = \angle CAD$ (by construction)

$\therefore \triangle ADB \cong \triangle ADC$ (AAS rule)

$AB = AC$ (CPCT)

Hence, proved.

1. P is a point in the interior of square $ABCD$, such that the base angles of $\triangle PAB$ are equal. Show that $\triangle PCD$ is an isosceles triangle.



Suggested Answer:

Given: Square $ABCD$, with point P in its interior. In $\triangle PAB$, $\angle 1 = \angle 2$. **To prove:** $\triangle PCD$ is an isosceles triangle or $PD = PC$.

Proof:

In $\triangle PAB$

$\angle 1 = \angle 2$ (given)

$\therefore PA = PB$ (Sides opposite to equal angles) ----(i)

Therefore $\angle 3 = 90^\circ - \angle 1$ and $\angle 4 = 90^\circ - \angle 2$ (ABCD is square, $\angle A = \angle B = 90^\circ$)

$$\Rightarrow \angle 3 = \angle 4 \quad (\angle 1 = \angle 2) \quad \text{(ii)}$$

In $\triangle PAD$ and $\triangle PBC$

$AD = BC$ (sides of square) $PA = PB$ [From (i)]

$\angle 3 = \angle 4$ (From (ii))

By SAS rule, $\triangle PAD \cong \triangle PBC$ $PD = PC$ (CPCT)

Hence, $\triangle PCD$ is an isosceles triangle.

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Main Script	Triangle	Properties of triangles

QA_Isosceles Triangle Property

Objective: The student will be able to apply the properties of the isosceles triangle in solving questions.

Teaching Aids: PPT and Worksheet

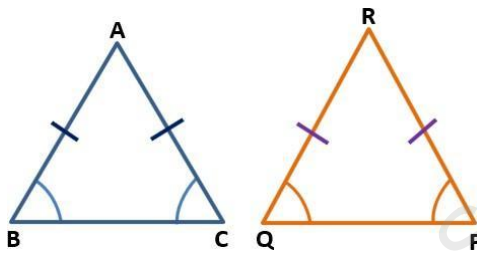
Notes to the teacher: Teacher may present the questions and allow the students to write or say the answers before revealing them to the students.

I. Choose the correct option:

1. In triangles ABC and RQP, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are:

- isosceles but not congruent
- isosceles and congruent
- congruent but not isosceles
- neither congruent nor isosceles

Answer: a) Isosceles but not congruent



Suggested Solution :

In ΔABC

$$\angle B = \angle C \text{ (AB = AC)}$$

$$\angle C = \angle P \text{ and } \angle B = \angle Q \text{ (Given)}$$

$$\therefore \angle P = \angle Q$$

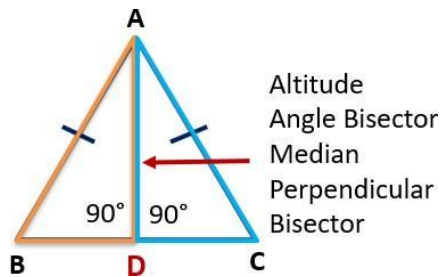
$$\Rightarrow RQ = RP \text{ (Sides opposite to equal angles)}$$

Hence, ΔRQP is isosceles. BC and QP may be equal or unequal. So, ΔABC and ΔRPQ need not be congruent.

2. In the isosceles triangle ABC, $AB = AC$. If AD is a median on side BC, then AD is also:

- Angle bisector
- Altitude
- Perpendicular bisector
- All three, a), b) and c)

Answer: d) All three



Suggested Solution:

$$\triangle ABD \cong \triangle ACD \text{ (SAS rule)}$$

$$\Rightarrow \angle ADB = \angle ADC \text{ (CPCT)}$$

$$\angle ADB = \angle ADC = 90^\circ \text{ (}\angle ADB \text{ and } \angle ADC \text{ form a linear pair)}$$

$\therefore$ AD is the altitude on BC Similarly $\angle BAD = \angle CAD$

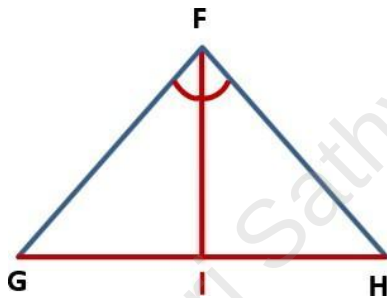
$\therefore$ AD is the bisector of $\angle A$

AD is the median and altitude on BC

$\therefore$ It is the perpendicular bisector of BC

Therefore AD is all three, an altitude, an angle bisector and a perpendicular bisector.

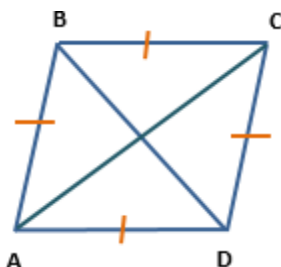
II. In $\triangle FGH$, $HI = GI$. Is $\angle GFI = \angle HFI$ by isosceles triangle property? Give reasons.



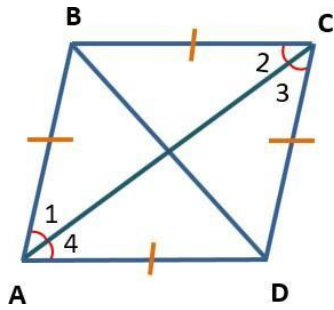
Suggested Answer:

No, HI and GI are equal sides in two different triangles ($\triangle FHI$ and $\triangle FGI$ respectively). Angles opposite to equal sides of the same triangle are equal.

