

SQUARES AND SQUARE ROOTS

In this lesson, students link the geometric concepts of side length and area of a square to the algebra concepts of squares and square roots of numbers. They create a table of perfect squares. They use the table to find square roots of perfect squares, and they approximate the square root of a whole number.

This lesson is the first of a lesson cluster where students develop and practice rules for exponents. In future lessons, students will use the definition of exponents and inductive reasoning to make conjectures about rules for exponents. Then the rules for exponents will be formalized, and students will simplify expressions that include exponents and roots.

Math Goals

(Standards for
posting in **bold**)

- Understand geometrically and numerically the connection between squaring a number and finding the square root of a number.
(ALG 2.0)
- Approximate a square root by locating it between two consecutive integers.
(Gr7 NS2.4; Gr7 MR2.7)
- Use fractions and decimals to approximate square roots.
(Gr7 NS1.2; Gr7 NS1.3; Gr7 MR2.1; Gr7 MR2.7)

Summative Assessment

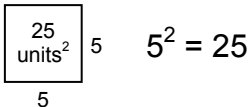
Future Week

- Week 25: Exponents and Roots
(Gr7 NS1.2; Gr7 NS2.4)

21.2 Squares and Square Roots

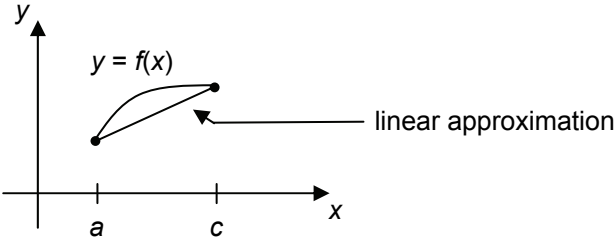
PLANNING INFORMATION

Estimated Time: 45 – 60 Minutes

Student Pages * SP10: Ready, Set, Go * SP11: Table of Squares * SP12-13: Estimating Square Roots SP14: More Square Root Estimates	Materials Calculators (optional)	Reproducibles
Homework SP14: More Square Root Estimates	Prepare Ahead	Management Reminders
Assessment * SP25: Knowledge Check 21 R86: Knowledge Challenge 21 A101-102: Weekly Quiz 21	Strategies for English Learners Give a visual review of why square numbers are called square numbers. 	Strategies for Special Learners Refer often to a number line with numbers and their square roots so that students see why the linear interpolation method makes sense.

* Recommended transparency: Blackline masters for overheads 245-248 and 251 can be found in the Teacher Resource Binder.

21.2 Squares and Square Roots

THE WORD BANK	
exponential notation	The <u>exponential notation</u> b^n (read as “b to the power n”) is used to express the product of b with itself n times: $b^n = b \cdot b \cdot \dots \cdot b$ (n times). The number b is the <u>base</u>, and the natural number n is the <u>exponent</u>. Exponential notation is extended to arbitrary integer exponents by setting $b^0 = 1$ and $b^{-n} = \frac{1}{b^n}$. Example: $2^3 = 2 \cdot 2 \cdot 2 = 8$. (The base is 2 and the exponent is 3.) $3^2 5^3 = 3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 = 1,125$. (The bases are 3 and 5.) $2^0 = 1$. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.
square of a number	The <u>square of a number</u> is the product of the number with itself. Example: The square of 5 is 25, since $5^2 = 5 \times 5 = 25$. The square of -5 is also 25, since $(-5)^2 = (-5) \times (-5) = 25$.
perfect square	A <u>perfect square</u>, or <u>square number</u>, is a number that is a square of a natural number. Example: The area of a square with integral side-length is a perfect square. The perfect squares are $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, ...
square root of a number	A <u>square root</u> of a number n is a number whose square is equal to n, that is, a solution of the equation $x^2 = n$. The <u>positive square root</u> of a number n, written $\sqrt{n}$, is the positive number whose square is n. Example: Both 5 and -5 are square roots of 25, because $5^2 = 25$ and $(-5)^2 = 25$. The positive square root of 25 is 5.
linear interpolation	<u>Linear interpolation</u> refers to a method of approximating the values of a function f at points of an interval $a < x < c$ by the values of the linear function that coincides with f at the endpoints a and c.  Example: We may approximate $\sqrt{x}$ for $25 < x < 36$ by the linear function $y = 5 + \left(\frac{1}{11}\right)(x - 25)$, which has values $y = \sqrt{25} = 5$ at $x = 25$ and $y = \sqrt{36} = 6$ at $x = 36$.

