

2.3 Subtraction of Integers

The key to subtraction of real numbers is to forget what you might have learned in elementary school about “taking away”. With positive numbers you can reasonably say “take 3 away from 7, to get 4”, but the reasoning becomes difficult if we want to take -5 away from 8. For **subtraction of integers**, we will work with a more formal (or abstract) definition. If we denote $-y$ as the opposite of y , then subtraction can be defined as:

$$x - y = x + (-y)$$

That is, to subtract two real numbers, we add the opposite of the second number (number being subtracted) to the first number. Thus, to compute $5 - 9$, we convert the problem to $5 + (-9)$ (since -9 is the opposite of 9), and thus:

$$\begin{array}{ll} 5 - 9 = 5 + (-9) & \text{converting to addition} \\ = -4 & \text{using the number line} \end{array}$$

Similarly, to compute $6 - (-5)$, we convert the problem to $6 + 5$ (since 5 is the opposite of -5), and thus:

$$\begin{array}{ll} 6 - (-5) = 6 + 5 & \text{converting to addition} \\ = 11 & \text{using the number line} \end{array}$$

Thus subtraction of integers consists of two steps:

1. Convert subtraction to addition by adding the opposite of the second number.
2. Using the number line to perform the addition.

The next example will provide additional practice.

Example 1 Compute each subtraction.

- a. $12 - 21$
- b. $-4 - 7$
- c. $-9 - (-4)$
- d. $-6 - (-13)$

Solution a. First convert the subtraction to an addition problem (note that -21 is the opposite of 21), then use the number line to compute the addition:

$$\begin{array}{ll} 12 - 21 = 12 + (-21) & \text{converting to addition} \\ = -9 & \text{using the number line} \end{array}$$

b. First convert the subtraction to an addition problem (note that -7 is the opposite of 7), then use the number line to compute the addition:

$$\begin{array}{ll} -4 - 7 = -4 + (-7) & \text{converting to addition} \\ = -11 & \text{using the number line} \end{array}$$

c. First convert the subtraction to an addition problem (note that 4 is the opposite of -4), then use the number line to compute the addition:

$$\begin{array}{ll} -9 - (-4) = -9 + 4 & \text{converting to addition} \\ = -5 & \text{using the number line} \end{array}$$

d. First convert the subtraction to an addition problem (note that 13 is the opposite of -13), then use the number line to compute the addition:

$$\begin{array}{ll} -6 - (-13) = -6 + 13 & \text{converting to addition} \\ = 7 & \text{using the number line} \end{array}$$

When students first learn subtraction, there is a tendency to try to skip the step of rewriting the subtraction as an addition, and just do the problem mentally. ***Avoid this tendency!!*** The real key to subtracting integers is to consistently apply the step of rewriting to an addition, then just adding as you did in the previous section. Even students taking calculus will often be seen rewriting these subtractions, rather than doing the problem in their head.

In a situation with a number of subtractions, such as $-5 - 7 - (-3) + 4 - 6 + (-2)$, there are two approaches. One approach, naturally, is to work the problem from left to right, performing each computation as a separate step. In this problem that approach would lead to five different computations, which is time-consuming and likely to produce errors. An alternate approach, which we will use, is to start by rewriting the entire group of subtractions as additions. After that, we will add positive numbers and negative numbers separately (as in the previous section), then use the number line for the final addition.

Following this procedure:

$$\begin{aligned}-5 - 7 - (-3) + 4 - 6 + (-2) &= -5 + (-7) + 3 + 4 + (-6) + (-2) \\ &= (-5) + (-7) + (-6) + (-2) + 3 + 4 \\ &= -20 + 7 \\ &= -13\end{aligned}$$

Note that only subtractions are rewritten as addition in the first step. Also note the second step, in which we reorganized the additions using the commutative property. Often in algebra you will see steps or reorganization, even though nothing was computed during the step. The third step involved adding all negative numbers and positive numbers separately, and the fourth step uses the number line to compute the final answer.

