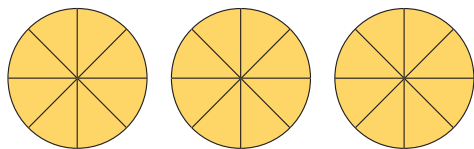


Suppose you serve three large cheese quesadillas at a party. You divide the three quesadillas into eighths. How many pieces do you have?

You can use circle models to represent each quesadilla.



Each circle is divided into eighths. You see that there are 24 pieces.
So, $3 \div \frac{1}{8} = 24$.

Activity

- 1. a. Draw 3 circles. Divide each circle in half. How many halves are there? **6 halves**
b. What is $3 \div \frac{1}{2}$? **6**
- 2. Divide 4, 5, and 6 circles into halves. Copy and complete the table.

Number of Circles	Fraction	Number of Pieces	Division Problem
3	$\frac{1}{2}$	■ 6	$3 \div \frac{1}{2} = \blacksquare$ 6
4	$\frac{1}{2}$	■ 8	$4 \div \frac{1}{2} = \blacksquare$ 8
5	$\frac{1}{2}$	■ 10	$5 \div \frac{1}{2} = \blacksquare$ 10
6	$\frac{1}{2}$	■ 12	$6 \div \frac{1}{2} = \blacksquare$ 12

- 3. a. **Patterns** How do the number of pieces relate to the number of circles in the table? **The number of pieces is double the number of circles.**
b. What happens when you divide a number by $\frac{1}{2}$? **You get twice the number.**
- 4. **Number Sense** How are dividing by $\frac{1}{2}$ and multiplying by 2 related? Explain. **They produce the same result.**

EXERCISES

Find each quotient.

5. $4 \div \frac{1}{3}$ **12**
6. $5 \div \frac{1}{3}$ **15**
7. $4 \div \frac{1}{4}$ **16**
8. $5 \div \frac{1}{4}$ **20**

Investigation

Fraction Division

Students model division of a whole number by a fraction in preparation for learning an algorithm for dividing by fractions in Lesson 5-3 that follows.

Optional Materials

- compass
- straightedge

Teaching Notes

Discuss the 3 circle models that are each divided into eight equal-sized pieces. Make sure students understand how it represents $3 \div \frac{1}{8}$. Ask: *What can you say about the quotient when you divide a whole number by a fraction?* **The quotient is greater than the whole number.**

Inclusion

Students with impaired coordination might have difficulty drawing the circles for Step 1. Provide them with a sheet containing empty circles that they can use to make the models.

Tactile Learners

Some students may benefit from cutting out the semi-circles and then counting them to model the division.

English Learners

When a quantity is divided into two equal parts, people often say it has been “divided in half.” This might lead students to believe that “dividing by 2” and “dividing by $\frac{1}{2}$ ” have the same meaning. Contrast $3 \div 2$, or $1\frac{1}{2}$, with $3 \div \frac{1}{2}$, or 6.

Exercises

Have students work independently on the exercises.

5-3

1. Plan

Lesson Preview

Check Skills You'll Need

Multiplying Fractions
Lesson 5-1: Examples 2–3. Extra Practice p. 646.

Lesson Resources

Teaching Resources
Practice, Reteaching, Enrichment

Reaching All Students
Practice Workbook 5-3
Spanish Practice Workbook 5-3
Guided Problem Solving 5-3
Technology Activities 10
Hands-On Activities 15, 18, 19

Presentation Assistant Plus!

Transparencies
• Check Skills You'll Need 5-3
• Problem of the Day 5-3
• Additional Examples 5-3
• Student Edition Answers 5-3
• Lesson Quiz 5-3
PH Presentation Pro CD-ROM 5-3

ASSESSMENT SYSTEM

Computer Test Generator CD



Technology

Resource Pro® CD-ROM
Computer Test Generator CD
PH Presentation Pro CD-ROM



www.PHSchool.com

Student Site
• Teacher Web Code: aak-5500
• Algebra Readiness Puzzles 5, 55
• Self-grading Lesson Quiz

PH SuccessNet Teacher Center

• Lesson Planner
• Resources

Plus **iTEXT**

5-3

Dividing Fractions

What You'll Learn

- OBJECTIVE 1** To divide whole numbers by fractions
OBJECTIVE 2 To divide fractions by fractions

... And Why

To fill birdfeeders, as in Example 4

Check Skills You'll Need

For help, go to Lesson 5-1.

Find each product.

