

CHAPTER 86 MEAN, MEDIAN, MODE AND STANDARD DEVIATION

EXERCISE 326 Page 919

1. Determine the mean, median and modal values for the set: {3, 8, 10, 7, 5, 14, 2, 9, 8}

$$\text{Mean} = \frac{3+8+10+7+5+14+2+9+8}{9} = \frac{66}{9} = 7.33$$

Ranking gives: 2 3 5 7 8 8 9 10 14

Median = middle value = 8

Most commonly occurring value, i.e. mode = 8

2. Determine the mean, median and modal values for the set: {26, 31, 21, 29, 32, 26, 25, 28}

$$\text{Mean} = \frac{26+31+21+29+32+26+25+28}{8} = \frac{218}{8} = 27.25$$

Ranking gives: 21 25 26 26 28 29 31 32

$$\text{Median} = \text{middle value} = \frac{26+28}{2} = 27$$

Most commonly occurring value, i.e. mode = 26

3. Determine the mean, median and modal values for the set:

$$\{4.72, 4.71, 4.74, 4.73, 4.72, 4.71, 4.73, 4.72\}$$

$$\text{Mean} = \frac{4.72+4.71+4.74+4.73+4.72+4.71+4.73+4.72}{8} = \frac{37.78}{8} = 4.7225$$

Ranking gives: 4.71 4.71 4.72 4.72 4.72 4.73 4.73 4.74

$$\text{Median} = \text{middle value} = \frac{4.72+4.72}{2} = 4.72$$

Most commonly occurring value, i.e. mode = 4.72

4. Determine the mean, median and modal values for the set:

{73.8, 126.4, 40.7, 141.7, 28.5, 237.4, 157.9}

$$\text{Mean} = \frac{73.8 + 126.4 + 40.7 + 141.7 + 28.5 + 237.4 + 157.9}{7} = \frac{806.4}{7} = 115.2$$

Ranking gives: 28.5 40.7 73.8 126.4 141.7 157.9 237.4

Middle value = median = 126.4

There is no mode since all the values are different

EXERCISE 327 Page 920

1. 21 bricks have a mean mass of 24.2 kg, and 29 similar bricks have a mass of 23.6 kg. Determine the mean mass of the 50 bricks.

$$\text{Mean value} = \frac{(21 \times 24.2) + (29 \times 23.6)}{21 + 29} = \frac{1192.6}{50} = 23.85 \text{ kg}$$

2. The frequency distribution given below refers to the heights in centimetres of 100 people.

Determine the mean value of the distribution, correct to the nearest millimetre.

150–156	5,	157–163	18,	164–170	20
171–177	27,	178–184	22,	185–191	8

$$\begin{aligned}\text{Mean value} &= \frac{(5 \times 153) + (18 \times 160) + (20 \times 167) + (27 \times 174) + (22 \times 181) + (8 \times 188)}{100} \\ &= \frac{17169}{100} = 171.7 \text{ cm}\end{aligned}$$

3. The gain of 90 similar transistors is measured and the results are as shown.

83.5–85.5	6,	86.5–88.5	39,	89.5–91.5	27,
92.5–94.5	15,	95.5–97.5	3		

By drawing a histogram of this frequency distribution, determine the mean, median and modal values of the distribution.

The histogram is shown below

The mean value lies at the centroid of the histogram. With reference to axis YY at 2.010 cm,

