

Square

In [geometry](#), a **square** is a [regular quadrilateral](#), which means that it has four equal sides and four equal [angles](#) (90-degree angles, or (100-gradian angles or [right angles](#))).^[1] It can also be defined as a [rectangle](#) in which two adjacent sides have equal length. A square with [vertices](#) *ABCD* would be denoted□ *ABCD*.

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Characterizations

A [convex quadrilateral](#) is a square [if and only if](#) it is any one of the following^{[2][3]}

- a [rectangle](#) with two adjacent equal sides
- a [rhombus](#) with a right vertex angle
- a [rhombus](#) with all angles equal
- a [parallelogram](#) with one right vertex angle and two adjacent equal sides
- a [quadrilateral](#) with four equal sides and fourright angles
- a quadrilateral where the diagonals are equal and are the perpendicular bisectors of each other: a rhombus with equal diagonals
- a convex quadrilateral with successive sides*a*, *b*, *c*, *d* whose area is *A* = 12 (*a*² + *c*²) = 12 (*b*² + *d*²).^[4]:Corollary 15

Properties

A square is a special case of a [rhombus](#) (equal sides, opposite equal angles), a [kite](#) (two pairs of adjacent equal sides), a [trapezoid](#) (one pair of opposite sides parallel), a [parallelogram](#) (all opposite sides parallel), a [quadrilateral](#) or tetragon (four-sided polygon), and a [rectangle](#) (opposite sides equal, right-angles) and therefore has all the properties of all these shapes, namely^[5]

- The [diagonals](#) of a square [bisect](#) each other and meet at 90°
- The [diagonals](#) of a square bisect its angles.
- Opposite sides of a square are bothparallel and equal in length.
- All four angles of a square are equal. (Each is 360°/4 = 90°, so every angle of a square is a right angle.)
- All four sides of a square are equal.
- The diagonals of a square are equal.
- The square is the n=2 case of the families of [hypercubes](#) and [n-orthoplexes](#).
- A square has [Schläfli symbol](#) {4}. A [truncated square](#), t{4}, is an [octagon](#), {8}. An [alternated square](#), h{4}, is a [digon](#), {2}.

Perimeter and area

The [perimeter](#) of a square whose four sides have length*ℓ* is

P

= 4*ℓ*

Square	
 90° 90°	
A regular quadrilateral (tetragon)	
Type	Regular polygon
Edges and vertices	4
Schläfli symbol	{4}
Coxeter diagram	

and the area A is

$$A = \ell^2.$$

In classical times, the second power was described in terms of the area of a square, as in the above formula. This led to the use of the term square to mean raising to the second power

The area can also be calculated using the diagonal d according to

$$A = \frac{d^2}{2}.$$

In terms of the circumradius R , the area of a square is

$$A = 2R^2;$$

since the area of the circle is πR^2 , the square fills approximately 0.6366 of its circumscribed circle

In terms of the inradius r , the area of the square is

$$A = 4r^2.$$

Because it is a regular polygon, a square is the quadrilateral of least perimeter enclosing a given area. Dually, a square is the quadrilateral containing the largest area within a given perimeter^[6] Indeed, if A and P are the area and perimeter enclosed by a quadrilateral, then the following isoperimetric inequality holds:

$$16A \leq P^2$$

with equality if and only if the quadrilateral is a square.

Other facts

- The diagonals of a square are $\sqrt{2}$ (about 1.414) times the length of a side of the square. This value, known as the square root of 2 or Pythagoras' constant, was the first number proven to be irrational.
- A square can also be defined as a parallelogram with equal diagonals that bisect the angles.
- If a figure is both a rectangle (right angles) and a rhombus (equal edge lengths), then it is a square.
- If a circle is circumscribed around a square, the area of the circle is $\pi/2$ (about 1.5708) times the area of the square.
- If a circle is inscribed in the square, the area of the circle is $\pi/4$ (about 0.7854) times the area of the square.
- A square has a larger area than any other quadrilateral with the same perimeter^[7]
- A square tiling is one of three regular tilings of the plane (the others are the equilateral triangle and the regular hexagon).
- The square is in two families of polytopes in two dimensions: hypercube and the cross-polytope. The Schläfli symbol for the square is {4}.
- The square is a highly symmetric object. There are four lines of reflectional symmetry and it has rotational symmetry of order 4 (through 90°, 180° and 270°). Its symmetry group is the dihedral group D_4 .
- If the inscribed circle of a square $ABCD$ has tangency points E on AB , F on BC , G on CD , and H on DA , then for any point P on the inscribed circle,^[8]

$$2(PH^2 - PE^2) = PD^2 - PB^2.$$

- If d_i is the distance from an arbitrary point in the plane to the i -th vertex of a square and R is the circumradius of the square, then^[9]

$$\frac{d_1^4 + d_2^4 + d_3^4 + d_4^4}{4} + 3R^4 = \left(\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{4} + R^2 \right)^2.$$

