

1.2 Operations with Whole Numbers

In this section we will consider the four basic operations with whole numbers: addition, subtraction, multiplication, and division. It is assumed that you are familiar with computations, which are reviewed only briefly here. Our emphasis will be on how the operations are related to each other, as well as their application to real-world situations.

Addition represents the idea of finding a total count, or summing up, of values. Since we use only ten digits in our system (remember base 10), it is often necessary to use place value to “carry” digits.

Example 1 Find the sum: $458 + 375 + 296$

Solution Unless you are using a calculator, it is easier to organize this problem vertically:

$$\begin{array}{r} 458 \\ 375 \\ +296 \\ \hline \end{array}$$

Starting in the ones place value, $8 + 5 + 6 = 19$. Representing this sum as “carrying” into the tens place:

$$\begin{array}{r} 1 \\ 458 \\ 375 \\ +296 \\ \hline 9 \end{array}$$

Now working in the tens place value, $1 + 5 + 7 + 9 = 22$. Representing this sum as “carrying” into the hundreds place:

$$\begin{array}{r} 21 \\ 458 \\ 375 \\ +296 \\ \hline 29 \end{array}$$

Finally working in the hundreds place value, $2 + 4 + 3 + 2 = 11$. Representing this sum as “carrying” into the thousands place:

$$\begin{array}{r} 21 \\ 458 \\ 375 \\ +296 \\ \hline 1129 \end{array}$$

Thus $458 + 375 + 296 = 1129$.

Example 2 The following table lists the monthly expenditures for food for the Jamison family in 1999:

month	expense (\$)
January	386
February	335
March	287
April	264
May	279
June	315
July	298
August	214
September	269
October	314
November	368
December	416

Find the total expenditure in food for the Jamison family in 1999.

Solution The total expenditure represents the sum of all twelve values in this table.
Representing this sum vertically:

$$\begin{array}{r}
 67 \\
 386 \\
 335 \\
 287 \\
 264 \\
 279 \\
 315 \\
 298 \\
 214 \\
 269 \\
 314 \\
 368 \\
 +416 \\
 \hline
 3745
 \end{array}$$

The Jamison's total expenditure in 1999 is \$3,745.

Note the following **properties of addition**:

Property Name	Property	Example
Commutative property	$a + b = b + a$	$5 + 8 = 8 + 5$
Associative property	$a + (b + c) = (a + b) + c$	$6 + (4 + 5) = (6 + 4) + 5$
Identity property	$a + 0 = 0 + a = a$	$8 + 0 = 0 + 8 = 8$

These properties are our first example of formalizing things we know about numbers, which is common in algebra. For example, the commutative property merely states we can add two numbers in either order. The associative property states that when adding more than two numbers, the method of grouping the numbers does not matter. And finally, the identity property states that adding 0 to a number does not change the number's value. The use of properties in algebra is commonplace, so I will introduce them in this textbook to familiarize you with their names and uses.

Subtraction of whole numbers is a natural result of an addition, called the **inverse operation** of addition. Given the addition statement $6 + 4 = 10$, there are two associated subtraction statements:

$$10 - 6 = 4 \quad \text{and} \quad 10 - 4 = 6$$

Thus subtraction represents the idea of “undoing” addition. In general, if $a + b = c$, then $c - a = b$ and $c - b = a$.

Example 3 Find the difference: $1426 - 548$

Solution As with addition, we align the problem vertically and then “borrow” from the next higher place value, when necessary. Borrowing 1 ten = 10 ones, then subtracting $16 - 8 = 8$:

$$\begin{array}{r} 1 \\ 1426 \\ -548 \\ \hline 8 \end{array}$$

Now borrowing 1 hundred = 10 tens, then subtracting $11 - 4 = 7$:

$$\begin{array}{r} 31 \\ 1426 \\ -548 \\ \hline 78 \end{array}$$

Finally borrowing 1 thousand = 10 hundreds, then subtracting $13 - 5 = 8$:

$$\begin{array}{r} 31 \\ 878 \\ 1426 \\ -548 \\ \hline 878 \end{array}$$

Thus $1426 - 548 = 878$. Since subtraction is the inverse operation of addition, this answer can be checked by performing the addition $878 + 548 = 1426$.

Example 4 After depositing his \$428 weekly paycheck, Jose has \$1145 in his checking account. How much was in his account before the deposit?

Solution Thinking of B as the balance prior to the deposit, the statement $B + 428 = 1145$ correctly interprets the deposit and final balance. Since subtraction is the inverse operation of addition, then $B = 1145 - 428$. Computing the subtraction:

$$\begin{array}{r} 3 \\ 1145 \\ -428 \\ \hline 717 \end{array}$$

Thus \$717 was in the account before the deposit. Again, note that this answer can be checked by computing the sum $717 + 428 = 1145$.