1. $8 \times \frac{3}{4}$ 6 2. $\frac{2}{3} \times 6$ 4 3. $\frac{4}{5} \cdot \frac{1}{4}$ $\frac{1}{5}$
4. $8 \cdot \frac{1}{8}$ 1 5. $\frac{1}{3}$ of $\frac{3}{7}$ $\frac{1}{7}$ 6. $\frac{10}{11} \cdot \frac{2}{5}$ $\frac{4}{11}$

New Vocabulary • reciprocal

OBJECTIVE

1

Dividing Whole Numbers by Fractions



Interactive lesson includes instant self-check, tutorials, and activities.

Reading Math

Reciprocal comes from a Latin word meaning alternating.

1

EXAMPLE Writing a Reciprocal

Write the reciprocal of each number.

a. $\frac{7}{8}$

b. 9

The reciprocal is $\frac{8}{7}$.

Write 9 as $\frac{9}{1}$. The reciprocal is $\frac{1}{9}$.

Check $\frac{7}{8} \times \frac{8}{7} = \frac{56}{56}$, or 1

Check $\frac{9}{1} \times \frac{1}{9} = \frac{9}{9}$, or 1



Check Understanding 1 Write the reciprocal of each number.

a. $\frac{3}{4}$ $\frac{4}{3}$ or $1\frac{1}{3}$

b. 7 $\frac{1}{7}$

c. $\frac{8}{7}$ $\frac{7}{8}$

To divide by a fraction, multiply by the reciprocal of the fraction. You can remember this by thinking “invert and multiply.”

Key Concepts

Dividing Fractions

Arithmetic

$$\frac{3}{5} \div \frac{1}{3} = \frac{3}{5} \times \frac{3}{1}$$

Algebra

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \text{ where } b, c, \text{ and } d \text{ are not } 0.$$



Ongoing Assessment and Intervention

Before the Lesson

Diagnose prerequisite skills using:

- Check Skills You'll Need

During the Lesson

Monitor progress using:

- Check Understanding
- Additional Examples
- Test Prep

After the Lesson

Assess knowledge using:

- Lesson Quiz
- Computer Test Generator CD

2 EXAMPLE Using Reciprocals to Divide by a Fraction

Find $12 \div \frac{8}{9}$.

$$\begin{aligned}
 12 \div \frac{8}{9} &= 12 \times \frac{9}{8} && \leftarrow \text{Multiply 12 by } \frac{9}{8}, \text{ the reciprocal of } \frac{8}{9}. \\
 &= \frac{12}{1} \times \frac{9}{8} && \leftarrow \text{Write 12 as } \frac{12}{1}. \\
 &= \frac{3 \cancel{12}}{1} \times \frac{9}{\cancel{8}_2} && \leftarrow \text{Divide 12 and 8 by their GCF, 4.} \\
 &= \frac{27}{2} && \leftarrow \text{Multiply.} \\
 &= 13\frac{1}{2} && \leftarrow \text{Write as a mixed number.}
 \end{aligned}$$

Check Understanding 2 Find each quotient.

a. $8 \div \frac{3}{4}$ **$10\frac{2}{3}$**

b. $7 \div \frac{2}{9}$ **$31\frac{1}{2}$**

c. $12 \div \frac{8}{7}$ **$10\frac{1}{2}$**

OBJECTIVE

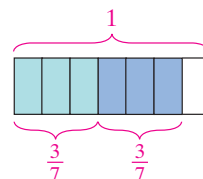
2

Dividing Fractions by Fractions

In the diagram at the right, a rectangle is divided into seven pieces. There are two groups of $\frac{3}{7}$ in $\frac{6}{7}$.

So, $\frac{6}{7} \div \frac{3}{7} = 2$.

$$\begin{aligned}
 \frac{6}{7} \div \frac{3}{7} &= \frac{6}{7} \cdot \frac{7}{3} && \leftarrow \text{Multiply } \frac{6}{7} \text{ by } \frac{7}{3}, \text{ the reciprocal of } \frac{3}{7}. \\
 &= \frac{2 \cancel{6}}{1 \cancel{7}} \cdot \frac{\cancel{7}^1}{3} && \leftarrow \text{Divide 6 and 3 by their GCF, 3.} \\
 &= \frac{2}{1} \cdot \frac{1}{3} && \leftarrow \text{Divide the numerator 7 and denominator 7 by their GCF, 7.} \\
 &= \frac{2}{1}, \text{ or } 2 && \leftarrow \text{Multiply and simplify.}
 \end{aligned}$$