Coordinates and equations

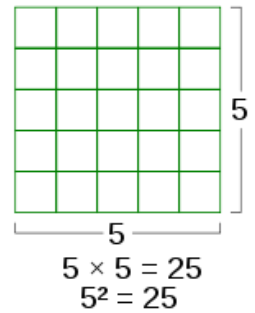
The coordinates for the vertices of a square with vertical and horizontal sides, centered at the origin and with side length 2 are $(\pm 1, \pm 1)$, while the interior of this square consists of all points (x_i, y_i) with $-1 < x_i < 1$ and $-1 < y_i < 1$. The equation

$$\max(x^2, y^2) = 1$$

specifies the boundary of this square. This equation means "x² or y², whichever is larger, equals 1." The circumradius of this square (the radius of a circle drawn through the square's vertices) is half the square's diagonal, and equals $\sqrt{2}$. Then the circumcircle has the equation

$$x^2 + y^2 = 2.$$

Alternatively the equation



The area of a square is the product of the length of its sides.

$$|x - a| + |y - b| = r.$$

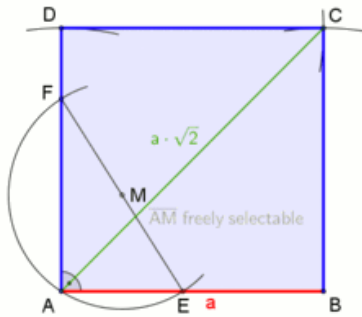
can also be used to describe the boundary of a square with center coordinates (a, b) and a horizontal or vertical radius of r .

Construction

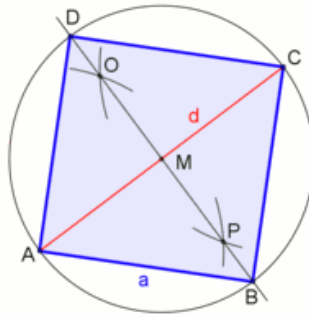
The following animations show how to construct a square using a compass and straightedge. This is possible as $4 = 2^2$, a power of two.



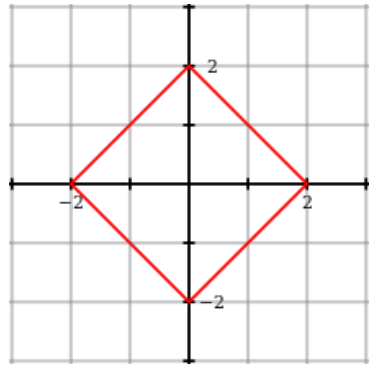
Square at a given circumscribed circle



Square at a given side length, right angle by using Thales' theorem



Square at a given diagonal



$|x| + |y| = 2$ plotted on Cartesian coordinates.

Symmetry

The *square* has D_{4h} symmetry, order 8. There are 2 dihedral subgroups: D_{2h} , D_{2h} , and 3 cyclic subgroups: Z_4 , Z_2 , and Z_2 .

A square is a special case of many lower symmetry quadrilaterals:

- a rectangle with two adjacent equal sides
- a quadrilateral with four equal sides and four right angles
- a parallelogram with one right angle and two adjacent equal sides
- a rhombus with a right angle
- a rhombus with all angles equal
- a rhombus with equal diagonals

These 6 symmetries express 8 distinct symmetries on a square. John Conway labels these by a letter and group order^[10]

Each subgroup symmetry allows one or more degrees of freedom for irregular quadrilaterals. **r8** is full symmetry of the square, and **a1** is no symmetry. **d4**, is the symmetry of a rectangle and **p4**, is the symmetry of a rhombus. These two forms are duals of each other and have half the symmetry order of the square. **d2** is the symmetry of an isosceles trapezoid and **p2** is the symmetry of a kite. **g2** defines the geometry of a parallelogram.

Only the **g4** subgroup has no degrees of freedom but can be seen as a square with directed edges.

Squares inscribed in triangles

Every acute triangle has three inscribed squares (squares in its interior such that all four of a square's vertices lie on a side of the triangle, so two of them lie on the same side and hence one side of the square coincides with part of a side of the triangle). In a right triangle two of the squares coincide and have a vertex at the triangle's right angle, so a right triangle has only two *distinct* inscribed squares. An obtuse triangle has only one inscribed square, with a side coinciding with part of the triangle's longest side.

The fraction of the triangle's area that is filled by the square is no more than 1/2.

