

Subtracting Integers

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CONCEPT

1

Subtracting Integers

Introduction

The Shark Dive



The next day, Cameron decided to take the day off from diving and play volleyball on the beach. His Dad decided to do a deep dive to hopefully see some sharks. Cameron would love to do a deep dive, but he isn't old enough yet, so this gave his Dad a chance to dive on his own.

When Cameron's Dad returned, he told Cameron the story of how he went down to a depth of 80 feet with hopes of seeing a shark. After ten minutes or so, he spotted a beautiful shark swimming above him. Cameron's Dad went up about 20 feet to try to take a picture of the shark.

He did get a few good shots before the shark swam away.

"What depth did you see the shark at?" Cameron asked.

Do you know? To figure this out, you will need to subtract integers. Subtracting integers is the focus of this lesson. By the end of it, you will know how to figure out at what depth Cameron's dad saw the shark.

What You Will Learn

By the end of this lesson, you will be able to demonstrate the following:

- Find differences of integers on a number line
- Subtract integers using opposites
- Evaluate variable expressions involving integer subtraction
- Model and solve real-world problems using simple equations involving integer change.

Explore Subtracting Integers with the [Interactive +/- Chips](#).

Watch this [KhanAcademyVideo: Subtracting Integers](#)

Teaching Time

I. Find Differences of Integers on a Number Line

We can subtract integers by using a strategy. Using a strategy will allow us to find the *difference* between two integers. Remember that the word "difference" is a key word that means the answer in a subtraction problem.

Do you remember what an integer is?

An **integer** is the set of whole numbers and their opposites. Essentially, we can think of integers as positive and negative whole numbers.

You may recall that one way to add integers is to use a number line. A similar strategy can be used to subtract integers as well. Let's look at how to do this.

In the first strategy, we will explore how to subtract integers using a number line. When using a number line to model subtraction, imagine a person standing at 0, facing the positive numbers on the line. The person will then move forward or backward to show the first quantity in the problem.

To model subtracting a number from that quantity, imagine that person turning and facing the negative numbers. Imagining the person turning around distinguishing the operation of subtraction from the operation of addition. (In Lesson 4.2, the person never turned around and faced the opposite direction because you were adding.)

To subtract a positive number, the person moves forward. To subtract a negative number, the person moves backward.

This first example shows how we can use a number line to model the subtraction of two positive integers.

Example

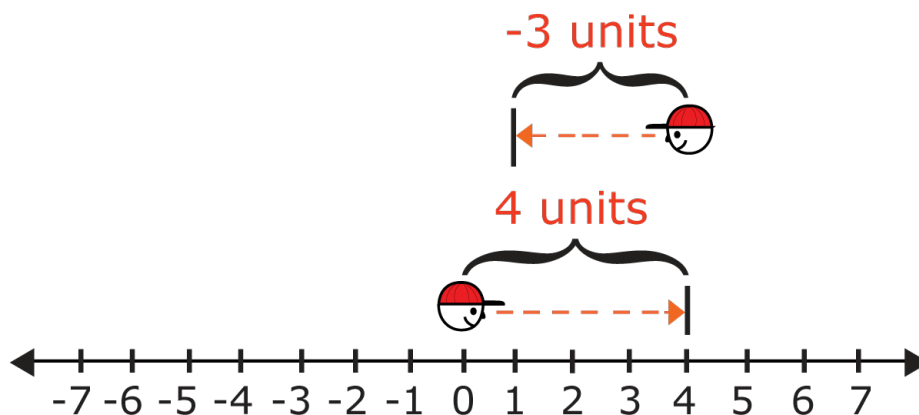
Use a number line to find the difference: $4 - 3$.

You are subtracting two positive integers.

So, to model $4 - 3$, imagine the person moving 4 units to the right of zero. This shows the quantity, 4.

Next, imagine the person turning around.

Since you are subtracting a positive integer, 3, the person moves forward. After turning around, imagine that person moving 3 units forward, or to the left.



The person ends up at 1. So, $4 - 3 = 1$.