Note in this past example how we used a symbol B (called a **variable**) to represent a quantity we were trying to find. We will frequently do this in applications if it makes the interpretation and resulting solution of the problem easier.

If you refer back to the three addition properties we listed (commutative, associative, identity), note that none of them are true when the $+$ symbol is replaced with $-$. That is, $a - b \neq b - a$, $a - (b - c) \neq (a - b) - c$, and $a - 0 \neq 0 - a \neq a$ (although $a - 0 = a$ is true). We say that subtraction is not commutative nor associative, and there is no identity property for subtraction.

Multiplication of whole numbers represents the idea of repeated addition. For example:

$$4 \cdot 5 = 5 + 5 + 5 + 5 = 20$$

$$5 \cdot 4 = 4 + 4 + 4 + 4 + 4 = 20$$

Note that this illustrates that multiplication is commutative, as was addition. Summarizing the **properties of multiplication**:

Property Name	Property	Example
Commutative property	$a \cdot b = b \cdot a$	$5 \cdot 4 = 4 \cdot 5$
Associative property	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$5 \cdot (2 \cdot 3) = (5 \cdot 2) \cdot 3$
Identity property	$a \cdot 1 = 1 \cdot a = a$	$12 \cdot 1 = 1 \cdot 12 = 12$
Multiplication property of 0	$a \cdot 0 = 0 \cdot a = 0$	$8 \cdot 0 = 0 \cdot 8 = 0$
Zero factor property	If $a \cdot b = 0$, $a = 0$ or $b = 0$	If $5 \cdot x = 0$, $x = 0$

Note that the commutative and associative properties are similar to those of addition. The identity property for multiplication is similar to that of addition, except that 1 is the number used for the identity (rather than 0 as was used in addition). Notice the two new properties for multiplication involving the number 0. The multiplication property of 0 states that multiplication of anything by 0 always results in 0, and the zero factor property states that if a product results in 0, then one of the original numbers being multiplied (called **factors**) must be 0.

Since we cannot re-write every multiplication problem as addition, we use memorized values from elementary school and carry as in addition.

Example 5 Compute the product: $256 \cdot 47$

Solution As with addition, start by writing the product vertically:

$$256$$

$$\times \quad 47$$

Starting with the ones place, $6 \cdot 7 = 42$. Carrying and continuing the multiplication:

$$34$$

$$256$$

$$\times \quad 47$$

$$1792$$

Now moving to the tens place (remember the 4 represents 40):

$$122$$

$$256$$

$$\times \quad 47$$

$$1792$$

$$\underline{10240}$$

Finally adding the two products:

$$256$$

$$\times \quad 47$$

$$1792$$

$$\underline{10240}$$

$$12032$$

Thus $256 \cdot 47 = 12,032$.

Example 6 The Wheeler family makes monthly mortgage payments of \$846 for a total of 15 years. What is the total amount of their payments?

Solution First we need to find the number of payments they make during the 15 years. Since they make a payment each month, and there are 12 months in a year, they make a total of

$$15 \cdot 12 = 180 \text{ payments}$$

during the 15 year period of time. Since each payment is \$846 and they make 180 payments, they pay a total of

$$180 \cdot \$846 = \$152,280$$

The total amount of the Wheeler's payments is \$152,280.

Division of whole numbers represents the idea of repeated subtraction. For example:

$$36 \div 12 = 3 \text{ since } 36 - 12 - 12 - 12 = 0$$

$$15 \div 3 = 5 \text{ since } 15 - 3 - 3 - 3 - 3 - 3 = 0$$

Division is more commonly thought of as the inverse operation for multiplication. For example:

$$36 \div 12 = 3 \text{ since } 3 \cdot 12 = 36$$

$$36 \div 3 = 12 \text{ since } 12 \cdot 3 = 36$$

$$15 \div 3 = 5 \text{ since } 5 \cdot 3 = 15$$

$$15 \div 5 = 3 \text{ since } 3 \cdot 5 = 15$$

In general, if $a \cdot b = c$, then $c \div a = b$ and $c \div b = a$.

Example 7 Compute the quotient: $6120 \div 45$

Solution Writing the quotient in our more traditional form, and using the “guess and subtract” technique learned in elementary school:

$$\begin{array}{r} 136 \\ 45 \overline{)6120} \\ \underline{45} \\ 162 \\ \underline{135} \\ 270 \\ \underline{270} \\ 0 \end{array}$$

Thus $6120 \div 45 = 136$. Note that we can check this quotient with the multiplication $136 \cdot 45 = 6120$. We say that the **quotient** is 136 and the **remainder** is 0. Often when the remainder is 0 we merely say the quotient is 136 (and the remainder is assumed to be 0).