III. Show that the diagonals of a rhombus bisect the vertex angles.



Suggested Answer:



Given: Rhombus ABCD, with diagonals AC and BD

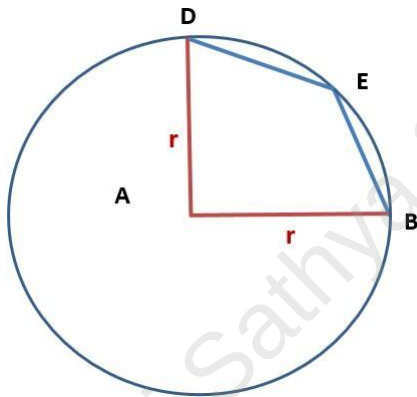
To Prove : AC bisects $\angle A$ and $\angle C$. i.e. $\angle 1 = \angle 4$ and $\angle 2 = \angle 3$ Proof : In $\triangle ABC$

$\angle 1 = \angle 2$ (BA = BC) and $\angle 1 = \angle 3$ (Alternate angles) Therefore $\angle 2 = \angle 3$

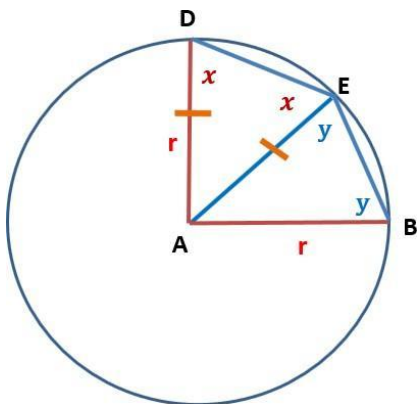
Similarly , $\angle 1 = \angle 4$

Similarly, we can prove BD bisects angles B and D Hence proved.

IV. In the figure, $\angle A = 90^\circ$. DAB is a quadrant of a circle with radius r . Find $\angle DEB$.



Suggested Answer:



In $\triangle ADE$, $AD = AE = r$

Hence $\angle ADE = \angle AED = x$ (Isosceles triangle property) Similarly in $\triangle AEB$, $\angle AEB = \angle ABE = y$

In quadrilateral ABED, sum of all angles is 360°

$$\Rightarrow x + x + y + y + 90^\circ = 360^\circ$$

$$\Rightarrow 2x + 2y = 270^\circ$$

$$\Rightarrow x + y = 270/2 = 135^\circ$$

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Questions to Assess	Triangle	Congruent triangles Special triangles(Equilateral & Isosceles triangle)

VC_Equality leads to Harmony

Objective: The student will be able to accept that treating all human beings as equal leads to harmony in a society.

Teaching Aids: N/A

Concept- Value Connection: Triangles are said to be congruent if they have the same shape and size. This is determined by equality of corresponding sides and angles. So the general definition of congruence is identical in form. That is, there is a quality of coinciding with one another. This harmony in triangles can be established by using one or the other rule i.e. SAS, ASA, AAS, SSS and RHS.

Students! We have learnt what it means for triangles to be congruent.

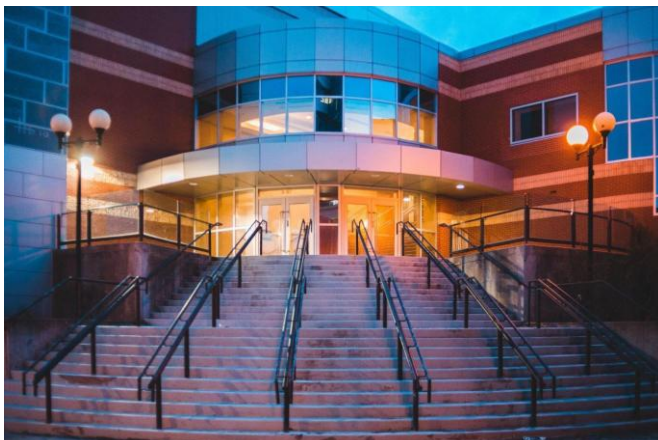
Let's apply this approach to our life and try to understand what congruence means in life. All of us may not be identical in form but we are equals as human beings. In our life too, we can make efforts to bring about harmony by treating everyone equal, without any discrimination, in order to have peace and harmony at home and in the society at large.

Notes to the teacher:

1. The following case studies can be written on the board
2. Students may be divided into two groups, as convenient.
3. Each group can take one case study, discuss the points for 5 minutes and try to arrive at a consensus response for each.
4. After 5 minutes, responses of each group can be briefly stated by their chosen leader.
5. The teacher can sum up the responses and conclude with points for further reflection by students.

Activity: Case Study—Is this Fair?**1. Discrimination based on Disability**

In a public building, the only way to enter the building is by stairs. No ramp is provided for people with disabilities. A person using a wheelchair is unable to enter the building.



Public building: pexels.com/photo/stairs-near-entrance-of-modern-building-in-evening-4529795/ - Attribution: Erik Mclean

2. Discrimination based on gender

In some rural areas in our country, most often girls are discriminated against educational opportunities and are made to do farm work or housework.

Discussion Questions

1. What according to you is the reason for these discriminations? Possible answers:

Case study 1: Very often discrimination based on disability stems from the following:

- Ignorance about the needs of the disabled.
- Insensitivity towards them.
- Lack of accommodation & respect for them.

Case study 2: Discrimination based on girl child education stems from the following:

- Patriarchal setup in our society.
- Poverty and Illiteracy.
- Social customs and evils such as the dowry system that makes parents think twice about educating girls.

2. What other kinds of discrimination do you observe around in the society, which disturb the harmony?

Possible answers:

- **Discrimination based on religion/race:**
 - Men and women from a particular religion in a few states of India say that home owners do not rent their flats to them due to their religious status.
- **Discrimination at workplace with respect to wages:**
 - We can see wage inequalities that occur prominently across gender. Many times an employer pays a woman less than a man. This is not limited to any particular strata of society but exists across sectors and manifests in various ways.
- **Age discrimination:**
 - A healthy 50 year old man applied for a caretaker post in an apartment building. He was rejected because the interviewer thought he was too old for that job.

3. What small steps can we take to cultivate harmony by treating everyone as equal? Possible answers:

- Putting ourselves in others' shoes might help us understand their perspective, views and opinions.
- By changing our mindset, we will be able to change our attitude more easily.
- In all our daily activities, we can consciously look for instances where someone is discriminated against for some reason, and try to address them.
- Raising an awareness in the society about their role in ending discrimination.