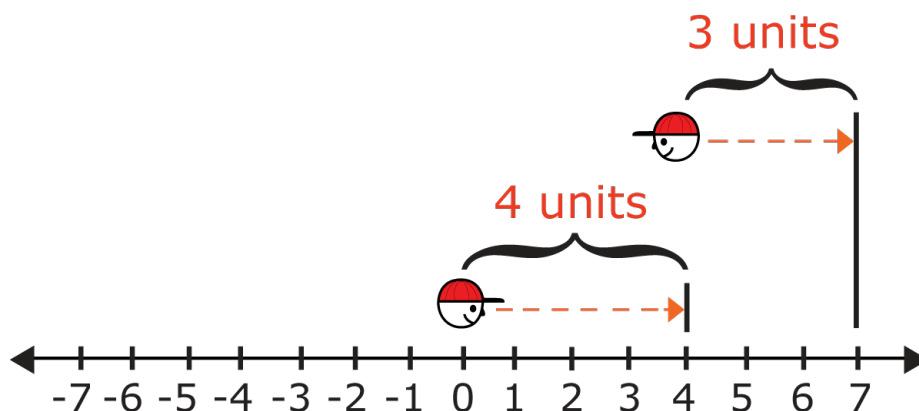
Our answer is 1.

Now, let's imagine subtracting a negative integer from a positive integer.

Example

Use a number line to find the difference of $4 - (-3)$.

To model $4 - (-3)$, imagine the person moving 4 units forward and to the right of zero. Then imagine the person turning around. Since a negative integer, -3 , is being subtracted, imagine the person moving 3 units backward. That means the person will be moving to the left.



The person ends up at 7. So, $4 - (-3) = 7$.

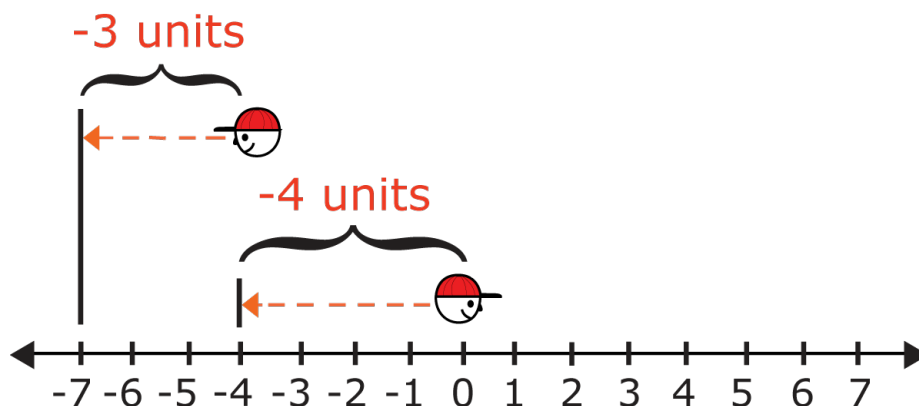
Our answer is 7.

Example

Use a number line to find the difference of $-4 - 3$.

To model $-4 - 3$, imagine the person moving 4 units backward and to the left. The person moves backward because the initial quantity, -4, is a negative integer.

Next, imagine the person turning around. Since a positive integer, 3, is being subtracted, imagine the person moving 3 units forward and to the left.



The person ends up at -7. So, $-4 - 3 = -7$.

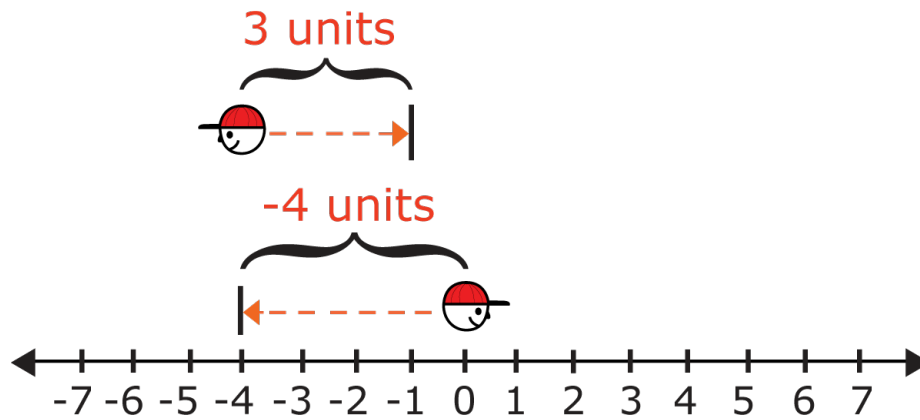
Our answer is -7.