Example 8 Compute the quotient: $8495 \div 27$

Solution Using the same approach as in Example 7:

$$\begin{array}{r} 314 \\ 27 \overline{)8495} \\ \underline{81} \\ 39 \\ \underline{27} \\ 125 \\ \underline{108} \\ 17 \end{array}$$

Thus $8495 \div 27 = 314$, with a remainder of 17. This answer can be summarized with the notation $314 \text{ R } 17$. To check this quotient, compute $27 \cdot 314 = 8478$, then add on the remainder $8478 + 17 = 8495$. That is, $27 \cdot 314 + 17 = 8495$.

Example 9 The Almos family borrows \$20,880 to purchase a new car at a special 0% interest rate (we'll deal with interest later in Chapter 6). The car dealer allows them 5 years to pay back the amount they borrow, and requires equal monthly payments. How much are their monthly payments?

Solution Since there are 12 months in each year, they must make a total of $5 \cdot 12 = 60$ payments on the loan. Dividing \$20,880 by 60 will result in the monthly payment:

$$\begin{array}{r} 348 \\ 60 \overline{)20880} \\ \underline{180} \\ 288 \\ \underline{240} \\ 480 \\ \underline{480} \\ 0 \end{array}$$

The Almos' monthly payment will be \$348. As a check $60 \cdot 348 = 20,880$.

Terminology

addition
subtraction
variable
properties of multiplication
division
remainder

properties of addition
inverse operation
multiplication
factors
quotient

Exercise Set 1.2

Perform the following additions and subtractions.

- | | |
|------------------------|------------------------|
| 1. $2,456 + 8,946$ | 2. $892 + 5,688$ |
| 3. $98 + 1,856$ | 4. $9,568 + 5,487$ |
| 5. $17,847 + 6,879$ | 6. $6,956 + 65,462$ |
| 7. $534 - 276$ | 8. $1,002 - 453$ |
| 9. $2,231 - 859$ | 10. $12,458 - 5,674$ |
| 11. $101,200 - 53,432$ | 12. $102,101 - 57,234$ |

Perform the following multiplications.

- | | |
|------------------------|------------------------|
| 13. $56 \cdot 35$ | 14. $28 \cdot 57$ |
| 15. 154×87 | 16. 268×67 |
| 17. $1,859 \cdot 68$ | 18. $2,695 \cdot 465$ |
| 19. $10,000 \cdot 64$ | 20. $95 \cdot 100,000$ |
| 21. 101×1001 | 22. 111×1011 |
| 23. $5,305 \times 132$ | 24. $6,487 \times 328$ |

Perform the following divisions.

- | | |
|---------------------------|--------------------------|
| 25. $2,668 \div 58$ | 26. $7,743 \div 89$ |
| 27. $12,549 \div 47$ | 28. $38,090 \div 65$ |
| 29. $8,365 \div 27$ | 30. $9,740 \div 48$ |
| 31. $14,846 \div 124$ | 32. $33,429 \div 132$ |
| 33. $1,450,000 \div 1000$ | 34. $560,000 \div 100$ |
| 35. $105,812 \div 1,740$ | 36. $867,594 \div 2,317$ |

In the following exercises, a property of whole numbers is illustrated. Give the name of the property being illustrated.

- | | |
|--|--|
| 37. $45 + 87 = 87 + 45$ | 38. $223 + 0 = 0 + 223 = 223$ |
| 39. $19 \cdot (5 \cdot 24) = (19 \cdot 5) \cdot 24$ | 40. $12 \cdot 95 = 95 \cdot 12$ |
| 41. $(15 + 23) + 19 = 15 + (23 + 19)$ | 42. Given $25 \cdot x = 0$, then $x = 0$. |
| 43. $33 \cdot 100 = 100 \cdot 33$ | 44. $12 \cdot 1 = 1 \cdot 12 = 12$ |
| 45. $29 \cdot 0 = 0 \cdot 29 = 0$ | 46. $17 + 820 = 820 + 17$ |
| 47. $68 + 0 = 0 + 68 = 68$ | 48. $12 \cdot (5 \cdot x) = (12 \cdot 5) \cdot x$ |
| 49. $19 \cdot 1 = 1 \cdot 19 = 19$ | 50. $52 \cdot 0 = 0 \cdot 52 = 0$ |
| 51. Given $x \cdot 33 = 0$, then $x = 0$. | 52. $16 + (12 + 23) = (16 + 12) + 23$ |

Answer each of the following application questions. Be sure to read the question, interpret the problem mathematically, solve the problem, then answer the question. You should answer the question in the form of a sentence.