21.2 Squares and Square Roots

MATH BACKGROUND

Approximating Square Roots by Linear Interpolation

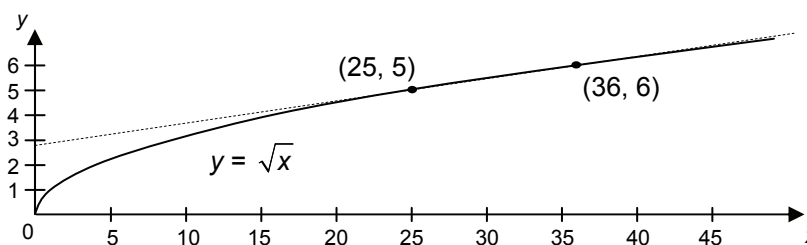
Linear interpolation is a method by which the values $y = f(x)$ of a function f on an interval $x_1 < x < x_2$ are estimated by the values of the linear function $y = mx + b$ that matches the values of f at the endpoints of the interval. The parameters m and b satisfy the two equations $f(x_1) = mx_1 + b$ and $f(x_2) = mx_2 + b$. The graph of the linear approximation $y = mx + b$ is then the straight line segment joining the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ on the graph of f .

The linear approximation can be found directly through proportional reasoning, without writing down any equations of lines. To illustrate, we approximate values of $\sqrt{x}$.

To find an approximate value for $\sqrt{27}$:

First find the closest perfect square (25) that is less than 27 and the closest perfect square (36) that is greater than 27. Then $25 < 27 < 36$. The points 25 and 36 will be the endpoints of the interval of interpolation.

Take the square root of each number: $\sqrt{25} = 5$, $\sqrt{36} = 6$. We aim to approximate $\sqrt{27}$ using the y -coordinate of the straight line through (25,5) and (36,6).



Now 27 is 2 units larger than 25, and 36 is 11 units larger than 25. Thus 27 is two elevenths ($\frac{2}{11}$) of the distance from 25 to 36. We approximate $\sqrt{27}$ by the number that is two elevenths of the distance from $\sqrt{25}$ to $\sqrt{36}$, that is, two elevenths of the distance from 5 to 6 (proportional reasoning!!). This number is $5 + \frac{2}{11} = 5\frac{2}{11}$.

Summary: On a number line, 27 is $\frac{2}{11}$ of the distance from 25 to 36. Therefore, $\sqrt{27}$ is approximately equal to $\frac{2}{11}$ of the distance from $\sqrt{25} = 5$ to $\sqrt{36} = 6$, which is $5 + \frac{2}{11}$. The approximation to $\sqrt{27}$ is $5\frac{2}{11}$.

By linear interpolation to the nearest thousandth: $5\frac{2}{11} \approx 5.182$

Accurate to the nearest thousandth: $\sqrt{27} \approx 5.196$

Math Background 1
Teacher
Mathematical Insight

21.2 Squares and Square Roots

PREVIEW / WARMUP

Whole Class

➤ SP10*
Ready, Set, Go

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant.
- Students draw several squares and record both their side lengths and areas. Share and discuss. Stress the relationship between side length and area, the definitions of squares and square roots (numerically and geometrically), and the notations for squares and square roots.

INTRODUCE

Whole Class

➤ SP11*
Table of Squares

➤ SP12-13*
Estimating Square
Roots

Calculators

- Students complete the table of perfect squares. Encourage students to keep this table handy (or even memorize some of the perfect squares) because it will make computations that involve square roots easier.
- Students put the square roots of perfect squares on a number line.