Now, let's imagine subtracting a negative integer from a negative integer.

Example

Use a number line to find the difference of $-4 - (-3)$.

To model $-4 - (-3)$, imagine the person moving 4 units backward and to the left. Then imagine the person turning around. Since a negative integer, -3, is being subtracted, imagine the person moving 3 units backward and to the right.



The person ends up at -1. So, $-4 - (-3) = -1$.

Our answer is -1.

1.3 G. Lesson Exercises

Subtract the following integers. Use a number line or +/- chips to help you.

1. $-5 - 2$
2. $7 - (-2)$
3. $-9 - (-5)$

Go back to the

Watch the

Practice with



Take a few minutes to check your work with a partner. Are your answers accurate?

II. Subtract Integers Using Opposites

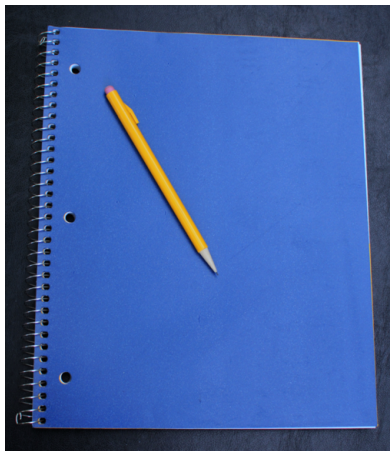
Another strategy for subtracting integers involves using **opposites**. Remember, you can find the opposite of an integer by changing the sign of an integer. The opposite of any integer, b , would be $-b$.

For any two integers, a and b , the difference of $a - b$ can be found by adding $a + (-b)$. So, to subtract two integers, take the opposite of the integer being subtracted and then add that opposite to the first integer.



Sure.

For any two integers, a and b , the difference of $a - b$ can be found by adding $a + (-b)$. So, to subtract two integers, take the opposite of the integer being subtracted and then add that opposite to the first integer.



Write this down in your notebook and then continue with the lesson.

Example

Find the difference of $5 - (-8)$.

The integer being subtracted is -8 . The opposite of that integer is 8 , so add 8 to 5 .

$$5 - (-8) = 5 + 8 = 13.$$

So, the difference of $5 - (-8)$ is 13 .

Our answer is 13.

Example

Find the difference of $-12 - (-2)$.

The integer being subtracted is -2 . The opposite of that integer is 2 , so add 2 to -12 .

$$-12 - (-2) = -12 + 2.$$

Add as you would add any integers with different signs.

$|-12| = 12$ and $|2| = 2$, so subtract the lesser absolute value from the greater absolute value.

$$12 - 2 = 10$$

Give that answer the same sign as the integer with the greater absolute value. $12 > 2$, so -12 has a greater absolute value than 2. Give the answer a negative sign.

So, the difference of $-12 - (-2)$ is -10.

Our answer is -10.

Remember the Rules for Adding Signed Numbers

Same Signs: Add and sign on the answer stays the same.

Example: $-6 + -7 = -13$

Different Signs: Subtract and use the sign that has the most

Example: $-6 + 7 \Rightarrow$ Subtract $7 - 6 = 1 \Rightarrow$ more + with 7 $\Rightarrow$ so answer is +1

$-9 + 5 \Rightarrow$ subtract $9 - 5 = 4 \Rightarrow$ more neg with 9 $\Rightarrow$ so answer is -4

Example

Find the difference of $-20 - 3$.

The integer being subtracted is 3. The opposite of that integer is -3, so add -3 to -20.

$$-20 - 3 = -20 + (-3).$$

Add as you would add any integers with the same sign. In this case, a negative sign.