3 EXAMPLE Dividing a Fraction by a Fraction

Find $\frac{5}{10} \div \frac{5}{6}$.

$$\begin{aligned}
 \frac{5}{10} \div \frac{5}{6} &= \frac{5}{10} \times \frac{6}{5} && \leftarrow \text{Multiply by } \frac{6}{5}, \text{ the reciprocal of } \frac{5}{6}. \\
 &= \frac{1 \cancel{5}}{2 \cancel{10}} \times \frac{6^3}{\cancel{5}_1} && \leftarrow \text{Divide 5 and 5 by their GCF, 5.} \\
 &= \frac{3}{5} && \leftarrow \text{Divide 10 and 6 by their GCF, 2.} \\
 &= \frac{3}{5} && \leftarrow \text{Multiply.}
 \end{aligned}$$

3d. $\frac{2}{3}$ is between $\frac{1}{2}$ and 1.
There are 2 fourths in $\frac{1}{2}$, and 4 fourths in 1.
So there must be between 2 and 4 fourths in $\frac{2}{3}$.

Check Understanding 3 Find each quotient.

a. $\frac{9}{16} \div \frac{3}{4}$ **$\frac{3}{4}$**

b. $\frac{5}{8} \div \frac{5}{6}$ **$\frac{3}{4}$**

c. $\frac{8}{15} \div \frac{2}{3}$ **$\frac{4}{5}$**

d. **Number Sense** Without dividing, how can you tell whether $\frac{2}{3} \div \frac{1}{4}$ is greater than or less than 2? **Answers may vary. See above left for sample.**

2. Teach

Professional Development

Math Background

Algebraically, division is defined in terms of multiplication. That is, the quotient $m \div n$ is defined as $m \times \frac{1}{n}$, provided that the divisor n is not zero. The number $\frac{1}{n}$ is called the *reciprocal* of n . This definition of division is stated less formally as follows: *Dividing by a nonzero number is the same as multiplying by the reciprocal of that number.*

The reciprocal of a fraction $\frac{c}{d}$ is $\frac{d}{c}$. So, when a division involves fractions, the definition of division leads to the familiar rule for dividing fractions: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, provided that b , c , and d are not zero.

Teaching Notes

1 EXAMPLE English Learners

To help students remember the word *reciprocal*, discuss the related word *reciprocate*. Point out that, when you do a favor for a friend, the friend reciprocates by doing a favor for you. Tell students to think of a reciprocal as the result when the numerator and denominator of a fraction do a "favor" for each other: The numerator takes the place of the denominator, and the denominator takes the place of the numerator.

2 EXAMPLE Auditory Learners

Students might find it helpful to recite the "invert the second fraction and multiply" procedure softly as they work.

In Check Understanding 2a, for instance, they might say: "Eight divided by three fourths" as they copy the exercise, then "Eight multiplied by four thirds" as they write the multiplication.

PowerPoint

Additional Examples

1 Write the reciprocal of each number.

a. $\frac{4}{9}$ **$\frac{9}{4}$** b. 5 **$\frac{1}{5}$**

2 Find $4 \div \frac{6}{7}$. **$4\frac{2}{3}$**

3 Find $\frac{3}{8} \div \frac{7}{12}$. **$\frac{9}{14}$**



Reaching All Students

Below Level Give students several blank forms like the one below to use for organizing their work.

$$\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square \cdot \square}{\square \cdot \square} = \frac{\square}{\square}$$

Advanced Learners Find the missing divisor.

$$\begin{aligned}
 \frac{3}{8} \div \square &= \frac{15}{16} && \text{2} \\
 \frac{2}{5} \div \square &= \frac{1}{15} && \text{6}
 \end{aligned}$$

English Learners See note on page 231.

Auditory Learners See note on page 231.

Error Prevention!

When dividing by a fraction, students might change the operation to multiplication but forget to use the reciprocal of the divisor. Suggest that they copy the division, then write the related multiplication directly beneath. For instance, they should begin their work on Check Understanding 4a as follows.

$$\begin{array}{r} \frac{3}{8} \div \frac{12}{1} \\ \downarrow \quad \downarrow \quad \downarrow \\ \frac{3}{8} \times \frac{1}{12} \end{array}$$

PowerPoint

Additional Examples

- 4 Six people will equally share $\frac{3}{4}$ of a cake. How much of the cake will each person get?
 $\frac{1}{8}$ of the cake

Closure

- What does it mean for two numbers to be reciprocals?
Their product is 1.
- How do you use reciprocals when dividing by a fraction?
To divide by a fraction, you multiply by the reciprocal of that fraction.