Points to reflect upon

- We may differ in appearances, language and social status, but at the core we are all humans and so are equals.
- Discrimination against a set of people has an impact on not just the individuals, but also on the society at large.
- The best way to promote fairness and equality is to be proactive against unfair discrimination.
- Working collectively with a sense of belonging as part of a community leads to harmony.

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Value Content	Peace	Harmony

MS_SSS Congruence Rule

Objective: The student will be able to identify congruent triangles using SSS congruence rule.

Teaching Aids: PPT

Teaching Learning Material (TLM): Big compasses and ruler to draw on the black/white board.

(TLM 1)

Triangle cutouts. **((TLM 2))**

What do you need?: Big compasses and ruler, *two sheets of A4 size thick paper, a pair of scissors, coloured markers, ruler and a pencil.*

Get ready: Use the sheets of paper to prepare the cutouts of two congruent triangles and mark the three pairs of equal sides with different coloured markers.



How to use: The teacher may use the TLM to introduce the SSS rule to the students through:

- (i) construction of triangles using compasses and ruler and
- (ii) superimposition of the triangles cutouts upon one another for showing their congruence.

Notes to the teacher: The teacher may start with a recap of SAS and ASA rules.

You have learnt the **SAS** and **ASA** congruence rules where two triangles are congruent, if either:
Two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle. **(SAS)**

Or

Two angles and the included side of one triangle are equal to two angles and the included side of the other triangle. **(ASA)**

Both these rules involve equality of maximum two sides or two angles of the two triangles. What happens if three angles or three sides are respectively equal in the two triangles?

Case I: Three angles of one triangle are equal to the corresponding three angles of another.

[Notes to the teacher: The students may be allowed to construct different triangles with the same measure of three angles to give their opinion on what happens in this case and whether congruence holds or not, along with their reasoning. The teacher may use the **TLM 1 (Big compasses and ruler)** to do the same on the board.]

Two triangles are not always congruent if three angles of one are equal to three angles of another as we can draw an infinite number of triangles with three given angles.

Case II: Three sides of one triangle are equal to corresponding three sides of another triangle. To check, we can construct a triangle with sides 4 cm, 5 cm and 6 cm.

[**Notes to the teacher:** The teacher may encourage the students to use their compasses to construct the triangle in their notebook and also use the **TLM 1** to draw the triangle on the board with a longer measure of sides.]

Can you draw two different triangles with these measurements of sides? No, we get a unique triangle.

So, all the triangles with these measures will be congruent to this unique triangle. This is true for any measure of sides.

[**Notes to the teacher:** The teacher may use **TLM 2 (Cutouts of triangles)** to show by superimposing one triangle upon another that the two triangles are congruent.

Observe that the two triangles with same measurement of sides are congruent as their cutouts exactly overlap each other.]

We get another rule of congruence of triangles which involves the equality of three sides of two triangles:

SSS congruence rule (Side-Side-Side): If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

This is a theorem which can be proved using a suitable construction and SAS rule. Let's consider some examples.

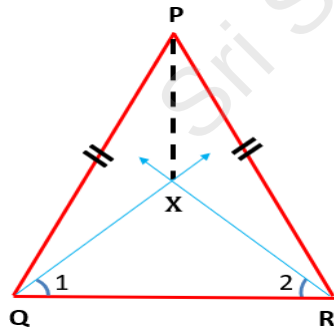
[**Notes to the teacher:** While solving problems, the teacher may encourage the students to:

- read the question and draw a labelled figure to represent the information given in the question.
- use the figure and break down the question into what is given (Given) and what is to be proved (To Prove).
- observe the triangles and check for existing equalities based on what is given and what can be derived from the given information to identify the congruence rule that can be applied.
- write them all step by step and get the 'Proof'.]

Example 1

ΔPQR is isosceles with $PQ = PR$, bisectors of $\angle Q$ and $\angle R$ intersect at point X . Prove that Δ

$PXQ \cong \Delta PXR$



Suggested Answer:

Given: ΔPQR with $PQ = PR$ and QX bisects $\angle Q$ and RX bisects $\angle R$. PX is joined.

To Prove: $\Delta PXQ \cong \Delta PXR$.

Proof:

In ΔPQR

$PQ = PR$ (Given)

$\Rightarrow \angle PQR = \angle PRQ$ (Angles opposite to equal sides)

$\Rightarrow \frac{1}{2} \angle PQR = \frac{1}{2} \angle PRQ$

$\Rightarrow \angle 1 = \angle 2$ (QX bisects $\angle Q$ and RX bisects $\angle R$)

$\therefore$ In ΔXQR

$XQ = XR$ (Sides opposite equal angles) (i)

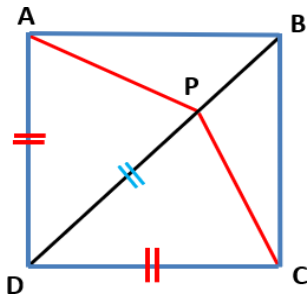
In ΔPXQ and ΔPXR

$PQ = PR$ (Given) $PX = PX$ (Common) $XQ = XR$ [From (i)]

$\therefore \Delta PXQ \cong \Delta PXR$ (SSS rule) Hence proved.

Example 2

P is a point in the interior of a square ABCD such that it is equidistant from vertices A and C. Prove that point P lies on the diagonal BD.



Suggested Answer:

Given: Square ABCD, with point P in its interior such that $AP = CP$

To prove: Point P lies on BD or points D, P and B are collinear.

Proof:

In ΔADP and ΔCDP

$AD = CD$ (Sides of square ABCD) $DP = DP$ (Common)

$AP = CP$ (Given)

$\therefore \Delta ADP \cong \Delta CDP$ (SSS rule)

$\Rightarrow \angle APD = \angle CPD$ (CPCT) (i)

Similarly $\Delta ABP \cong \Delta CBP$ (SSS rule) and $\angle APB = \angle CPB$ (CPCT) (ii)

$\angle APD + \angle CPD + \angle APB + \angle CPB = 360^\circ$ (Angles at same point P)

$\Rightarrow 2\angle APD + 2\angle APB = 360^\circ$ [From (i) and (ii)]

$\Rightarrow \angle APD + \angle APB = 180^\circ$

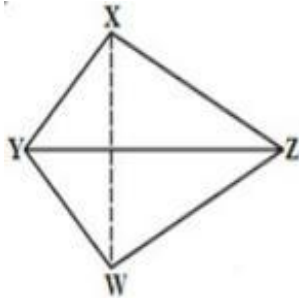
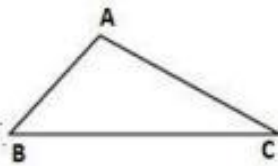
$\Rightarrow$ D, P and B are collinear. So, point P lies on BD. Hence Proved.