$|-20| = 20$ and $|-3| = 3$, so add their absolute values:

$$20 + 3 = 23$$

Give that answer the same sign as the two original integers, a negative sign.

So, the difference of $-20 - 3$ is -23.

Our answer is -23.

Examples

$$-2 - 3 = -2 + -3$$

$$4 - 5 = 4 + -5$$

$$8 - -8 = 8 + +8$$

Now take a few minutes to practice what you have learned.

1.3 H. Lesson Exercises

Rewrite each difference as adding the opposite and then use the rules for adding and solve.

1. $-5 - 7$

2. $8 - (-4)$

3. $-12 - (-8)$



Take a few minutes to check your answers with a friend.

III. Evaluate Variable Expressions Involving Integer Subtraction

Do you remember what a **variable expression** is? A variable expression is a number sentence that has numbers, variables and operations in it. In a variable expression we can have like and unlike terms. Like terms can be combined or simplified and unlike terms can not be combined or simplified.

Example

$$6x + (-4x)$$

These two terms are alike because they both have x 's with them. In the last lesson, you learned how to add like terms in a variable expression.

The answer to this example is $2x$.

You can use what you know about how to subtract integers to help you find the value of expressions with variables, too.

Remember, you can only subtract like terms in an expression that has variables.

Example

$$\text{Find the difference } -10n - (-8n)$$

Since $-10n$ and $-8n$ both have the same variable, they are like terms. Use what you know about how to subtract integers to help you.

The term being subtracted is $-8n$. The integer -8 is part of that term. The opposite of that integer is 8 , so add $8n$ to $-10n$.

$$-10n - (-8n) = -10n + 8n$$

Both like terms have different signs. So, find the absolute values of both integers. Then subtract the term whose integer has the lesser absolute value from the term with the larger absolute value.

$$|-10| = 10 \text{ and } |8| = 8$$

$$\text{Now subtract } 10n - 8n = 2n.$$

Since $10 > 8$, and $-10n$ has a negative sign, give the answer a negative sign.

The difference of $-10n - (-8n)$ is $-2n$.

Remember you can only combine terms that are alike.

1.3 I. Lesson Exercises

Use what you have learned to simplify each expression.

1. $-4y - 6y$
2. $18x - (-4x)$
3. $-9a - (-3a)$



Take a few minutes to check your work with a friend.

IV. Model and Solve Real-World Problems Using Simple Equations Involving Integer Change

Knowing how to subtract integers can also help you solve many problems in real life. To solve a real-world problem, write an expression or an equation that can be used to solve the problem, then solve. Let's practice this a bit and then return to our introductory problem.



Example

The temperature inside a laboratory freezer was -10° Celsius. A scientist at the lab then lowered the temperature inside the freezer by 5° Celsius. What was the new temperature inside the freezer?

The problem says that the temperature was *lowered*. This means that the temperature *decreased*, so you should subtract. To find the new temperature, you can *subtract* the amount by which the temperature was lowered from the original temperature, using one of these equations.

$$\begin{array}{l} -10^{\circ}\text{C} - 5^{\circ}\text{C} = ? \\ \text{or} \quad -10 - 5 = ? \end{array}$$

The integer being subtracted is 5. The opposite of that integer is -5, so add -5 to -10.

$$-10 - 5 = -10 + (-5).$$

Add as you would add any integers with the same sign—a negative sign.

$|-10| = 10$ and $|-5| = 5$, so add their absolute values.

Give that answer the same sign as the two original integers, a negative sign.

$$10 + 5 = 15$$

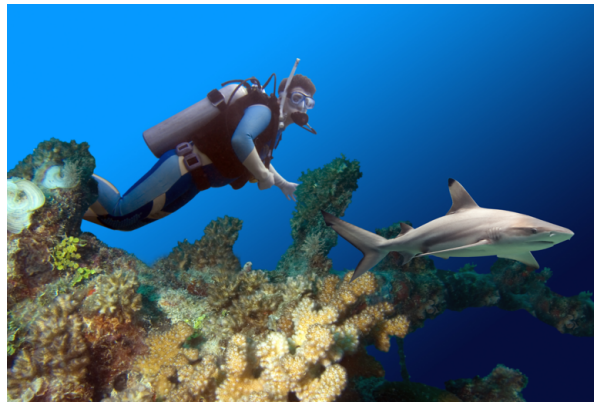
So, the difference of $-10 - 5$ is -15.