To divide a fraction by a whole number, first write the whole number as an improper fraction with a denominator of 1.

4 EXAMPLE Dividing a Fraction by a Whole Number

Feeding Birds Suppose you plan to put the same amount of seed in three feeders. You have $\frac{7}{8}$ pound of seed. How much seed will each feeder get?

The $\frac{7}{8}$ pound of seed must be distributed evenly among three feeders, so divide $\frac{7}{8}$ by 3.

$$\begin{aligned} \frac{7}{8} \div 3 &= \frac{7}{8} \div \frac{3}{1} \quad \leftarrow \text{Write 3 as } \frac{3}{1}. \\ &= \frac{7}{8} \times \frac{1}{3} \quad \leftarrow \text{Multiply by } \frac{1}{3}, \text{ the reciprocal of } \frac{3}{1}. \\ &= \frac{7}{24} \quad \leftarrow \text{Multiply.} \end{aligned}$$

Each feeder will get $\frac{7}{24}$ pound of seed.

-  **Check Understanding** 4 Find each quotient. a. $\frac{3}{8} \div 12$ $\frac{1}{32}$ b. $\frac{11}{15} \div 110$ $\frac{1}{150}$
c. Your art teacher must cut $\frac{5}{6}$ yard of fabric into five equal pieces for his students. How much fabric does each student get?
Answers may vary. Sample: $\frac{1}{6}$ of the original piece, or $\frac{1}{6}$ yd.

EXERCISES

 For more practice, see *Extra Practice*.

A Practice by Example

Write the reciprocal of each number.

Example 1
(page 230)

1. $\frac{2}{5}$ $\frac{5}{2}$, or $2\frac{1}{2}$ 2. $\frac{1}{7}$ 7 3. 11 $\frac{1}{11}$ 4. $\frac{5}{3}$ $\frac{3}{5}$ 5. $\frac{4}{11}$ $\frac{11}{4}$, or $2\frac{3}{4}$

Example 2
(page 231)

Find each quotient. Exercise 6 has been started for you.

6. $5 \div \frac{3}{8} = 5 \times \frac{8}{3}$ $13\frac{1}{3}$ 7. $4 \div \frac{3}{5}$ $6\frac{2}{3}$ 8. $5 \div \frac{5}{16}$ 16
9. $7 \div \frac{3}{5}$ $11\frac{2}{3}$ 10. $9 \div \frac{4}{9}$ $20\frac{1}{4}$ 11. $6 \div \frac{2}{5}$ 15 12. $8 \div \frac{3}{7}$ $18\frac{2}{3}$

Example 3
(page 231)

13. $\frac{8}{9} \div \frac{1}{3}$ $2\frac{2}{3}$ 14. $\frac{1}{4} \div \frac{1}{4}$ 1 15. $\frac{11}{2} \div \frac{3}{4}$ $7\frac{1}{3}$ 16. $\frac{1}{5} \div \frac{1}{4}$ $\frac{4}{5}$
17. $\frac{4}{9} \div \frac{2}{3}$ $\frac{2}{3}$ 18. $\frac{9}{2} \div \frac{1}{2}$ 9 19. $\frac{8}{9} \div \frac{4}{5}$ $1\frac{1}{9}$ 20. $\frac{3}{4} \div \frac{1}{8}$ 6

Example 4
(page 232)

21. $\frac{3}{4} \div 3$ $\frac{1}{4}$ 22. $\frac{1}{2} \div 5$ $\frac{1}{10}$ 23. $\frac{11}{3} \div 4$ $2\frac{11}{12}$ 24. $\frac{5}{12} \div 15$ $\frac{1}{36}$

-  25. **Construction** Sam has $\frac{3}{4}$ ton of stones to divide evenly among four sidewalks. How much stone will be used in each sidewalk? $\frac{3}{16}$ t

44. 24;
four $\frac{1}{4}$ inches



46. Answers may vary.
Sample: When you divide a number by 2, the quotient is less than the dividend. When you divide a number by $\frac{1}{2}$, the quotient is greater than

the dividend. For instance: $3 \div 2 = 1\frac{1}{2}$

B Apply Your Skills

Find each quotient.