Example 2 Compute the expression: $-6 - 8 - (-5) - (-2) + 4 + (-6) - 1$

Solution First rewrite all subtractions as addition of opposites, then add positive and negative numbers separately, then use the number line for the final addition. The steps are:

$$\begin{aligned}-6 - 8 - (-5) - (-2) + 4 + (-6) - 1 &= -6 + (-8) + 5 + 2 + 4 + (-6) + (-1) \\ &= (-6) + (-8) + (-6) + (-1) + 5 + 2 + 4 \\ &= -21 + 11 \\ &= -10\end{aligned}$$

Recall that solutions to equations are numbers which, when substituted for the variable in the equation, produce a true statement. We can now determine solutions to equations which involve subtraction, as the following example illustrates.

Example 3 Determine whether or not the given integer value is a solution to the equation.

- a. $x - 4 = 2$; $x = 6$
- b. $x - 3 = -5$; $x = -8$
- c. $y - 3 = -5$; $y = -2$
- d. $a - (-4) = 0$; $a = 4$

Solution a. Substitute $x = 6$ into the equation and rewrite the subtraction as an addition:

$$6 - 4 = 2$$

$$6 + (-4) = 2$$

$$2 = 2$$

Note the last addition was performed using the number line. Since this last statement ($2 = 2$) is true, $x = 6$ is a solution to the equation $x - 4 = 2$.

- b. Substitute $x = -8$ into the equation and rewrite the subtraction as an addition:

$$-8 - 3 = -5$$

$$-8 + (-3) = -5$$

$$-11 = -5$$

Note the last addition was performed using the number line. Since this last statement ($-11 = -5$) is false, $x = -8$ is not a solution to the equation $x - 3 = -5$.

- c. Substitute $y = -2$ into the equation and rewrite the subtraction as an addition:

$$-2 - 3 = -5$$

$$-2 + (-3) = -5$$

$$-5 = -5$$

Since this last statement ($-5 = -5$) is true, $y = -2$ is a solution to the equation $y - 3 = -5$.

- d. Substitute $a = 4$ into the equation and rewrite the subtraction as an addition:

$$4 - (-4) = 0$$

$$4 + 4 = 0$$

$$8 = 0$$

Since this last statement ($8 = 0$) is false, $a = 4$ is not a solution to the equation $a - (-4) = 0$.

Recall that an arithmetic sequence is a sequence of numbers (called terms) in which the same number (called the common difference) is added on to each term to obtain the next term. For example, with the sequence $-5, -3, -1, 1, \dots$ from the last section, the common difference was noted to be 2. Another way to find the common difference is by using subtraction. Note that:

$$-3 - (-5) = -3 + 5 = 2$$

$$-1 - (-3) = -1 + 3 = 2$$

$$1 - (-1) = 1 + 1 = 2$$

Since each successive subtraction results in the number 2, the common difference is 2, and thus the next term in the sequence is $1 + 2 = 3$. This provides an alternate approach to finding terms of an arithmetic sequence.

Example 4 Find the common difference and the next term for each arithmetic sequence.

- a. $2, -3, -8, \dots$
- b. $-12, -10, -8, \dots$
- c. $-17, -36, -55, \dots$
- d. $-103, -76, -49, \dots$

Solution

- a. Subtracting terms to find the common difference:

$$-3 - 2 = -3 + (-2) = -5$$

$$-8 - (-3) = -8 + 3 = -5$$

The common difference is -5 , so the next term is $-8 + (-5) = -13$.

- b. Subtracting terms to find the common difference:

$$-10 - (-12) = -10 + 12 = 2$$

$$-8 - (-10) = -8 + 10 = 2$$

The common difference is 2, so the next term is $-8 + 2 = -6$.

- c. Subtracting terms to find the common difference:

$$-36 - (-17) = -36 + 17 = -19$$

$$-55 - (-36) = -55 + 36 = -19$$

The common difference is -19 , so the next term is $-55 + (-19) = -74$.

- d. Subtracting terms to find the common difference:

$$-76 - (-103) = -76 + 103 = 27$$

$$-49 - (-76) = -49 + 76 = 27$$

The common difference is 27, so the next term is $-49 + 27 = -22$.