This means that the new temperature inside the freezer must be -15° Celsius.

Now, we can apply what we have learned to our introduction problem.

Real-Life Example Completed

The Shark Dive



Here is the original problem once again. Reread it and underline any important information.

The next day, Cameron decided to take the day off from diving and play volleyball on the beach. His Dad decided to do a deep dive to hopefully see some sharks. Cameron would love to do a deep dive, but he isn't old enough yet, so this gave his Dad a chance to dive on his own.

When Cameron's Dad returned, he told Cameron the story of how he went down to a depth of 80 feet with hopes of seeing a shark. After ten minutes or so, he spotted a beautiful shark swimming above him. Cameron's Dad went up about 20 feet to try to take a picture of the shark.

He did get a few good shots before the shark swam away.

"What depth did you see the shark at?" Cameron asked.

To find the depth that Cameron's Dad saw the shark, we need to write a subtraction problem and solve it. Remember that depth has to do with below the surface, so we use negative integers to represent different depths.

-80 was his starting depth, then he went up -20 so we take away 20 feet.

$$-80 - (-20) = -80 + +20 = -60 \text{ feet}$$

Cameron's Dad saw the shark at 60 feet below the surface.

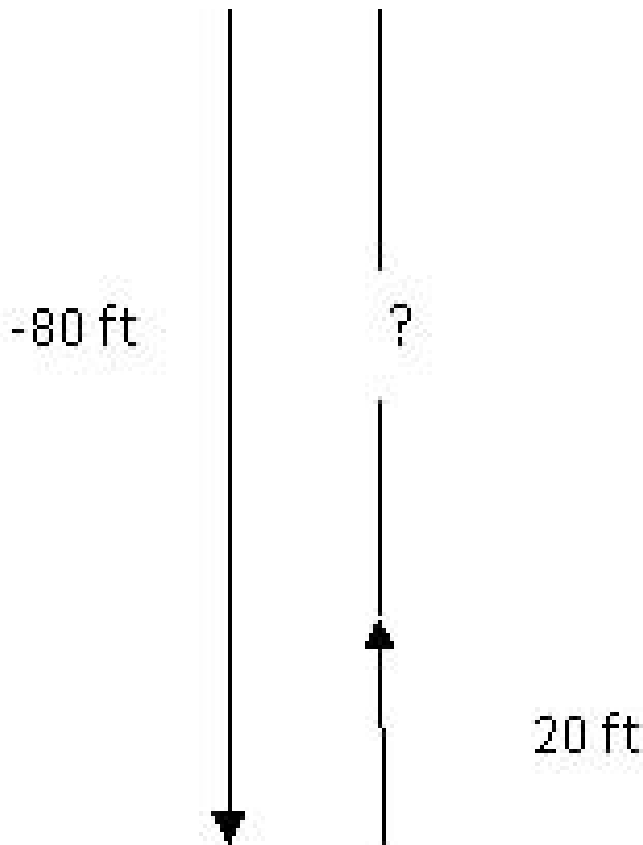


FIGURE 1.1

Vocabulary

Here are the vocabulary words that are found in this lesson.

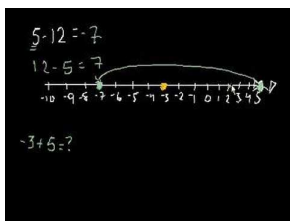
Difference the answer in a subtraction problem.

Integer the set of whole numbers and their opposites.

Variable Expression a number sentence that uses numbers, variables and operations, without an equal sign.

Technology Integration

This video presents examples of addition and subtraction of negative numbers.

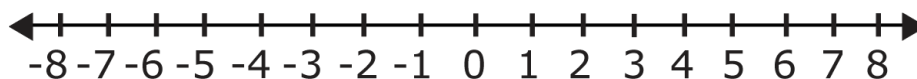
**MEDIA**

Click image to the left for more content.