$$AM = \sum(am)$$

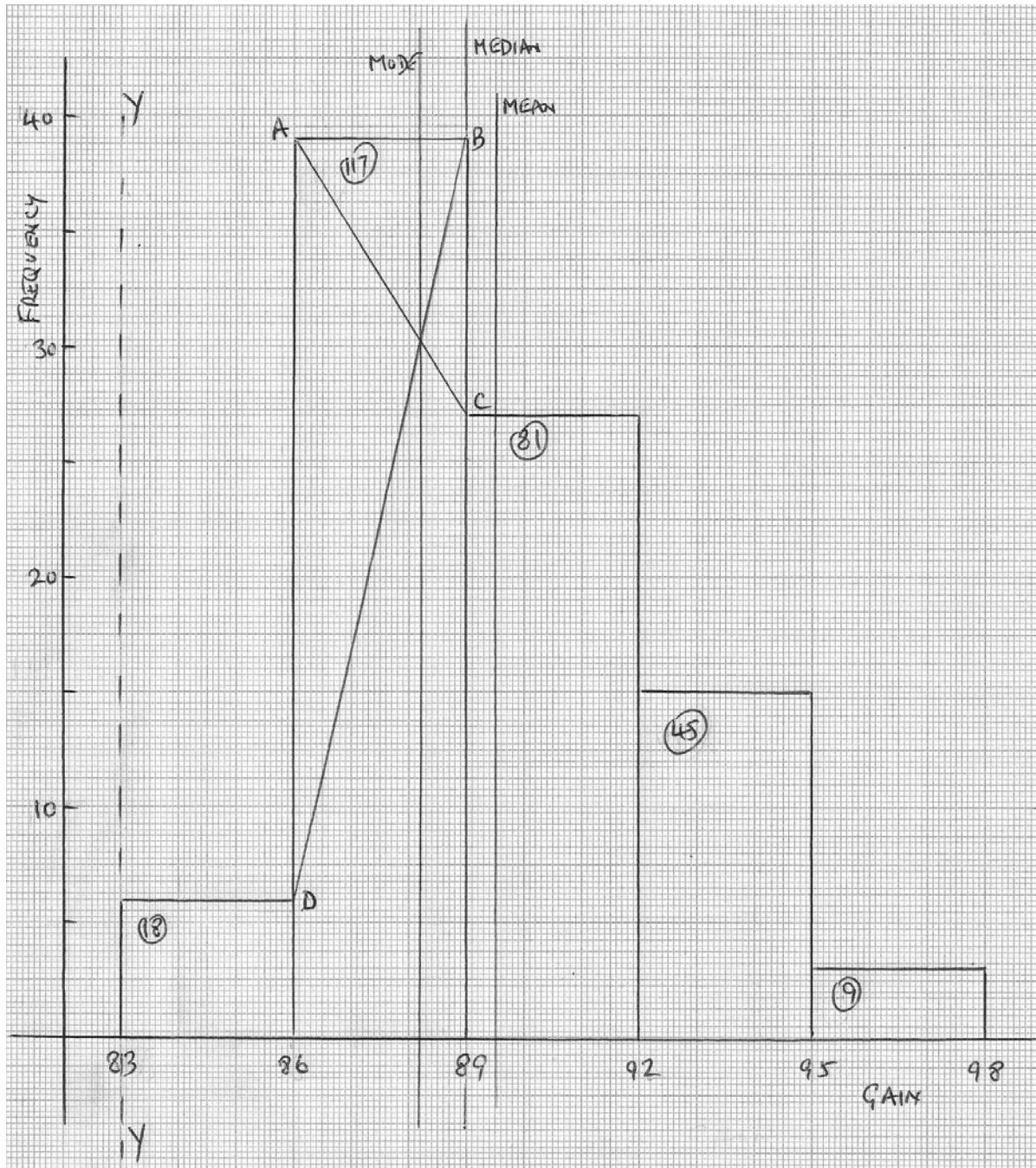
where A = area of histogram = $18 + 117 + 81 + 45 + 9 = 270$ and M = horizontal distance of centroid from YY.

$$\begin{aligned}\text{Hence,} \quad 270 M &= (18 \times 1.5) + (117 \times 4.5) + (81 \times 7.5) \\ &\quad + (45 \times 10.5) + (9 \times 13.5)\end{aligned}$$

i.e. $270 M = 1755$

i.e.
$$M = \frac{1755}{270} = 6.5 \text{ cm}$$

Thus, the mean gain is at $83 + 6.5 = 89.5$



The median is the gain where the area on each side of it is the same, i.e. $270/2$, i.e. 135 square units on each side.