[Notes to the teacher: The teacher may explain the proof of the theorem SSS as follows, if required:

Given: ΔABC and ΔXYZ with $AB = XY$, $BC = YZ$ and $AC = XZ$.

To Prove: $\Delta ABC \cong \Delta XYZ$

Construction: Draw YW such that $\angle ZYW = \angle B$ and $WY = AB$. Join XW and WZ.



Proof: In $\triangle ABC$ and $\triangle WYZ$

$BC = YZ$ (Given)

$\angle B = \angle ZYW$ (Construction) $AB = YW$ (Construction)

$\therefore \triangle ABC \cong \triangle WYZ$ (SAS congruence axiom)

$\Rightarrow \angle A = \angle W$ and $AC = WZ$ (CPCT) _____ (i)

We have, $AB = XY$ (Given) and $AB = YW$ (Construction)

$\therefore XY = YW$ and similarly $XZ = WZ$ (ii)

In $\triangle XYW$

$XY = YW$ [From (ii)]

$\therefore \angle YWX = \angle YXW$ (Angles opposite to equal sides) (iii)

Similarly $\angle ZWX = \angle ZWX$ (iv)

Adding steps (iii) and (iv)] we get $\angle W = \angle X$

Also $\angle W = \angle A$ (From (i)]

$\therefore \angle A = \angle X$

Now, in $\triangle ABC$ and $\triangle XYZ$

$AB = XY$ and $AC = XZ$ (Given)

$\angle A = \angle X$

$\therefore \triangle ABC \cong \triangle XYZ$ (SAS congruence axiom)]

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Triangle	Criterion for SSS congruence

MS_RHS Congruence Rule

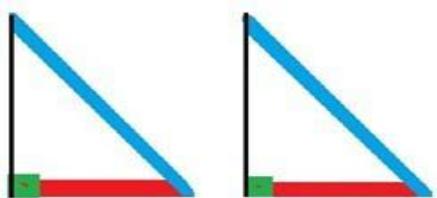
Objective: The student will be able identify congruent triangles using the RHS congruence rule.

Teaching Aids: PPT

Teaching Learning Material (TLM): Cut-outs of Right Triangles

What do you need?: Two sheets of A4 size thick papers, a pair of scissors, coloured markers, a ruler and a pencil.

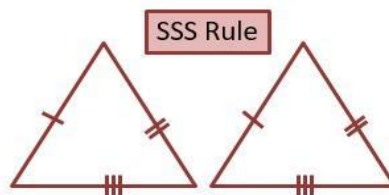
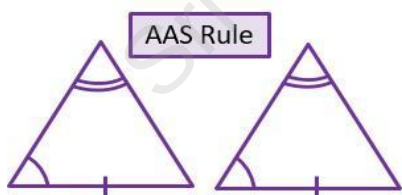
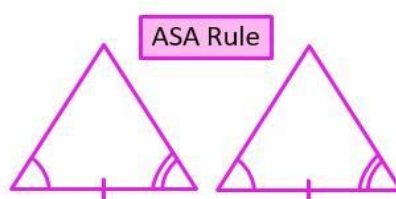
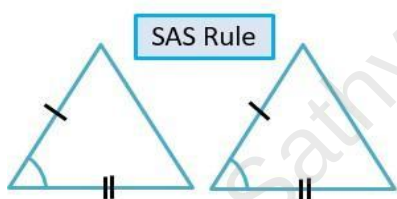
Get ready: Use the two sheets of paper to prepare cutouts of two congruent right angled triangles and mark the right angle and also the hypotenuse and the base of the triangles cutouts with different coloured markers.



How to use: The teacher may use the TLM to introduce the RHS rules to the students by superimposing one cutout upon another for showing their congruence.

Notes to the teacher: The teacher may start with a recap of the congruence rules learnt by them.

You have learnt about four criteria for congruence of triangles which are **SAS**, **ASA**, **AAS** and **SSS**. These criteria of congruence can be used to identify congruent triangles of all kinds.



You will find it interesting to know that there is one more criteria for congruence of triangles which is applicable only to right triangles. So, while there are four rules of congruence for triangles of other kinds, right triangles have five congruence rules.

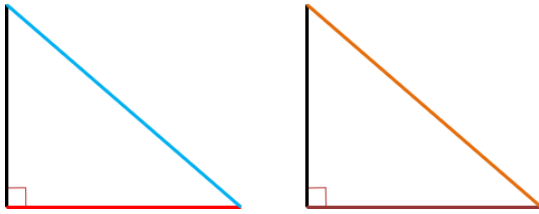
[**Notes to the teacher:** The teacher may use the TLM to show by superimposing the triangles cutouts, the RHS congruence rule.]

If we superimpose two right triangles with one pair of sides and hypotenuse equal, they cover each other exactly even though the equal right angle is not an included angle.

We get the following congruence rule:

RHS congruence rule (Right-Hypotenuse-Side): Two right triangles are congruent if the

hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle.



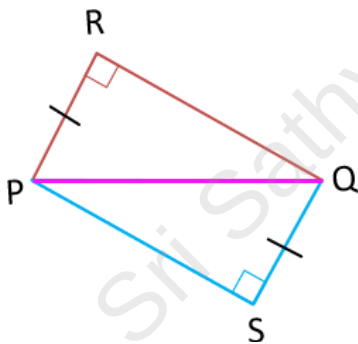
This result is a theorem that can be proved using the SAS axiom and a suitable construction. Let's consider some examples.