26. $\frac{4}{3} \div \frac{5}{3}$ **$\frac{4}{5}$** 27. $11 \div \frac{121}{10}$ **$\frac{10}{11}$** 28. $\frac{9}{8} \div 4$ **$\frac{9}{32}$** 29. $\frac{11}{9} \div 3$ **$\frac{11}{27}$**
 30. $2 \div \frac{9}{7}$ **$1\frac{5}{9}$** 31. $\frac{5}{3} \div \frac{4}{3}$ **$1\frac{1}{4}$** 32. $\frac{15}{8} \div \frac{9}{5}$ **$1\frac{1}{24}$** 33. $8 \div \frac{15}{6}$ **$3\frac{1}{5}$**

Geography Use the table at the right.

34. About how many times more people live in Argentina than in Peru?
about $1\frac{1}{3}$ times more
35. About how many times more people live in Brazil than in Colombia?
about $4\frac{1}{2}$ times more
36. The population of Brasilia, the capital of Brazil, is about $\frac{1}{85}$ of the population of Brazil. What fraction of the total population of South America lives in Brasilia? **about $\frac{1}{170}$**

Country	Portion of South America's Population
Brazil	$\frac{1}{2}$
Colombia	$\frac{1}{9}$
Argentina	$\frac{1}{10}$
Peru	$\frac{1}{13}$

SOURCE: U.S. Census Bureau. Go to www.PHSchool.com for a data update.
Web Code: aag-2041

43. Answers may vary.
Sample: The reciprocal of $\frac{11}{9}$ was multiplied by $\frac{2}{3}$ instead of by the reciprocal of $\frac{2}{3}$. The correct answer is $1\frac{5}{6}$.

Writing in Math

For help with writing to explain, as in Exercise 43, see p. 235.

Algebra Evaluate each expression for $x = \frac{5}{6}$ and $n = 3$.

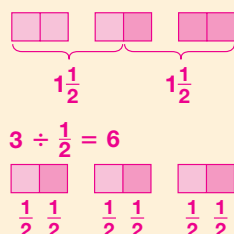
37. $30 \div x$ **36** 38. $\frac{3}{5} \div x$ **$\frac{18}{25}$** 39. $x \div 2$ **$\frac{5}{12}$**
 40. $\frac{n}{5} \div \frac{1}{3}$ **$1\frac{4}{5}$** 41. $\frac{51}{16} \div n$ **$1\frac{1}{16}$** 42. $\frac{1}{3} \div \frac{5}{n}$ **$\frac{1}{5}$**

43. **Error Analysis** Explain and correct the error in the work at the right.
See above left.
44. **Measurement** How many $\frac{1}{4}$ inches are in $\frac{1}{2}$ foot? Draw a diagram that shows the problem and your solution. **See margin.**
45. **Baking** You are baking an apple pie. The recipe calls for eight sliced apples. Suppose you cut the apples into eighths. How many pieces of apple would you have? **64 pieces**
46. **Writing in Math** How are dividing by 2 and dividing by $\frac{1}{2}$ different? Include a diagram. **See margin.**

Find the number that completes each equation.

47. $\frac{3}{2} \div \frac{1}{2} = \square$ **3** 48. $\frac{3}{2} \div \square = 3$ **$\frac{1}{2}$** 49. $\frac{3}{2} \div \square = \frac{3}{4}$ **2**

- 50. Baking** A recipe for a loaf of banana bread requires $\frac{2}{3}$ cup of vegetable oil. You have 3 cups of oil but need 1 cup for a different recipe. How many loaves of banana bread can you make with the rest of the oil?
3 loaves



GPS Use the Guided Problem Solving worksheet with Exercise 50.

3. Practice

Assignment Guide

1 Objective 1

A B Core 1–12, 27, 30, 33

2 Objective 2

A B Core 13–26, 28–29, 31–32, 34–50

C Extension 51–55

Test Prep 56–59

Mixed Review 60–67

Practice 5-3

Dividing Fractions

Write the reciprocal of each number.

1. $\frac{10}{7}$ 2. $\frac{1}{4}$ 3. $\frac{1}{3}$ 4. $\frac{1}{12}$

5. Draw a diagram to show how many $\frac{1}{2}$ -ft pieces of string can be cut from a piece of string $4\frac{1}{2}$ ft long.

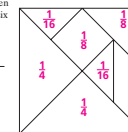


Find each quotient.

6. $\frac{3}{10} \div \frac{2}{5}$ 7. $\frac{3}{24} \div 3$ 8. $\frac{1}{3} \div 2$
 9. $\frac{1}{4} \div \frac{1}{2}$ 10. $\frac{2}{24} \div \frac{3}{16}$ 11. $\frac{1}{4} \div \frac{1}{8}$
 12. $\frac{1}{2} \div \frac{1}{3}$ 13. $\frac{6}{9} \div \frac{1}{2}$ 14. $3 \div \frac{3}{8}$

Solve.