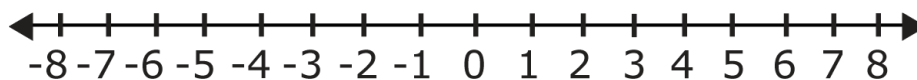
[KhanAcademyAdding/Subtracting NegativeNumbers](#)**Time to Practice**

Directions: Subtract the following integers using a number line and draw a picture of the problems using the +/- chips.

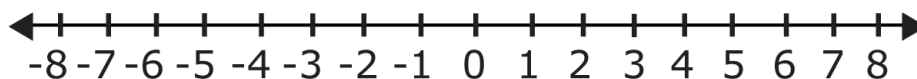
1. $8 - 6$



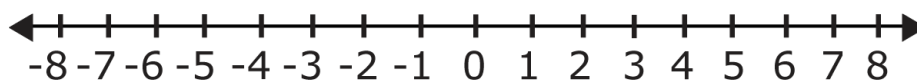
2. $-5 - 3$



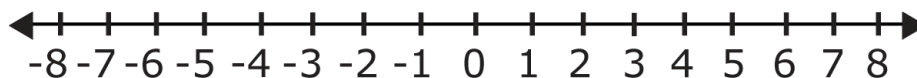
3. $-3 - (-6)$



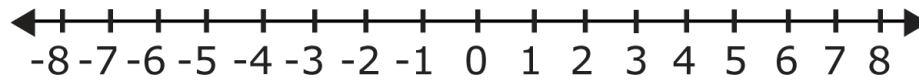
4. $2 - (-5)$



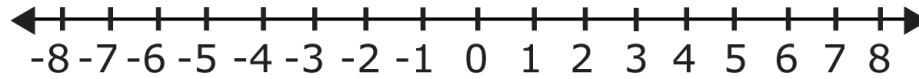
5. $-6 - 3$



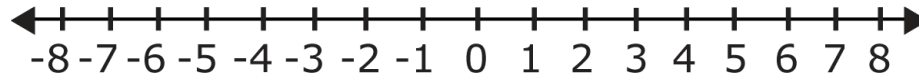
6. $8 - (-3)$



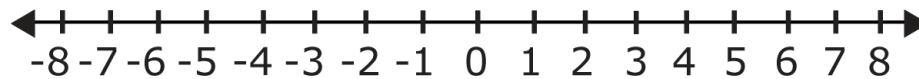
7. $-7 - (-7)$



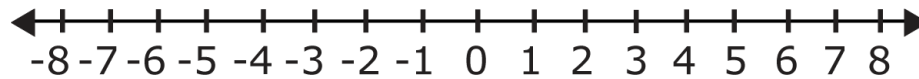
8. $-5 - 4$



9. $1 - (-8)$



10. $-4 - 7$



Directions: Find each difference by changing to adding the opposite.

11. $15 - 7$

12. $-7 - 12$

13. $0 - 4$

14. $13 - (-9)$

15. $-21 - 4$

16. $33 - (-4)$

17. $-11 - (-8)$

18. $18 - 8$

Directions: Simplify each variable expression.

19. $-8m - 3m$

20. $(-7c) - (-c)$

21. $-19a - (-4a)$

22. $33b - (-18b) + 7$

Directions: Solve this real-world problem by drawing a picture of a thermometer.

23. Carl lives in Minneapolis, Minnesota. The low temperature in his hometown on February 1 was $-2^{\circ}F$. The next day, the low temperature in his hometown was $6^{\circ}F$ less than that. What was the temperature the next day?

Review Practice:

Write an integer to represent each value

24. Using your debit card to spend \$42.

25. Earning \$75

26. Diving 90 feet under water

Fill in each with $<$, $>$, or $=$

27. $8 \underline{\hspace{1cm}} - 2$

28. $| -6 | \underline{\hspace{1cm}} - 6$

29. $-9 \underline{\hspace{1cm}} -12$

30. Order the following numbers on a number line: 9, -3 , -7 , 0, 3

Evaluate each

31. $2 + -8$

32. $-2 + 8$

33. $-2 + -8$