The terminology used with subtraction is different than that used for addition. If we say “add 7 to 10”, clearly we mean to compute $10 + 7 = 17$. If we reverse this as $7 + 10 = 17$, it really does not matter, since the commutative property states that both values are the same. With subtraction, however, this is not the case. If we say “subtract 12 from 5”, we mean to compute

$$5 - 12 = 5 + (-12) = -7$$

This is not the same as computing

$$12 - 5 = 12 + (-5) = 7$$

The key is the phrase says “subtract 12 *from* 5”, so we must start with 5 and subtract 12 from it. The following example will provide other examples of words used to represent subtraction, as well as the order of use.

Example 5 Convert each written phrase to a mathematical computation, then compute the required quantity.

- a. Subtract -4 from -15 .
- b. Subtract 6 from -9 .
- c. Reduce 6 by 15.
- d. The difference of -5 and 8.
- e. The difference of -7 and -15 .
- f. Diminish -7 by -4 .

Solution a. We will start with -15 (note the word *from*), then subtract -4 from that number. The expression and computation are:

$$\begin{array}{ll} -15 - (-4) = -15 + 4 & \text{rewrite as an addition} \\ = -11 & \text{computed on the number line} \end{array}$$

b. We will start with -9 (again note the word *from*), then subtract 6 from that number. The expression and computation are:

$$\begin{array}{ll} -9 - 6 = -9 + (-6) & \text{rewrite as an addition} \\ = -15 & \text{computed on the number line} \end{array}$$

c. The phrase indicates we should start with 6, then subtract 15 *from* it. The expression and computation are:

$$\begin{array}{ll} 6 - 15 = 6 + (-15) & \text{rewrite as an addition} \\ = -9 & \text{computed on the number line} \end{array}$$

- d. With the phrase “the difference of”, we interpret the subtraction in exact order as the numbers are given. The expression and computation are:
 $-5 - 8 = -5 + (-8)$ rewrite as an addition
 $= -13$ computed on the number line
- e. Again, interpreting “the difference of” in exact order, we start with -7 then subtract -15 . The expression and computation are:
 $-7 - (-15) = -7 + 15$ rewrite as an addition
 $= 8$ computed on the number line
- f. The phrase “diminish -7 by -4 ” indicates we should start with -7 , then subtract -4 from it. The expression and computation are:
 $-7 - (-4) = -7 + 4$ rewrite as an addition
 $= -3$ computed on the number line

Terminology

subtraction of integers

Exercise Set 2.3

Subtract the two integers. Remember to rewrite all subtractions as addition for your first step.

- | | |
|-------------------|-------------------|
| 1. $12 - 18$ | 2. $13 - 25$ |
| 3. $-4 - 8$ | 4. $-3 - 9$ |
| 5. $6 - (-3)$ | 6. $8 - (-4)$ |
| 7. $-12 - (-9)$ | 8. $-15 - (-7)$ |
| 9. $-43 - 62$ | 10. $-67 - 38$ |
| 11. $-29 - (-40)$ | 12. $-15 - (-32)$ |
| 13. $29 - 15$ | 14. $81 - 47$ |
| 15. $-15 - 28$ | 16. $-27 - 19$ |
| 17. $42 - (-19)$ | 18. $26 - (-48)$ |
| 19. $-30 - (-17)$ | 20. $-26 - (-18)$ |

Compute each expression.