[Notes to the teacher: While solving problems, the teacher may encourage the students to:

- read the question and draw a labelled figure to represent the information given in the question.
- use the figure and break down the question into what is given (Given) and what is to be proved (To Prove).
- observe the triangles and check for existing equalities based on what is given and what can be derived from the given information to identify the congruence rule that can be applied.
- write them all step by step and get the 'Proof'.

Example 1

ΔPRQ and ΔPSQ are right angled at R and S respectively. If $PR = QS$, prove that $\angle PQR = \angle QPS$.



Suggested Answer:

Given: ΔPRQ and ΔPSQ with $\angle R = \angle S = 90^\circ$ and $PR = QS$.

To prove: $\angle PQR = \angle QPS$. **Proof:** In ΔPRQ and ΔPSQ $PR = QS$ (Given)

$PQ = PQ$ (Common)

$\angle R = \angle S = 90^\circ$ (Given)

$\therefore \Delta PRQ \cong \Delta PSQ$ (RHS rule)

$\Rightarrow \angle PQR = \angle QPS$ (CPCT)

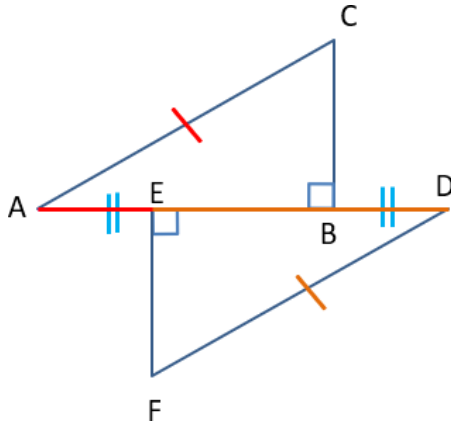
Hence proved.

Example 2

ΔABC and ΔDEF are right angled at B and E respectively. If $AC = DF$ and $AE = DB$, prove that $\Delta ABC \cong \Delta DEF$.

Suggested Answer:

Given: ΔABC and ΔDEF with $\angle B = \angle E = 90^\circ$, $AC = DF$ and $AE = DB$.



To prove: $\Delta ABC \cong \Delta DEF$ **Proof:** In ΔABC and ΔDEF $AC = DF$ (Given)

$AE = DB$ (Given)

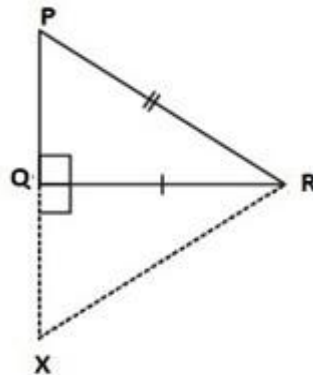
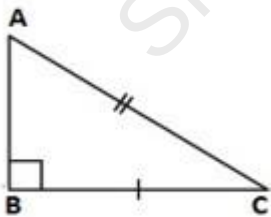
$\Rightarrow AE + EB = DB + EB$ (Adding EB on both sides)

$\Rightarrow AB = DE$

$\angle B = \angle E = 90^\circ$ (Given)

$\therefore \Delta ABC \cong \Delta DEF$ (RHS rule) Hence proved.

[Notes to the teacher: If required, the teacher may explain the proof of the RHS theorem to the students as follows:



Given: Two right triangles, ΔABC and ΔPQR with $\angle B = 90^\circ$ and $\angle Q = 90^\circ$; $AC = PR$ and $BC = QR$.

To prove: $\Delta ABC \cong \Delta PQR$

Construction: Produce PQ to X so that QX = AB. Join XR.

Proof:

In $\triangle ABC$ and $\triangle XQR$ AB = QX (Construction)

$$\angle ABC = \angle XQR = 90^\circ$$

BC = QR (Given)

$\therefore \triangle ABC \cong \triangle XQR$ (SAS rule)

$\Rightarrow$ So $\angle A = \angle X$ (CPCT) (1)

AC = XR (CPCT) (2)

Also, AC = PR (Given)

Therefore PR = XR (From the above) So, in $\triangle RPX$

$\angle P = \angle X$ (Angles opposite to equal sides) (3)

We get $\angle A = \angle P$ (From (1) and (3)) (4)

Thus, in $\triangle ABC$ and $\triangle PQR$

$\angle A = \angle P$ [From (4)]

$\angle B = \angle Q$ (Given)

So, $\angle A + \angle B = \angle P + \angle Q$ (on adding)

But $\angle A + \angle B + \angle C = 180^\circ$ and $\angle P + \angle Q + \angle R = 180^\circ$ (angle sum property of triangle)

$\Rightarrow \angle A + \angle B = 180^\circ - \angle C$ and $\angle P + \angle Q = 180^\circ - \angle R$ But $\angle A + \angle B = \angle P + \angle Q$

$\Rightarrow 180^\circ - \angle C = 180^\circ - \angle R$

$\Rightarrow \angle C = \angle R$ (Cancellation laws) (5)

Now, in $\triangle ABC$ and $\triangle PQR$ BC = QR (given)

$\angle C = \angle R$ [from (5)] AC = PR (given)

$\therefore \triangle ABC \cong \triangle PQR$ (SAS axiom of congruence) Hence Proved.]

Time to teach	Asset type	Theme	Sub Theme
10 minutes	Main Script	Triangle	Criterion for RHS congruence

SA_Project Congruent Triangles

Objective: The student will be able to build a 3-D model or an art design based on the concept of congruence of triangles.

Teaching Aids: PPT and Project report template.

Notes to the teacher: The teacher may ask the students to select one of the following topics for the project:

To use congruent triangles in creating an art design or to build a 3-D model of a truss bridge using a framework of congruent triangles made up of straight pieces called members. The project may be graded for internal assessment.

The teacher may give the instructions and guide the students on how to do the project and provide 1 week to submit the model and project report. The students may be guided to prepare the report according to the template.

Aim:

A) To design and build a model of a truss bridge. Or

B) To prepare an art design for tiling the wall of a room, using cutouts of congruent triangles.

Setting for the Activity: To be done as homework.

Type of Activity: Group project.

