

5.2 Multiplication and Division of Integers

Integer Multiplication

Pattern Model for Addition

A pattern with previously known facts is established and continued to produce new facts.

Example:

$$3(-3) = -9$$

$$2(-3) = -6$$

$$1(-3) = -3$$

$$0(-3) = 0$$

$$-1(-3) =$$

$$-2(-3) =$$

$$-3(-3) =$$

Chip Model and Charged-Field Model for Multiplication

Positive and negative integers are represented by different colored chips or by positive and negative charges, where the – can mean to remove chips or charges.

Example: 4×3

Example: -4×-3

Number-Line Model

Repeated addition is used on the number line.

Example: 4×-3

Properties of Integer Multiplication

Theorem 5-5: Properties of Integer Multiplication

The set of integers I satisfies the following properties of multiplication for all integers $a, b, c \in I$.

Closure property of multiplication of integers
 ab is a unique integer.

Commutative property of multiplication of integers
 $ab = ba$

Associative property of multiplication of integers
 $(ab)c = a(bc)$

Multiplicative Identity Property
1 is the unique integer such that, for all integers a , $1 \cdot a = a = a \cdot 1$.

Distributive properties of multiplication over addition for integers
 $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

Zero multiplication property of integers
0 is the unique integer such that for all integers a , $a \cdot 0 = 0 = 0 \cdot a$.

Theorem 5-6
For every integer a , $(-1)a = -a$.

Theorem 5-7
For all integers a and b , $(-a)b = -(ab)$
 $(-a)(-b) = ab$

Theorem 5-8 Distributive Property of Multiplication over Subtraction for Integers
For any integers a, b , and c ,

$$a(b - c) = ab - ac \quad \text{and} \quad (b - c)a = ba - ca$$

Example: $(m + n)(m - n)$

Example: Factor $(x + y)^2 - m^2$

Integer Division

Definition of Integer Division

If a and b are integers, then $a \div b$ is the unique integer c , if it exists, such that $a = bc$.

Example: $-15 \div -3$

Example: $6 \div 0$

Order of Operations on Integers

As with whole numbers, a number line can be used to describe greater-than and less-than relations for the set of integers.

Definition of Less Than for Integers

For any integers a and b , a is less than b , written $a < b$, if, and only if, there exists a positive integer k such that $a + k = b$.

Theorem 5-9

For integers a and b , $a < b$ (or equivalently $b > a$) if, and only if, $b - a$ is equal to a positive integer; that is, $b - a$ is greater than 0.

Theorem 5-10

- If $x < y$ and n is any integer, then $x + n < y + n$.
- If $x < y$, then $-x > -y$.
- If $x < y$ and $n > 0$, then $nx < ny$.
- If $x < y$ and $n < 0$, then $nx > ny$.