15. How many $\frac{1}{2}$ -cup servings are there in a 6-cup package of rice?
8 servings
16. George cut 5 oranges into quarters. How many pieces of orange did he have?
20 pieces
17. A cake recipe calls for $\frac{3}{4}$ cup of butter. One tablespoon equals $\frac{1}{16}$ cup. How many tablespoons of butter are used to make the cake?
10 tablespoons
18. Maureen, Frank, Tashia, Zane, Eric, and Wesley are addressing envelopes for volunteer work at a local charity. They were given $\frac{2}{3}$ of an entire mailing to address to be evenly divided among six of them. What fraction of the entire mailing does each person address?
 $\frac{1}{9}$
19. Study the tangram pieces at the right. If the entire square is 1, find the fractional value of each piece. You can copy the tangram and cut the pieces to compare them.



Reteaching 5-3

Dividing Fractions

Find $8 \div \frac{4}{5}$.

① The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

$$\frac{4}{5} \times \frac{5}{4}$$

② Multiply 8 by the reciprocal.

$$8 \div \frac{4}{5} = 8 \times \frac{5}{4} = \frac{8}{1} \times \frac{5}{4} = \frac{2}{1} \times \frac{5}{1} = 10$$

$$8 \div \frac{4}{5} = 10$$

Find $\frac{2}{3} \div \frac{8}{15}$.

① The reciprocal of $\frac{8}{15}$ is $\frac{15}{8}$.

$$\frac{8}{15} \times \frac{15}{8}$$

② Multiply $\frac{2}{3}$ by the reciprocal.

$$\frac{2}{3} \div \frac{8}{15} = \frac{2}{3} \times \frac{15}{8} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

$$\frac{2}{3} \div \frac{8}{15} = \frac{5}{4}$$

Write the reciprocal of each number.

1. $\frac{1}{2}$ 2. $\frac{5}{3}$ 3. $\frac{1}{20}$
 4. $\frac{9}{8}$ 5. 14 6. $\frac{1}{18}$
 7. $\frac{9}{5}$ 8. $\frac{11}{3}$ 9. $\frac{7}{9}$
 10. $\frac{12}{11}$ 11. $\frac{7}{2}$ 12. $\frac{15}{13}$

Find each quotient.

13. $2 \div \frac{2}{3}$ 14. $7 \div \frac{8}{5}$ 15. $9 \div \frac{12}{10}$
 16. $6 \div \frac{15}{8}$ 17. $5 \div \frac{7}{2}$ 18. $14 \div \frac{16}{5}$
 19. $\frac{2}{3} \div \frac{12}{5}$ 20. $\frac{2}{3} \div \frac{11}{8}$ 21. $\frac{2}{3} \div 2$
 22. $\frac{2}{3} \div \frac{15}{8}$ 23. $\frac{1}{2} \div 4$ 24. $\frac{2}{3} \div \frac{8}{5}$
 25. $\frac{10}{11} \div 3$ 26. $\frac{3}{5} \div 5$ 27. $\frac{2}{3} \div 10$
 28. $\frac{1}{2} \div \frac{7}{8}$ 29. $\frac{5}{8} \div 3$ 30. $\frac{11}{12} \div \frac{1}{4}$

4. Assess



Lesson Quiz 5-3

Find each quotient.

- $\frac{4}{5} \div \frac{2}{3}$ **$1\frac{1}{5}$**
- $\frac{3}{8} \div \frac{5}{6}$ **$\frac{9}{20}$**
- $7 \div \frac{3}{10}$ **$23\frac{1}{3}$**
- $\frac{7}{12} \div 4$ **$\frac{7}{48}$**

Alternative Assessment

Each student in a pair writes a fraction and a whole number. Partners work together to divide each of their whole numbers by a fraction (4 possibilities) and each of their fractions by the other fraction (2 possibilities).

Test Prep

Resources

For additional practice with a variety of test item formats:

- Test-Prep, p. 263
- Test-Taking Strategies, p. 259
- Test-Taking Strategies With Transparencies

Exercise 56 Have students discuss the steps they would take and the operation they would use to find the answer.

Challenge

Simplify each expression.