Preparation of Activity: The class is to be divided into groups of minimum three students each.

Role of the Teacher: Mentor/Guide

Project: (A) Building a Bridge

Work in groups to design and build a 3-D model of a truss bridge, with length 30 cm - 60 cm and height 7 cm - 14 cm.

or **(B) Mathematics to Art:**

Work in groups to create an art design for tiling a wall of a room, using congruent triangles (paper cutting and pasting). The maximum size of the design can be 40 cm x 40 cm.

Resources Required:

S.NO	Building a Bridge	Mathematics to Art
1	Straws/broom sticks/rolled paper sticks/ice cream sticks - the number depends on the design and the size of the bridge.	Thick sheets of paper of different colours. Number depending on the design and the size of the art design.
2	A cardboard to mount the model design - Size as per the size and design.	

3	Craft Glue/adhesive tape.
4	Cutter/pair of scissors
5	A pencil and a ruler
6	White paper
7	Project Report File

Procedure:

1. Plan the design and dimensions of the model/design and also the type and the quantity of material to be used. Research different designs and ways of creating it before starting to create your own.
2. Prepare a workflow for completing the project, in consultation with the group members. Allocate and fix responsibilities within your team.
3. Make a simple and neat drawing of the design, which can be referred to while building the model/design.

- A. **Building a Bridge:** To guide you on the procedure of building the bridge model, the following simple design is used. (Figure 1). Challenge for you is to choose a more complex design and stronger material.

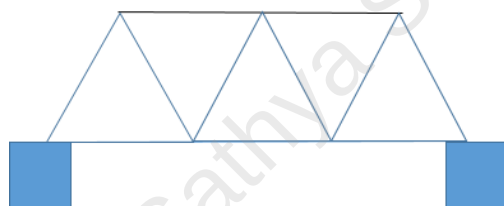


Figure 1

- a. Cut the straws into small pieces of two different lengths, say 3 cm and 6 cm. (Figure 2)

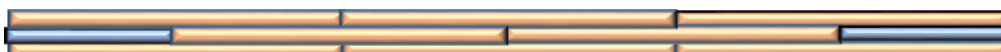


Figure 2

- b. Lay the pieces end to end in a line as shown in figure 3. Paste them with their ends overlapping to get the upper and lower beams for the side supports of the bridge. Make two sets of each. (Figure 3)



Upper Beam



Lower Beam

Figure 3

c. Fix double pieces of straw in the shape of congruent triangles, to the upper and lower beams of the side supports of the bridge. The pieces should have their ends slightly overlapping. Refer to the design of your model to decide the height and the type of triangles. Make two such side supports. (Figure 4)

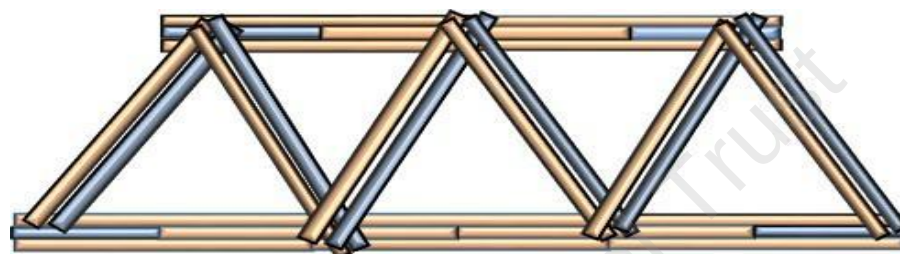


Figure 4

d. Make the side supports of your bridge stand erect and fix pieces of straws across the bottom beams of both the sides. Fix some straws on the top also. (Figure 5)

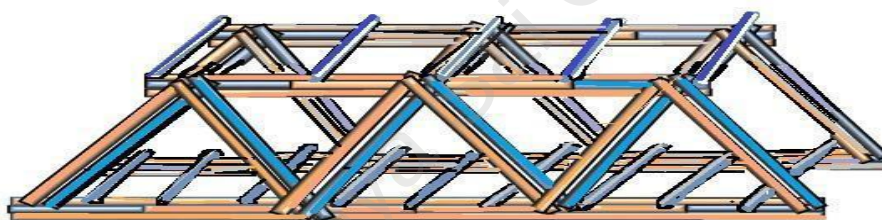


Figure 5

e. A deck made up of straw pieces can be placed on the bottom straws of the bridge for small toy cars to pass. Your bridge is ready!

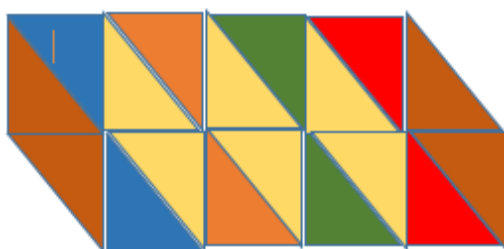
f. Place it with its ends resting on the edge of two chairs. You can test the strength of your bridge by putting small objects on its base and gradually increasing the weight. A spring scale available in your physics lab can also be used for the strength test.

(B) Mathematics to Art

a. Cut out congruent triangles from coloured sheets of paper. The colour, size and the type of the triangles depends on the design you have planned.



b. Prepare a cardboard mount and paste the triangles on it according to your design.



c. Your tile design is ready! Give it a title.

Model/design and the project reports to be submitted within the specified time

Follow-up activity:

- The students may use the concept of congruent triangles to create a model of geodesic dome using straws.
- The models prepared by the students may be displayed in the exhibitions conducted in the school.

Precautions: The group members must ensure that it is in the knowledge of their parents, where and when they work together on the home project.

Troubleshooting:

- ✓ The teacher should be available to guide the students during the assignment.
- ✓ The teacher must ensure that the students get the support of the school for undertaking research during the assignment. (e.g. internet access, access to library, etc.) and also in providing the resources for it as per the school policy.

Time to teach	Asset type	Theme	Sub Theme
15 minutes	Suggested Activity	Triangle	Congruent triangles

QA_Applying SSS and RHS Rules

Objective: The student will be able to solve problems using the SSS and RHS rules of congruence.