The first two rectangles have an area of $18 + 117 = 135$

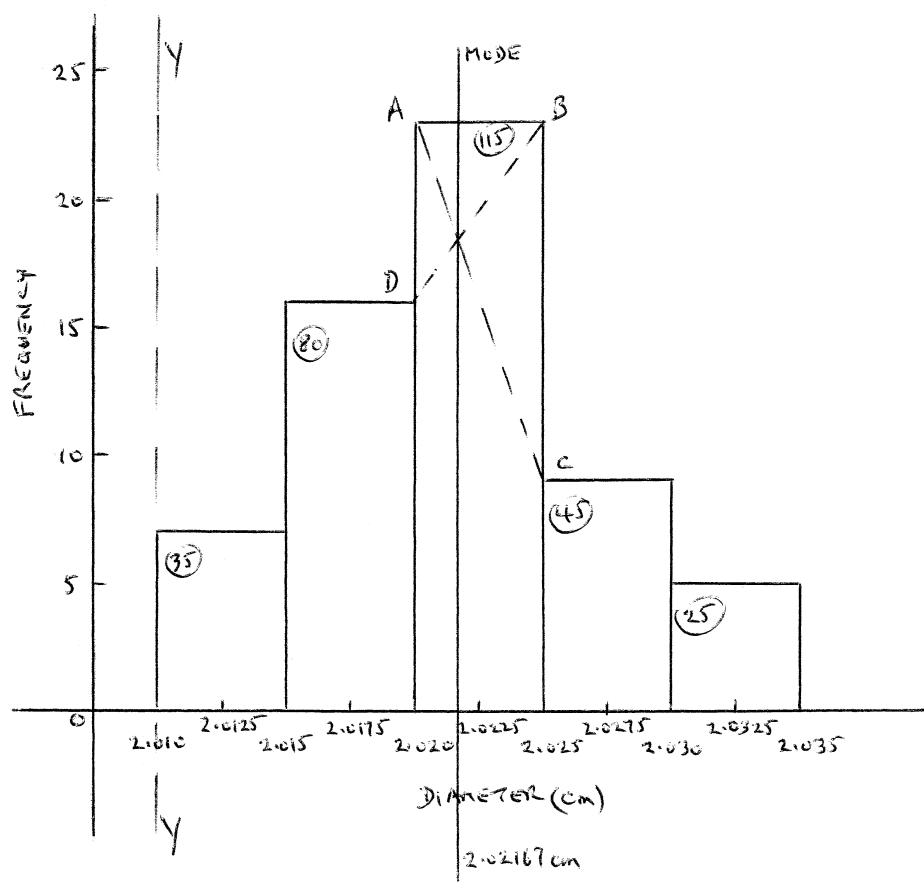
i.e. median gain occurs at 89

The mode is at the intersection of AC and BD , i.e. at 88.2

4. The diameters, in centimetres, of 60 holes bored in engine castings are measured and the results are as shown. Draw a histogram depicting these results and hence determine the mean, median and modal values of the distribution.

2.011–2.014	7,	2.016–2.019	16,	2.021–2.024	23,
2.026–2.029	9,	2.031–2.034	5		

The histogram is shown below



The mean value lies at the centroid of the histogram. With reference to axis YY at 2.010 cm,

$$AM = \sum(am)$$

where A = area of histogram = $35 + 80 + 115 + 45 + 25 = 300$ and M = horizontal distance of centroid from YY (actually, the area of, say, 35 square units is 35×10^{-3} square units; however, the 10^{-3} will cancel on each side of the equation so has been omitted)

Hence,

$$300M = (35 \times 0.0025) + (80 \times 0.0075) + (115 \times 0.0125)$$

$$+ (45 \times 0.0175) + (25 \times 0.0225)$$

i.e. $300M = 3.475$

i.e. $M = \frac{3.475}{300} = 0.01158 \text{ cm}$

Thus, the mean is at $2.010 + 0.01158 = 2.02158 \text{ cm}$

The median is the diameter where the area on each side of it is the same, i.e. $300/2$, i.e. 150 square units on each side

The first two rectangles have an area of $35 + 80 = 115$; hence, 35 more square units are needed from the third rectangle. $\frac{35}{115} \times 100\% = 30.43\%$ of the distance from 2.020 to 2.025

i.e. $0.3043 \times (2.025 - 2.020) = 0.00152$

i.e. median occurs at $2.020 + 0.00152 = 2.02152 \text{ cm}$

The mode is at the intersection of AC and BD , i.e. at 2.02167 cm

EXERCISE 328 Page 922

1. Determine the standard deviation from the mean of the set of numbers:

{35, 22, 25, 23, 28, 33, 30} correct to 3 significant figures.

$$\text{Mean, } \bar{x} = \frac{35 + 22 + 25 + 23 + 28 + 33 + 30}{7} = \frac{196}{7} = 28$$