51. $(\frac{2}{7} \times \frac{2}{7}) \div \frac{2}{7}$ **$\frac{2}{7}$** 52. $(\frac{2}{7})^2 \div \frac{2}{7}$ **$\frac{2}{7}$** 53. $(\frac{2}{7})^2 \div 2^2$ **$\frac{1}{49}$** 54. $(\frac{2}{7})^2 \div (\frac{1}{7})^2$ **4**

55. **Stretch Your Thinking** The numerator of a fraction is a two-digit prime number. The denominator is four more than the numerator. Three of the digits are the same prime number. What is the fraction? **$\frac{73}{77}$**



Test Prep

Multiple Choice

The table shows the different weights of the same object on Earth, on the moon, and on Venus.

Earth	Moon	Venus
1 lb	$\frac{1}{6}$ lb	$\frac{5}{6}$ lb

56. How many pounds would a 186-pound astronaut weigh on Venus? **B**
A. 31 B. 155 C. 186 D. 365

57. Suppose you weigh m pounds on the moon. Which expression represents your weight on Venus? **H**
F. $\frac{1}{5}m$ G. $1\frac{1}{5}m$ H. $5m$ I. $6m$

58. If Ezra weighs 150 pounds on Earth, what is his weight on the moon? **B**
A. 20 lb B. 25 lb C. 30 lb D. 35 lb

59. Jaime bought a carton of juice containing 192 fluid ounces, or 24 cups of juice. (a) If a serving is $\frac{3}{4}$ cup, how many servings of juice are in the carton? (b) Explain in words how you found your answer. **See margin.**



Take It to the NET
Online lesson quiz at
www.PHSchool.com
Web Code: aaa-0503

Short Response



Mixed Review

Lesson 5-2

Find each product.

60. $3\frac{3}{8} \times 2\frac{2}{9}$ **$7\frac{1}{2}$** 61. $8\frac{1}{3} \times 2\frac{2}{5}$ **20** 62. $5\frac{1}{3} \cdot 8\frac{1}{4}$ **44** 63. $4\frac{1}{2} \cdot 3\frac{1}{3}$ **15**

Lesson 3-1

Test each number for divisibility by 2, 3, 5, 9, or 10.

64. 1,250 **2, 5, 10** 65. 372 **2, 3** 66. 55,600 **2, 5, 10** 67. 445 **5**

Enrichment 5-3

Dividing Fractions

Patterns in Numbers

You have found rules for number sequences. For example, you know that the rule for the sequence 3, 9, 27, 81 is *Multiply by 3*.

Sample answers:
Exercises 1–10

You can also find rules for sequences that contain fractions. Some rules also contain fractions. Write the next three terms in each sequence below. Then write the rule.

1. 81, 27, 9, 3 ... Rule: **Divide by 3.**

2. 250, 50, 10, 2 ... Rule: **Divide by 5.**

3. 1, 8, 64, 512 ... Rule: **Multiply by 8.**

4. 1000, 100, 10, 1 ... Rule: **Divide by 10.**

5. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$... Rule: **Multiply by $\frac{1}{2}$.**

6. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}$... Rule: **Multiply by $\frac{2}{3}$.**

7. How could you rewrite the rule *Divide by 2* using a fraction in the rule? Think: $4 \div 2 = 2$, $4 \times \frac{1}{2} = 2$. **Multiply by $\frac{1}{2}$.**

8. Rewrite each rule for Exercises 1–6 above.

a. 81, 27, 9, ... **Multiply by $\frac{1}{3}$.**

b. 250, 50, 10, ... **Multiply by $\frac{1}{5}$.**

c. 1, 8, 64, ... **Multiply by $\frac{1}{8}$.**

d. 1000, 100, 10, ... **Multiply by $\frac{1}{10}$.**

e. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$... **Divide by $\frac{1}{2}$.**

f. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}$... **Divide by $\frac{2}{3}$.**

9. Draw the next figure in the pattern. Then write two rules for the pattern.

Multiply by $\frac{1}{2}$; divide by 2.

10. Draw the next figure in the pattern. Then write two rules for the pattern.

Multiply by 2; divide by $\frac{1}{2}$.

59. [2] 32 servings
To find how many servings of $\frac{3}{4}$ cup there are, divide the number of cups available, 24, by $\frac{3}{4}$.

$24 \div \frac{3}{4} = 24 \times \frac{4}{3} =$
 $\frac{8 \cdot 24}{1} \times \frac{4}{3} = \frac{32}{1} = 32.$
So there are 32 servings of $\frac{3}{4}$ cup each.

[1] correct expression, but wrong answer

In this book, there are many exercises that ask you to give an explanation. When you are asked to explain an error, consider the following:

- Find the error and explain what is wrong.
- Correct the error.
- Check your work, if possible.