Squaring the circle

Squaring the circle is the problem, proposed by ancient geometers, of constructing a square with the same area as a givencircle by using only a finite number of steps with compass and straightedge

In 1882, the task was proven to be impossible, as a consequence of the Lindemann–Weierstrass theorem which proves that π (π) is a transcendental number, rather than an algebraic irrational number that is, it is not the root of any polynomial with rational coefficients.

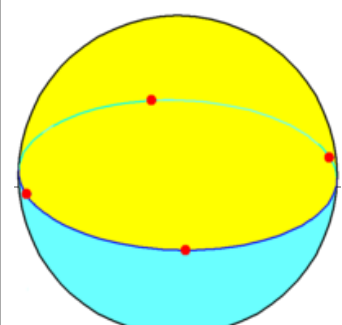
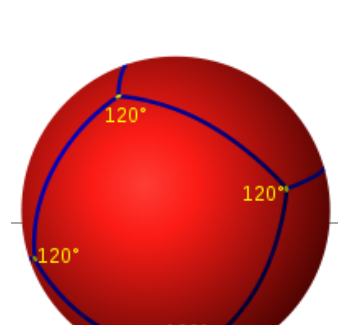
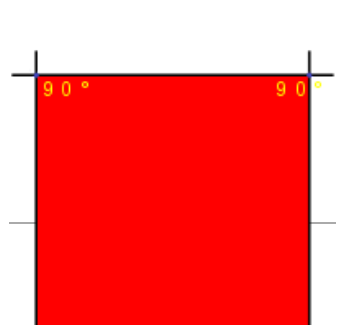
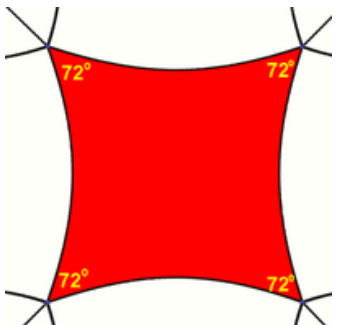
Non-Euclidean geometry

In non-Euclidean geometry squares are more generally polygons with 4 equal sides and equal angles.

In spherical geometry, a square is a polygon whose edges are great circle arcs of equal distance, which meet at equal angles. Unlike the square of plane geometry, the angles of such a square are larger than a right angle. Larger spherical squares have larger angles.

In hyperbolic geometry, squares with right angles do not exist. Rather, squares in hyperbolic geometry have angles of less than right angles. Larger hyperbolic squares have smaller angles.

Examples:

 Two squares can tile the sphere with 2 squares around each vertex and 180-degree internal angles. Each square covers an entire hemisphere and their vertices lie along a great circle. This is called a spherical square <u>dihedron</u>. The <u>Schläfli symbol</u> is {4,2}.	 Six squares can tile the sphere with 3 squares around each vertex and 120-degree internal angles. This is called a spherical cube. The <u>Schläfli symbol</u> is {4,3}.	 Squares can tile the <u>Euclidean plane</u> with 4 around each vertex, with each square having an internal angle of 90°. The <u>Schläfli symbol</u> is {4,4}.	 Squares can tile the <u>hyperbolic plane</u> with 5 around each vertex, with each square having 72-degree internal angles. The <u>Schläfli symbol</u> is {4,5}. In fact, for any $n \geq 5$ there is a hyperbolic tiling with n squares about each vertex.
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Crossed square

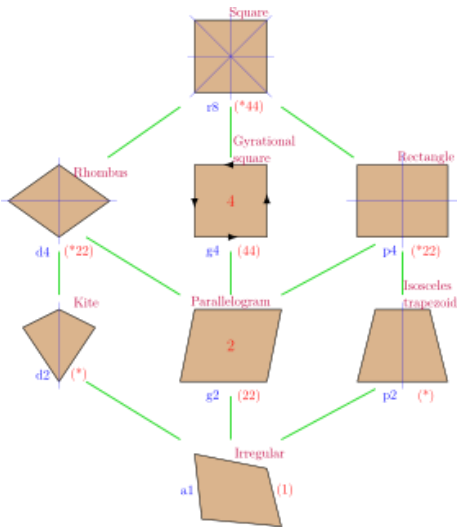
A **crossed square** is a faceting of the square, a self-intersecting polygon created by removing two opposite edges of a square and reconnecting by its two diagonals. It has half the symmetry of the square, D_{2h} order 4. It has the same vertex arrangement as the square, and is vertex-transitive. It appears as two 45-45-90 triangle with a common vertex, but the geometric intersection is not considered a vertex.