Standard deviation,

$$\begin{aligned} \sigma &= \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{(35-28)^2 + (22-28)^2 + (25-28)^2 + (23-28)^2 + (28-28)^2 + (33-28)^2 + (30-28)^2}{7} \right\}} \\ &= \sqrt{\frac{148}{7}} = \sqrt{21.143} = 4.60, \text{ correct to 3 significant figures} \end{aligned}$$

2. The values of capacitances, in microfarads, of ten capacitors selected at random from a large batch of similar capacitors are: 34.3, 25.0, 30.4, 34.6, 29.6, 28.7, 33.4, 32.7, 29.0 and 31.3

Determine the standard deviation from the mean for these capacitors, correct to 3 significant figures.

$$\text{Mean, } \bar{x} = \frac{34.3 + 25.0 + 30.4 + 34.6 + 29.6 + 28.7 + 33.4 + 32.7 + 29.0 + 31.3}{10} = \frac{310}{10} = 31$$

Standard deviation,

$$\begin{aligned} \sigma &= \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{(34.3-31)^2 + (25.0-31)^2 + (30.4-31)^2 + \dots + (31.3-31)^2}{10} \right\}} \\ &= \sqrt{\frac{80.2}{10}} = \sqrt{8.02} = 2.83 \mu\text{F} \end{aligned}$$

3. The tensile strength in megapascals for 15 samples of tin were determined and found to be:

34.61, 34.57, 34.40, 34.63, 34.63, 34.51, 34.49, 34.61,
34.52, 34.55, 34.58, 34.53, 34.44, 34.48 and 34.40

Calculate the mean and standard deviation from the mean for these 15 values, correct to 4

significant figures.

$$\text{Mean, } \bar{x} = \frac{34.61 + 34.57 + 34.40 + 34.63 + \dots}{15} = \frac{517.95}{15} = 34.53 \text{ MPa}$$

Standard deviation,

$$\begin{aligned}\sigma &= \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{(34.61 - 34.53)^2 + (34.57 - 34.53)^2 + (34.40 - 34.53)^2 + \dots}{15} \right\}} \\ &= \sqrt{\frac{0.0838}{15}} = \sqrt{0.005586666} = 0.07474 \text{ MPa}\end{aligned}$$

4. Calculate the standard deviation from the mean for the mass of the 50 bricks given in Problem 1 of Exercise 327, page 920, correct to 3 significant figures.

$$\text{Mean value} = \frac{(21 \times 24.2) + (29 \times 23.6)}{21 + 29} = \frac{1192.6}{50} = 23.85 \text{ kg from Problem 1 of Exercise 327}$$

$$\begin{aligned}\text{Standard deviation, } \sigma &= \sqrt{\left\{ \frac{\sum \{f(x - \bar{x})^2\}}{\sum f} \right\}} = \sqrt{\left\{ \frac{21(24.2 - 23.85)^2 + 29(23.6 - 23.85)^2}{50} \right\}} \\ &= \sqrt{\frac{4.385}{50}} = 0.296 \text{ kg}\end{aligned}$$

5. Determine the standard deviation from the mean, correct to 4 significant figures, for the heights of the 100 people given in Problem 2 of Exercise 327, page 920

$$\begin{aligned}\text{Mean value} &= \frac{(5 \times 153) + (18 \times 160) + (20 \times 167) + (27 \times 174) + (22 \times 181) + (8 \times 188)}{100} \\ &= \frac{17169}{100} = 171.7 \text{ cm from Problem 2 of Exercise 327}\end{aligned}$$

$$\text{Standard deviation, } \sigma = \sqrt{\left\{ \frac{\sum \{f(x - \bar{x})\}}{\sum f} \right\}}$$

$$= \sqrt{\left\{ \frac{5(153 - 171.7)^2 + 18(160 - 171.7)^2 + 20(167 - 171.7)^2 + 27(174 - 171.7)^2 + 22(181 - 171.7)^2 + 8(188 - 171.7)^2}{100} \right\}}$$

$$= \sqrt{\frac{8825.4}{100}} = 9.394 \text{ cm}$$

6. Calculate the standard deviation from the mean for the data given in Problem 4 of Exercise 327, page 920, correct to 3 decimal places.