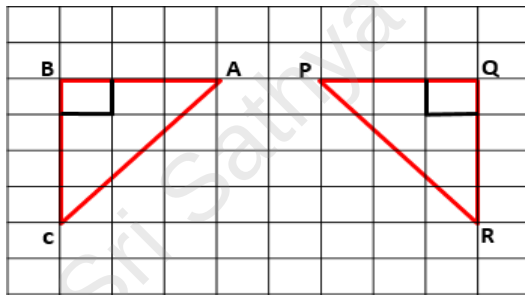
Teaching Aids: PPT/Worksheet.

Notes to the teacher: Teacher may present the questions and allow the students to write or say the answers before revealing them.

I. Choose the correct option in each of the following:

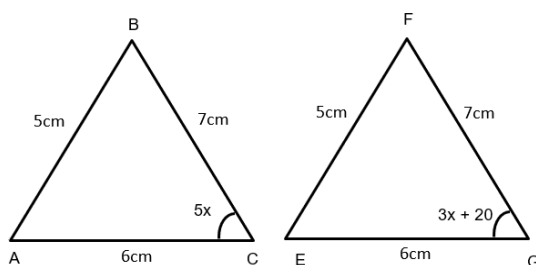
1) Observe the given figure and choose the correct option which represents the congruence of triangles in symbolic form.

- a) $\triangle ABC \cong \triangle PRQ$
- b) $\triangle ABC \cong \triangle PQR$
- c) $\triangle ABC \cong \triangle RPQ$
- d) $\triangle BCA \cong \triangle QPA$



Correct option: b) $\triangle ABC \cong \triangle PQR$

2) Find the value of x in the following figure



Suggested Answer:

In the $\triangle ABC$ and $\triangle EFG$, $AB = EF = 5\text{cm}$ (Given)

$BC = FG = 7\text{cm}$ (Given) $AC = EG = 6\text{cm}$ (Given)

Hence, $\triangle ABC \cong \triangle EFG$ (SSS criteria)

$\angle BCA = \angle FGE$ (by CPCT)

$$\therefore 5x = 3x + 20$$

$$\Rightarrow 5x - 3x = 20$$

$$\Rightarrow 2x = 20$$

$$\Rightarrow x = 10$$

3) If $AB = EF$, $BC = DF$ and $CA = DE$ then

a) $\triangle ABC \cong \triangle DEF$

b) $\triangle CBA \cong \triangle DFE$

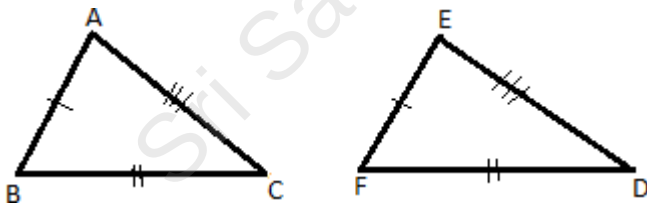
c) $\triangle BAC \cong \triangle FDE$

d) $\triangle DEF \cong \triangle BCA$

Correct option: b) $\triangle CBA \cong \triangle DFE$

Suggested Answer:

Given that $AB = EF$, $BC = FD$ and $CA = DE$ The triangles are as follows



Thus $A \rightarrow E$, $B \rightarrow F$ and $C \rightarrow D$

4) In $\triangle ABC$ and $\triangle PRQ$, $\angle B = \angle R = 90^\circ$ and $BC = RQ$

The two triangles are congruent by RHS criterion if

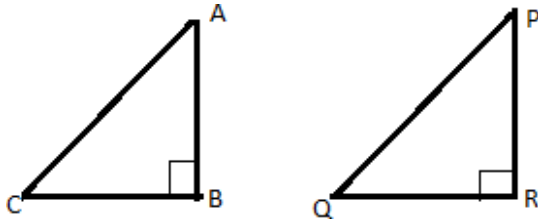
a) $AC = PQ$ b) $AB = PQ$ c) $AC = PR$ d) $AB = PR$

Correct option: a) $AC = PQ$

Suggested Answer:

$$\angle B = \angle R = 90^\circ \text{ and } BC = RQ$$

By RHS criteria of congruence, $\triangle ABC \cong \triangle PRQ$ if the hypotenuse of the triangles are also equal.



II. Bella went to Egypt with her family. She saw the pyramid and wondered if the triangular faces were congruent. Clear her confusion, by proving that the faces of the pyramid are congruent.

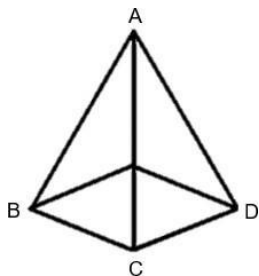
[Note: These are considered to be equilateral square pyramids]



Pyramid: flickr.com/photos/girolame/2427530661 - Attribution: Jerome Bon

Suggested Answer:

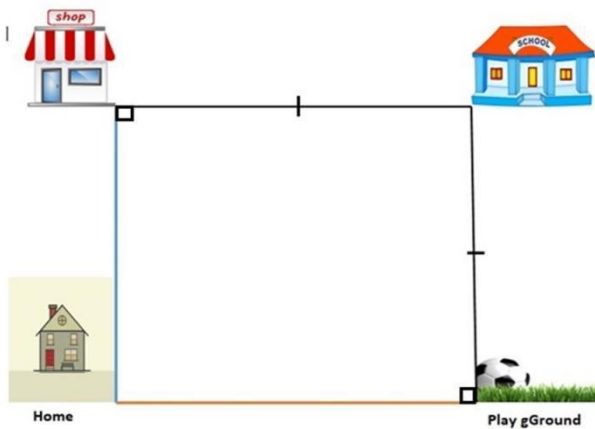
In the $\triangle ACB$ and $\triangle ACD$,



$AB = AD$ (Sides of equilateral triangle)

$CB = CD$ (Sides of equilateral triangle) $AC = AC$ (Common)

Hence, $\triangle ACB \cong \triangle ACD$ (by SSS criteria)



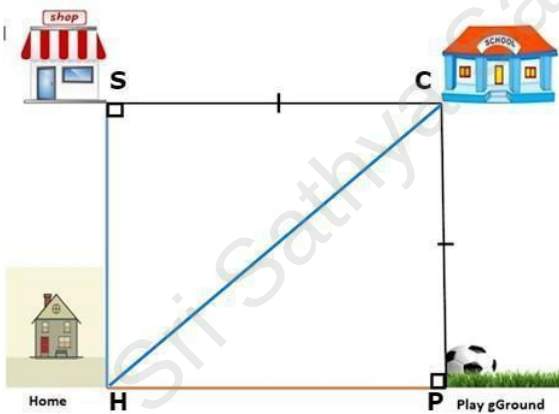
III. Rahul walks to his school. There is no direct road from his home to school. He usually takes the route to school via the playground. The shop and the playground are equidistant from the school. Show using a congruence rule that he walks the same distance if he takes the route via the shop.