- | | |
|--|--|
| 21. $5 - 8 - (-3) + 2$ | 22. $3 - 9 - (-6) + 7$ |
| 23. $-8 - 5 - (-3) + (-2)$ | 24. $-12 - 7 - (-6) + (-4)$ |
| 25. $-7 + (-3) - 4 - (-8)$ | 26. $-9 + (-6) - 7 - (-12)$ |
| 27. $4 - 12 + (-5) - 5 - (-6)$ | 28. $6 - 9 + (-4) - 9 - (-2)$ |
| 29. $-5 - 3 + (-6) - 11 - 6 - (-8)$ | 30. $-9 - 13 + (-9) - 7 + 5 - (-3)$ |
| 31. $-8 + 2 + (-5) - (-3) - 7 - 13 - (-6)$ | 32. $-13 + 7 + (-7) - (-5) - 6 - 8 - (-9)$ |
| 33. $-5 + (-2) - 5 - -4 $ | 34. $-14 + (-9) - 12 - -7 $ |
| 35. $ -15 - -7 - 45 $ | 36. $ -13 - -19 - 37 $ |

Determine whether or not the given integer value is a solution to the equation.

- | | |
|----------------------------|------------------------------|
| 37. $x - 5 = 4; x = 9$ | 38. $x - 5 = 4; x = -1$ |
| 39. $x - 6 = 3; x = -3$ | 40. $x - 6 = 3; x = 9$ |
| 41. $y - 12 = -7; y = -19$ | 42. $y - 12 = -7; y = 5$ |
| 43. $a - 7 = -15; a = -8$ | 44. $a - 7 = -15; a = -22$ |
| 45. $t - (-6) = 6; t = 0$ | 46. $t - (-6) = 6; t = 12$ |
| 47. $s - (-9) = -3; s = 6$ | 48. $s - (-9) = -3; s = -12$ |

Find the common difference and the next term in each sequence.

- | | |
|--------------------------|---------------------------|
| 49. 4, 6, 8, ... | 50. 5, 7, 9, ... |
| 51. -8, -5, -2, ... | 52. -9, -5, -1, ... |
| 53. 10, 6, 2, ... | 54. 8, 4, 0, ... |
| 55. -1, -5, -9, ... | 56. -8, -13, -18, ... |
| 57. -25, -39, -53, ... | 58. -37, -62, -87, ... |
| 59. -147, -116, -85, ... | 60. -161, -133, -105, ... |

Convert each written phrase to a mathematical computation, then compute the required quantity.

- | | |
|----------------------------------|----------------------------------|
| 61. Subtract 18 from 7. | 62. Subtract 25 from 9. |
| 63. Subtract 12 from -8. | 64. Subtract 15 from -18. |
| 65. Subtract -8 from -12. | 66. Subtract -14 from -19. |
| 67. Reduce 12 by 28. | 68. Reduce 5 by 18. |
| 69. Reduce -14 by 13. | 70. Reduce -25 by 12. |
| 71. Reduce -15 by -22. | 72. Reduce -18 by -27. |
| 73. The difference of -4 and 7. | 74. The difference of -6 and 9. |
| 75. The difference of -8 and -5. | 76. The difference of -9 and -3. |

- | | |
|--|--|
| 77. The difference of -15 and -29 . | 78. The difference of -23 and -42 . |
| 79. Diminish 24 by 15 . | 80. Diminish 26 by 12 . |
| 81. Diminish 14 by 27 . | 82. Diminish 13 by 52 . |
| 83. Diminish -14 by -5 . | 84. Diminish -56 by -31 . |
| 85. 7 less than 15 . | 86. 12 less than 26 . |
| 87. 24 less than -13 . | 88. 56 less than -16 . |
| 89. -19 less than -15 . | 90. -38 less than -22 . |

Answer the following questions. Give complete answers and explain your answer.

- 91.** Is subtraction a commutative property? That is, does $a - b = b - a$ for all integers a and b ? Give examples in your explanation.
- 92.** Is subtraction an associative property? That is, does $(a - b) - c = a - (b - c)$ for all integers a , b , and c ? Give examples in your explanation.
- 93.** When will it be the case that $x - y > 0$ for two integers x and y ? Give examples in your explanation.
- 94.** When will it be the case that $x - y < 0$ for two integers x and y ? Give examples in your explanation.
- 95.** What number can be added to -8 to yield -12 ?
- 96.** What number can be added to -14 to yield -9 ?
- 97.** What number can be subtracted from -14 to yield -18 ?
- 98.** What number can be subtracted from -23 to yield -15 ?