From Problem 4, Exercise 327, mean value, $\bar{x} = 2.02158 \text{ cm}$

$$\text{Standard deviation, } \sigma = \sqrt{\left\{ \frac{\sum \{f(x - \bar{x})\}}{\sum f} \right\}}$$

$$= \sqrt{\left\{ \frac{7(2.0125 - 2.02158)^2 + 16(2.0175 - 2.02158)^2 + 23(2.0225 - 2.02158)^2 + 9(2.0275 - 2.02158)^2 + 5(2.0325 - 2.02158)^2}{60} \right\}}$$

$$= \sqrt{\frac{0.000577124 + 0.000266342 + 0.000019467 + 0.000315417 + 0.000596232}{60}}$$

$$= \sqrt{\frac{0.001774582}{60}} = 0.00544 \text{ cm}$$

1. The number of working days lost due to accidents for each of 12 one-monthly periods are as shown. Determine the median and 1st and 3rd quartile values for this data.

27 37 40 28 23 30 35 24 30 32 31 28

$$\text{Median} = \text{middle value} = \frac{30+30}{2} = 30 \text{ days}$$

$$\text{3rd quartile value} = \frac{32+35}{2} = 33.5 \text{ faults}$$

- 30 27 25 24 27 37 31 27 35
- Determine the median and quartile values for the data.

Median = middle value = 27 faults

$$\text{3rd quartile value} = \frac{31+35}{2} = 33 \text{ faults}$$

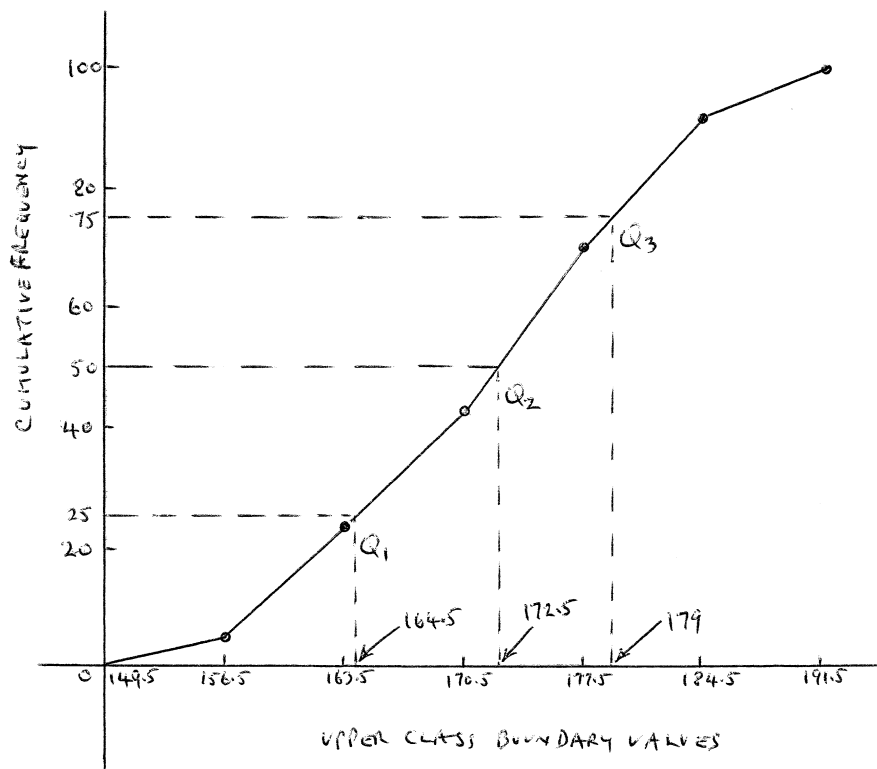
- The frequency distribution is shown below

Upper class boundary values	Frequency	Cumulative frequency
156.5	5	5
163.5	18	23
170.5	20	43
177.5	27	70
184.5	22	92
191.5	8	100

The ogive is shown below.

From the ogive, $Q_1 = 164.5\text{cm}$, $Q_2 = 172.5\text{cm}$ and $Q_3 = 179\text{cm}$

and semi-interquartile range = $\frac{179 - 164.5}{2} = \frac{14.5}{2} = 7.25\text{ cm}$



4. Determine the numbers contained in the 5th decile group and in the 61st to 70th percentile groups for the set of numbers: 40 46 28 32 37 42 50 31 48 45
- 32 38 27 33 40 35 25 42 38 41

Ranking gives:

(5th decile)