

Chapter 5: Understanding Integer Operations and Properties

5.2 Multiplication, Division, and Other Properties of Integers

5.2.1. Modeling Integer Multiplication

5.2.1.1. Using a Counters Model

5.2.1.1.1. See example p. 273

5.2.1.2. Using a Charged Field Model

5.2.1.2.1. See example p. 274-5

5.2.1.3. Using the Number Line Model

5.2.1.3.1. Good way of showing and explaining integer multiplication

5.2.1.3.2. See example p. 277

5.2.1.3.3. Only good for positive times a negative or positive times a positive – negative times anything does not make sense

5.2.1.4. Using Mathematical Relationships, Patterns, and Reasoning

5.2.1.4.1. Repeated addition model can be used to explain multiplication of integers

5.2.1.4.2. Patterns can be established to show why a negative times a negative is a positive – see example p. 278 (**good test question**)

5.2.1.4.3. Procedures for Multiplying Integers

- Multiplying two positive integers: Multiply digits, keep the sign (+)
- Multiplying two negative numbers: Multiply digits, change the sign to (+)
- Multiplying a positive and a negative: Multiply digits, change the sign to (-)

5.2.2. Properties of Integer Multiplication

5.2.2.1. Basic Properties of Integer Multiplication

- **Closure Property** – For all integers a and b , ab is a unique integer
- **Multiplicative Identity Property** – 1 is the unique integer such that for each integer a , $a \times 1 = 1 \times a = a$
- **Commutative Property** – For all integers a and b , $ab = ba$
- **Associative Property** – For all integers a , b , and c , $(ab)c = a(bc)$

5.2.2.2. Distributive Property

5.2.2.2.1. Ties integer addition and multiplication together

5.2.2.2.2. For all integers a , b , and c , $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$

5.2.2.3. **Zero Property of Multiplication** – For all integers a , $a(0) = 0(a) = 0$

5.2.2.4. Basic Properties of Integer Multiplication in a Proof – p. 282-3

5.2.2.5. **Your turn p. 265: Do the practice and the reflect**

5.2.3. Explaining Integer Division

5.2.3.1. Use “Factor $\times$ Missing Factor = Product” model from before

5.2.3.2. **Definition of Integer Division** – For all integers a , b , and c , $b \neq 0$, $a \div b = c$ if and only if $c \times b = a$

5.2.3.3. Procedure for Dividing Integers

- Dividing two positive integers: Divide digits, keep the sign (+)
- Dividing two negative integers: Divide digits, change the sign to (+)
- Division with one positive and one negative integer: Divide digits, change the sign to (-)

5.2.4. More Properties of Integer Multiplication and Division

5.2.4.1. Some Integer Division Properties

5.2.4.1.1. when $a \neq 0$, then $a \div a = 1$

5.2.4.1.2. $a \div 1 = a$

5.2.4.1.3. $0 \div a = 0$

5.2.4.1.4. $a \div 0$ is undefined for all a

5.2.4.1.5. **Properties of Integer Division** – For all integers a and b , $a \neq 0$, $a \div a = 1$, $a \div 1 = a$, $0 \div a = 0$, and $ab \div a = b$. You cannot divide an integer by zero because no unique quotient exists

5.2.4.2. Some Properties of Opposites for Integers

5.2.4.2.1. **Properties of Opposites**: For all integers a and b ,

- $-(-a) = a$
- $-a(-b) = ab$
- $(-a)b = a(-b) = -(ab)$
- $a(-1) = (-1)a = -a$

5.2.4.3. Some Distributive Properties for Integers

5.2.4.3.1. **Distributive Property for Multiplication over Subtraction** – For all integers a , b , and c , $a(b - c) = ab - ac$

5.2.4.3.2. **Distributive Property for Opposites over Addition** – For all integers a , b , and c , $-(a + b) = -1(a + b) = -a + (-b) = -a - b$

5.2.5. Problems and Exercises p. 289

5.2.5.1. Home work: 1, 2, 6-8, 14, 17-19, 20a, 21a, 23, 24-26, 29, 32