Candy House: pixabay.com/vectors/shop-candy-home-house-shopping-150362/ - Attribution: OpenClipart-Vectors

School: pixabay.com/vectors/school-building-education-property-295210/ - Attribution: Clker-Free-Vector-Images

Football: pixabay.com/photos/football-soccer-ball-sport-field-1396740/ - Attribution: AnnRos

House: pixabay.com/vectors/house-icon-symbol-architecture-2492054/ - Attribution: janjf93



Suggested Answer:

Let Home be represented as H, Shop as S, School as C and Playground as P. Join HC.

In the $\triangle HCP$ and $\triangle HCS$,

$$\angle HSC = \angle HPC = 90^\circ \text{ (Given)}$$

$$SC = PC \text{ (Given)}$$

$$HC = HC \text{ (Common)}$$

Hence, $\triangle HCP \cong \triangle HCS$ (RHS criteria)

$$HS = HP \text{ (by CPCT)}$$



Sri Sathya Sai
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Sri Sathya Sai Central Trust

Annexure to Master Lesson Plan

What is the purpose of this Master lesson Plan?	This Master Lesson Plan (MLP) prepared by Sri Sathya Sai Vidya Vahini (SSSVV), covers the entire chapter for the given board. The MLP for the chapter given in the textbook is broken down into smaller modules known as assets. Any chapter can be taught using the 7 asset types in SSSVV methodology. Several different assets together complete the entire chapter. Depending on the requirement, each asset is supported by teaching aids such as animated presentation, audio, video, and learning aids such as worksheets. The MLP is for the teacher's preparation and the teaching aids are for use in the classroom.
How to use it?	You can prepare for your class with this MLP by following 3 simple steps: 1. Read your textbook; 2. Go through the information given in the assets (document & multimedia); 3. Prepare your teaching notes. Great! Now you are ready to transact the chapter.
Duration	Kindly note the duration given for each asset. Due care has been taken in planning the assets, to ensure the chapter is completed within the time specified by the Board. Note: Preparation time is not included in the asset duration. The teaching time depends on the duration of the teaching aids and is not impacted by the length of the MLP.
What is a 'Main Script' (MS) Asset?	The Main Script asset is there for explaining the main concepts of the chapter clearly. The information given in the textbook is simplified, organised and structured to give more clarity. Additionally, you may find a video or mnemonics or a graphic organiser to deepen the understanding of the concept.
How to teach using the MS asset?	Please use the blackboard, slides and interaction to develop the concept.
What is an 'Inquisitive Questions' (IQ) asset?	The Inquisitive Questions asset uses questions to promote higher order thinking like the n th why, what-if, new perspectives, cause-effect, and others, creating curiosity.
How to teach using the IQ asset?	You may use the questions to connect with the students, encourage exploration to engage them in the learning process. You may allow multiple responses and instead of rejecting any of the responses, ask why, before revealing the suggested answer or hints.
What is a 'Day to day Relevance' (DD) Asset?	The Day to Day Relevance asset helps students understand the practical relevance of every topic, making them eager participants in the classroom. If students connect the concepts to their environment, they would learn meaningfully without dislike or simply memorizing for exams.
How to teach using the DD	You may use the asset by asking questions about their experience and use it to establish why they are learning the topic. If necessary you may substitute with a

asset?	recent or local example.
What is an 'Interesting Aside' (IA) asset?	The Interesting Aside asset uses attention grabbing titbits or anecdotes to bring joy and satisfy the child's hunger to know more. This is a quick/short asset intended to bring attention back to the main concept being taught.
How to teach using the IA asset?	You may use this asset to give a piece of interesting information relevant to the topic, without prolonged explanation/discussion. You may suggest appropriate books for further reading.
What is a 'Suggested Activity' (SA) Asset?	The Suggested Activity asset provides a detailed step by step procedure for the teacher to conduct a hands-on activity. This promotes Activity Based Learning.
How to teach using the SA asset?	You may use this asset to energise your students to learn by doing simple, fun-filled activities based on the topic. The asset includes - preparation, activity, and follow-up. You may engage the students in the preparation and follow-up stages to develop collaboration and responsibility. You could try it once, yourself, before the actual delivery to be more comfortable with it. Note: The duration mentioned in the asset is usually for the activity part only.
What is a 'Value Content' (VC) Asset?	The Value Content asset integrates Universal Human Values (such as empathy, inclusivity, respect for nature that translates into responsible behaviour) seamlessly into the curriculum, as the "End of education is character".
How to teach using the VC asset?	You may use this asset to engage with the students to bring out their innate values and connect with the intrinsic values in the topic/concept. The asset includes pointers on the specific pedagogical technique followed.
What is a 'Questions to Assess' (QA) asset?	The Questions to Assess asset uses an interactive approach to check learning of different types of learners, and provides feedback to the teacher for appropriate action. The questions cater to all the levels of Bloom's Taxonomy. Questions from 'Apply' level and above enable students to reflect on their learning.
How to teach using the QA asset?	You may go through the slides to understand its flow and know when to click for the answer (slides include suggested answers). While presenting in the class, you may display the question and encourage multiple responses, before revealing the answer. In some cases, this asset may include extra questions/worksheet that can be given as homework.