

Objective

To study the motion of a projectile for different launch angles, initial speeds, and projection heights, and to observe the variation in its trajectory, maximum height, horizontal range, and time of flight.

Theory

Projectile motion is the motion of an object projected into the air with an initial velocity and allowed to move under the action of gravity alone. No air resistance, the object follows a parabolic path.

The motion can be resolved into two independent components:

- **Horizontal motion**, which takes place with constant velocity.
- **Vertical motion**, which takes place with constant acceleration due to gravity (as 9.8m/sec^2).

If a projectile is launched with an initial speed v_0 at an angle θ with the horizontal, then

- Horizontal component of velocity:

$$v_x = v_0 \cos \theta$$

- Vertical component of velocity:

$$v_y = v_0 \sin \theta$$

At any instant t , the position of the projectile is given by

- **Horizontal displacement:**

$$x = v_0 \cos \theta \times t$$

- **Vertical displacement:**

$$y = v_0 \sin \theta \times t - \frac{1}{2}gt^2$$

The equation of the trajectory is

$$y = \frac{x \tan \theta - (gx^2)}{2(v_0)^2 \cos^2 \theta}$$

The important characteristics of projectile motion are:

- **Maximum Height**

$$H = \frac{(v_0^2 \sin^2 \theta)}{(2g)}$$

- **Time of Flight**

$$T = \frac{(2v_0 \sin \theta)}{(g)}$$

- **Horizontal Range**

$$R = \frac{(v_0^2 \sin 2\theta)}{(g)}$$

For a given initial speed, the horizontal range is maximum when the angle of projection is 45° . Two complementary angles θ and $(90^\circ - \theta)$ produce the same horizontal range but different maximum heights and trajectories.

Horizontal Projection

When an object is projected horizontally from a height h , its initial vertical velocity is zero.

The corresponding equations are:

- **Horizontal displacement:**

$$x = v_0 t$$

Vertical displacement:

$$x = h - \frac{1}{2}gt^2$$

- **Time of flight:**

$$T = \sqrt{\frac{2h}{g}}$$

- **Horizontal range:**

$$T = v_0 \sqrt{\frac{2h}{g}}$$

- **Speed at impact:**

$$v = \sqrt{v_0^2 + 2gh}$$

These relations show that horizontal and vertical motions are independent of each other and combine to produce a parabolic trajectory.

Learning Outcomes

After completing this activity, learners will be able to:

- Overview of the concept of projectile motion.
- Resolve velocity into horizontal and vertical components.

- Calculate maximum height, horizontal range, and time of flight.
- Describe the effect of launch angle and speed on the trajectory.
- Verify that complementary angles produce equal horizontal ranges.
- Interpret graphs of trajectory and velocity during projectile motion.