A crossed square is sometimes likened to a bow tie or butterfly. the crossed rectangle is related, as a faceting of the rectangle, both special cases of crossed quadrilaterals^[11]

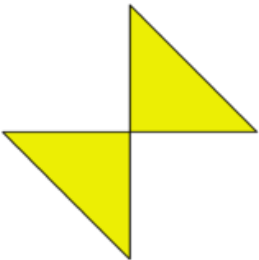
The interior of a crossed square can have a polygon density of ± 1 in each triangle, dependent upon the winding orientation as clockwise or counterclockwise.

A square and a crossed square have the following properties in common:

- Opposite sides are equal in length.
- The two diagonals are equal in length.
- It has two lines of reflectional symmetry and rotational symmetry of order 2 (through 180°).



The dihedral symmetries are divided depending on whether they pass through vertices **d** (for diagonal) or edges (**p** for perpendiculars) Cyclic symmetries in the middle column are labeled as **g** for their central gyration orders. Full symmetry of the square is **sr12** and no symmetry is labeled **a1**.

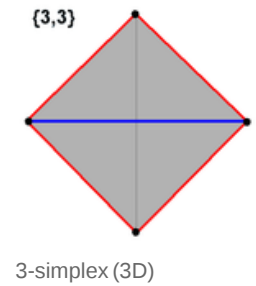


Crossed-square

It exists in the vertex figure of a uniform star polyhedra the tetrahemihexahedron

Graphs

The K_4 complete graph is often drawn as a square with all 6 possible edges connected, hence appearing as a square with both diagonals drawn. This graph also represents an orthographic projection of the 4 vertices and 6 edges of the regular 3-simplex (tetrahedron).



See also

- Cube
- Pythagorean theorem
- Square lattice
- Squircle
- Unit square

References

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10. John H. Conway, Heidi Burgiel, Chaim Goodman-Strauss, (2008) *The Symmetries of Things* ISBN 978-1-56881-220-5 (Chapter 20, Generalized Schaeffli symbols, Types of symmetry of a polygon pp. 275-278)
11. Wells, Christopher J. "Quadrilaterals" (<http://www.technologyuk.net/mathematics/geometry/quadrilaterals.shtml>) *www.technologyuk.net* Retrieved 2017-12-12

External links

- Animated course (Construction, Circumference, Area)
- Weisstein, Eric W. "Square". *MathWorld*.
- Definition and properties of a square With interactive applet
- Animated applet illustrating the area of a square

Fundamental convex <u>regular</u> and <u>uniform polytopes</u> in dimensions 2–10					
Family	A_n	B_n	$I_2(p) / D_n$	$E_6 / E_7 / E_8 / F_4 / G_2$	H_n
<u>Regular polygon</u>	<u>Triangle</u>	<u>Square</u>	<u>p-gon</u>	<u>Hexagon</u>	<u>Pentagon</u>
<u>Uniform polyhedron</u>	<u>Tetrahedron</u>	<u>Octahedron • Cube</u>	<u>Demicube</u>		<u>Dodecahedron • Icosahedron</u>
<u>Uniform 4-polytope</u>	<u>5-cell</u>	<u>16-cell • Tesseract</u>	<u>Demitesseract</u>	<u>24-cell</u>	<u>120-cell • 600-cell</u>
<u>Uniform 5-polytope</u>	<u>5-simplex</u>	<u>5-orthoplex • 5-cube</u>	<u>5-demicube</u>		
<u>Uniform 6-polytope</u>	<u>6-simplex</u>	<u>6-orthoplex • 6-cube</u>	<u>6-demicube</u>	<u>1₂₂ • 2₂₁</u>	
<u>Uniform 7-polytope</u>	<u>7-simplex</u>	<u>7-orthoplex • 7-cube</u>	<u>7-demicube</u>	<u>1₃₂ • 2₃₁ • 3₂₁</u>	
<u>Uniform 8-polytope</u>	<u>8-simplex</u>	<u>8-orthoplex • 8-cube</u>	<u>8-demicube</u>	<u>1₄₂ • 2₄₁ • 4₂₁</u>	
<u>Uniform 9-polytope</u>	<u>9-simplex</u>	<u>9-orthoplex • 9-cube</u>	<u>9-demicube</u>		
<u>Uniform 10-polytope</u>	<u>10-simplex</u>	<u>10-orthoplex • 10-cube</u>	<u>10-demicube</u>		
<u>Uniform <i>n</i>-polytope</u>	<u><i>n</i>-simplex</u>	<u><i>n</i>-orthoplex • <i>n</i>-cube</u>	<u><i>n</i>-demicube</u>	<u>1_{<i>k</i>2} • 2_{<i>k</i>1} • <i>k</i>₂₁</u>	<u><i>n</i>-pentagonal polytope</u>
Topics: <u>Polytope families</u> • <u>Regular polytope</u> • <u>List of regular polytopes and compounds</u>					

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