On page 233, you will find the following exercise.

43. **Error Analysis** Explain and correct the error in the work at the right.
- $$\frac{11}{9} \div \frac{2}{3} = \frac{3}{11} \times \frac{2}{3} = \frac{6}{11}$$

Here is one student's response.

The answer must be wrong because $\frac{11}{9}$ is about 1, and there is one group of $\frac{2}{3}$ in 1. The divisor is $\frac{2}{3}$. So $\frac{2}{3}$, not $\frac{11}{9}$, should be replaced with its reciprocal.

$$\frac{11}{9} \div \frac{2}{3} = \frac{11}{9} \times \frac{3}{2} = \frac{11}{6}, \text{ or } 1\frac{5}{6}$$

Since $1\frac{5}{6}$ is more than 1, this answer makes sense.

← Explain what is wrong.

← Correct the error.

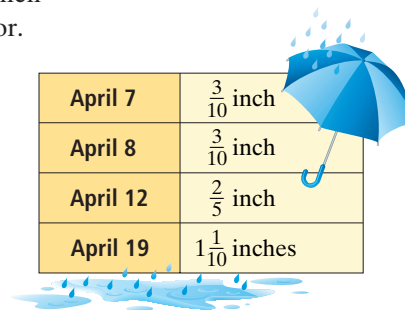
← Check for reasonableness.

EXERCISES

1–4. Answers may vary. See margin for samples.

- A student has a bottle containing 4 cups of laundry detergent. The student thinks that, if each load of laundry requires $\frac{1}{3}$ cup, then there is enough detergent for $4 \times \frac{1}{3}$, or $\frac{4}{3}$ loads. Explain the error.
- April rainfall is shown in the table. One person says the total rainfall is $2\left(\frac{3}{10}\right) + \frac{2}{5} + 1\frac{1}{10} = 3\frac{8}{10}$, or $3\frac{4}{5}$ inches. Explain the error.
- A student solves $10 \div \frac{5}{3}$ by finding $\frac{1}{10} \times \frac{3}{5}$. Her answer is $\frac{3}{50}$. Explain the error, and find the correct answer.
- A model airplane sits on a base that is $19\frac{11}{16}$ inches by $10\frac{7}{8}$ inches. One estimate of the area of the base is 190 square inches. Why is 220 square inches a better estimate?

April 7	$\frac{3}{10}$ inch
April 8	$\frac{3}{10}$ inch
April 12	$\frac{2}{5}$ inch
April 19	$1\frac{1}{10}$ inches



1. The answer is wrong because one cup of detergent will do 3 loads of laundry, already more than $\frac{4}{3}$. To find how many groups of $\frac{1}{3}$ are in 4, 4 should be divided by $\frac{1}{3}$.

not multiplied by $\frac{1}{3}$. So, $4 \div \frac{1}{3} = 4 \times 3 = 12$. Since 1 cup will do 3 loads, 4 cups will do 12 loads. The answer makes sense.

2, 3. See back of book.

4. 190 is 19×10 , so there is almost 1 inch in each measure that is ignored. The two measures were rounded down to the nearest whole number. However, $19\frac{11}{16}$ is closer

Writing to Explain

When students do an exercise that asks them to analyze an error, they may respond that an operation is inappropriate or that a particular calculation is wrong. Guide students to understand that error analysis involves three key tasks beyond simply identifying the error: (1) explaining why a particular calculation or operation used is incorrect, (2) correcting the error or errors, and (3) checking to see that their answer makes sense.

Teaching Notes

Begin the lesson by asking students to suggest what they think is involved in an error analysis exercise. Ask a volunteer to record responses on the board or on an overhead transparency.

Next, tell students to read the division exercise with the error, directing them to think about what has gone wrong. Then work through the student's response, step-by-step. Ask:

- Do you agree with the conclusion that the answer to the division is too small? **Yes. The answer must be greater than the dividend, $\frac{11}{9}$, because the divisor is a fraction.**
- Could you explain the error in another way? **Answers may vary; students may state that since multiplication and division are inverse operations, dividing by $\frac{2}{3}$ is equivalent to multiplying by its reciprocal, $\frac{3}{2}$.**

Inclusion

Make sure students understand the type of exercise this lesson examines. Tell them that in their text, exercises of this kind are signaled by the words "Error Analysis," which appear in blue type just after the exercise number. Encourage students to flip through the book to find other examples of this type of exercise.

to 20, and $10\frac{7}{8}$ is closer to 11. A closer estimate would be $20 \times 11 = 220$ in.².