- 53.** Hector has a balance in his checking account of \$859. He makes a deposit of \$638, then writes checks for \$92, \$337, and \$268. What is his new balance?
- 54.** Maria has a balance in her checking account of \$1,425. She makes two deposits of \$435 and \$169, then writes checks for \$209, \$148, and \$97. What is her new balance?
- 55.** Larry has \$46 cash in his wallet. If he just loaned \$35 to Moe, how much did he have before he loaned Moe the money?
- 56.** Ernst has \$287 in his savings account after he withdrew \$97 to buy new roller blades. How much did he have in his savings account before he withdrew the money?
- 57.** Mark loses four blackjack hands in which he bet \$7 for each hand. How much money did he lose? If he started with \$40, how much does he now have?
- 58.** Ross wins five blackjack hands in which he bet \$13 for each hand. How much money did he win? If he started with \$85, how much does he now have?
- 59.** Chris buys an RV for which she pays \$248 per month for 20 years. What is the total amount she paid for her RV?
- 60.** John buys a Jeep Cherokee for which he pays \$634 per month for 5 years. What is the total amount he paid for his Jeep?
- 61.** Steve runs 6 miles per day during the weekdays and 15 miles per day on the weekend. How many total miles does he run during one week? If he burns off 124 calories each mile he runs, how many calories does he burn during the week?
- 62.** Todd talks 45 minutes per day on the phone during the weekdays and 52 minutes per day on the weekend. How many total minutes does he talk during one week? How many full hours does he talk during the week?
- 63.** Sandy pays \$350 rent each month for five years. How much total rent does she pay during those five years?

64. Linda signs a lease to pay \$725 rent each month for two years. How much total rent does she owe for the lease?
65. Carolyn buys a new sports car at a total loan cost of \$32,400. If she makes monthly payments on her loan for five years, how much are her monthly payments?
66. Brad buys a used boat at a total loan cost of \$32,400. If he makes monthly payments on his loan for four years, how much are his monthly payments?
67. Sandra decides to buy a house and signs loan papers which require \$2,400 up-front (down payment), then payments of \$586 per month for 30 years. What is the total cost of the loan for Sandra?
68. Rosie decides to buy a house and signs loan papers which require \$3,950 up-front (down payment), then payments of \$643 per month for 30 years. What is the total cost of the loan for Rosie?
69. Sandra (from Exercise 67) is offered another loan with the same down payment, then payments of \$722 per month for 15 years. Find the total cost of this loan, and compare it with the loan from Exercise 67. How much money will she save with this new loan?
70. Rosie (from Exercise 68) is offered another loan with a down payment of \$6,500, then payments of \$876 per month for 20 years. Find the total cost of this loan, and compare it with the loan from Exercise 68. How much money will she save with this new loan?
71. After college, you accept a job which pays \$2,400 per month for the first year, then a raise of \$175 per month for the second year. What will your total income be for the first two years at your job?
72. For college, you rent an apartment for \$650 per month the first year, with an increase of \$25 per month for each of the second, third, and fourth years. What is the total rent paid for the four years at college?
73. You commute to (and from) work 218 times during the year. The distance from your home to work is 24 miles. What is the total distance commuting during the year?
74. Mary commutes to (and from) work 209 times during the year. The distance from her home to work is 8 miles. What is the total distance Mary commutes during the year?
75. Jerry packs walnuts into bags which hold 48 walnuts. If he needs to pack 8,900 walnuts, how many full bags will he be able to pack?
76. Jerry's walnut trees produce 80 pounds of walnuts per tree. He plants 50 trees per acre, and has 86 acres of walnuts. Find the total production of walnuts from his trees.
77. It takes Pete 2 hours to prepare for a marriage dissolution case and 5 hours to prepare for a murder case. Currently he has 8 marriage dissolution cases pending and 4 murder cases pending. How many hours of preparation does he have for his pending cases?
78. After preparing for all of his cases, Pete decides to sail to Hawaii. He estimates he can sail 120 miles per day, and that the total sailing trip is 5000 miles. If his water consumption is 3 gallons of water per day, how much water should he bring for the trip?

Answer the following questions.

- 79. What number multiplied by 18 gives a product of 846?
- 80. What number multiplied by 26 gives a product of 1014?
- 81. Show that 45 is a factor of 3690.
- 82. Show that 32 is a factor of 2144.
- 83. If x is a factor of y , and y is a factor of z , must x be a factor of z ? Give an example to justify your answer.
- 84. If x is a factor of y , and x is a factor of z , must y be a factor of z ? Give an example to justify your answer.
- 85. Can 0 be divided by 5? If so, give the quotient and if not, explain why.
- 86. Can 8 be divided by 0? If so, give the quotient and if not, explain why.