

5.1 Integers and the Operations of Addition and Subtraction

In *Principles and Standards* for grades 6-8, “Students can also work with whole numbers in their study of number theory. Tasks, such as the following, involving factors, multiples, prime numbers, and divisibility can afford opportunities for problem solving and reasoning

1. Explain why the sum of the digits of any multiple of 3 is itself divisible by 3.
2. A number of the form $abcabc$ always has several prim-number factors. Which prime numbers are always factors of a number of this form? Why?” (p. 217).

The set $\{1, 2, 3, 4, \dots\}$ is the set of **positive integers**. The set $\{-1, -2, -3, -4, \dots\}$ is the set of **negative integers**.

Representations of Integers

The negative integers are **opposites** of the positive integers. Note that $-x$ is the opposite of x and might not represent a negative numbers.

Integer Addition

Chip Model for Addition

Positive and negative integers are represented by different colored chips. One of each of the two colors makes a “zero pair.”

Example: $6 + -3$

Charged-Field Model for Addition

Positive and negative integers are represented by positive and negative charges.

Example: $6 + -2$

Number-Line Model

Start at zero and move forward or backward, depending on the sign.

Example: $7 + -4$

Thermometer Model

Like the number line model, start at zero and move up or down, depending on the sign.

Example: $7 + -3$

Pattern Model for Addition

A pattern with previously known facts is established and continued to produce new facts.

Example:

$$5 + 3 = 8$$

$$5 + 2 = 7$$

$$5 + 1 = 6$$

$$5 + 0 = 5$$

$$5 + -1 =$$

$$5 + -2 =$$

Note: Reasoning with a pattern is inductive reasoning and does not constitute a proof.

Absolute Value

The distance between the point corresponding to an integer and 0 is the **absolute value** of the integer.

$$|x| = x \text{ if } x \geq 0$$

$$|x| = -x \text{ if } x < 0$$

Example: $|x|$, given that $x > 3$

Example: $|x|$, given that $x < -2$

Properties of Integer Addition

Theorem 5-1: Properties

Given integers a , b , and c ,

Closure property of addition of integers

$a + b$ is a unique integer.

Commutative property of addition of integers

$$a + b = b + a$$

Associative property of addition of integers

$$(a + b) + c = a + (b + c)$$

Identity element of addition of integers

0 is the unique integer such that, for all integers a , $0 + a = a = a + 0$.

Theorem 5-2: Uniqueness of the Additive Inverse

For every integer a , there exists a unique integer $-a$, the additive inverse of a , such that $a + -a = 0 = -a + a$.

Theorem 5-3

For any integers, a and b ,

1. $-(-a) = a$
2. $-a + -b = -(a + b)$

Integer Subtraction

Chip Model for Subtraction

Example: $4 - -2$

Charged-Field Model for Subtraction

Example: $5 - -3$

Number-Line Model for Subtraction

Example: $5 - -2$

Pattern Model for Subtraction

Example:

$$4 - 3 = 1$$

$$4 - 4 = 0$$

$$4 - 5 =$$

$$4 - 6 =$$

$$4 - 7 =$$

$$4 - 3 = 1$$

$$4 - 2 = 2$$

$$4 - 1 = 3$$

$$4 - 0 = 4$$

$$4 - -1 =$$

Subtraction Using the Missing-Addend Approach

Subtraction of integers can be defined in terms of addition.

Example: $6 - 3 = n$ if, and only if, $6 = 3 + n$

Definition of Subtraction

For integers a and b , $a - b$ is the unique integer n such that $a = b + n$.

Subtraction Using Adding the Opposite Approach

Subtracting an integer is the same as adding its opposite.

Theorem 5-4

For all integers a and b , $a - b = a + -b$.

Order of Operations

Subtraction in the set of integers is neither commutative nor associative.