Why is the distance between $\sqrt{25}$ and $\sqrt{36}$ the same as the distance between $\sqrt{1}$ and $\sqrt{4}$? The distance between 5 and 6 ($\sqrt{25}$ and $\sqrt{36}$) is the same as the distance between 1 and 2 ($\sqrt{1}$ and $\sqrt{4}$).

- Have students estimate the location of $\sqrt{27}$ on the number line.

27 is between what two perfect squares? 25 and 36.

How can we use this information to approximate $\sqrt{27}$? Since $\sqrt{25} = 5$ and $\sqrt{36} = 6$, $\sqrt{27}$ is between 5 and 6. Since it is somewhat closer to 5, a reasonable approximation might be 5.1, 5.2, or 5.3.

- Show students the process of linear interpolation in order to find fraction and/or decimal approximations of the square roots of non-perfect squares.

On a number line, 27 represents what fraction of the distance from 25 to 36? Since the distance from 25 to 36 is 11 units, and 25 to 27 is 2 units, the fraction is $\frac{2}{11}$. Since $\sqrt{25} = 5$, we estimate $\sqrt{27}$ by $5 + \frac{2}{11} = 5\frac{2}{11}$. Since $\frac{2}{11} = 0.1818\ldots$ the estimation above, that $\sqrt{27}$ is approximately 5.2, is supported.

21.2 Squares and Square Roots

EXPLORE	
Pairs/Individuals ➤ SP12-13* Estimating Square Roots Calculators	Students continue to estimate the square roots of non-perfect squares, first by finding the whole numbers between which they lie on the number line, and then by using linear interpolation. Use calculators as a check for reasonableness.
PRACTICE	
Individuals ➤ SP14 More Square Root Estimates	Use for additional practice or homework. In order to help students practice number sense and estimation skills, stress that the calculator should be used for checking results only.
SUMMARIZE	
Whole Class ➤ SP12-13* Estimating Square Roots ➤ SP14 More Square Root Estimates	Discuss the problem below. <i>How do you know that $\sqrt{20}$ is between 4 and 5?</i> The two perfect squares closest to 20 below and above are 16 and 25, and we know that $\sqrt{16} = 4$ and $\sqrt{25} = 5$.
CLOSURE	
Whole Class ➤ SP10* Ready, Set	Review the goals and standards for the lesson.

21.2 Squares and Square Roots

SELECTED SOLUTIONS					
SP11 Table of Squares	$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
	$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
	$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
	$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
	$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$
SP12-13 Estimating Square Roots	1. $\sqrt{0} \quad \sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \quad \sqrt{49} \quad \sqrt{64} \quad \sqrt{81} \quad \sqrt{100}$ 2. $\sqrt{25} < \sqrt{27} < \sqrt{36}$, but $\sqrt{27}$ is closer to $\sqrt{25}$. $5 < \sqrt{27} < 6$, but $\sqrt{27}$ is closer to $\underline{5}$. $\frac{27-25}{36-25} = \frac{2}{11}$ Therefore, $\sqrt{27}$ is about $5\frac{2}{11}$ or <u>5.2</u> (number will vary) (calculator check: <u>5.2</u>) 3. $\sqrt{36} < \sqrt{40} < \sqrt{49}$, but $\sqrt{40}$ is closer to $\sqrt{36}$. $6 < \sqrt{40} < 7$, but $\sqrt{40}$ is closer to $\underline{6}$. $\frac{40-36}{49-36} = \frac{4}{13}$ Therefore, $\sqrt{40}$ is about $6\frac{4}{13}$ or <u>6.3</u> (number will vary) (calculator check: <u>6.3</u>) 4. $\sqrt{25} < \sqrt{30} < \sqrt{36}$, but $\sqrt{30}$ is closer to $\sqrt{25}$. $5 < \sqrt{30} < 6$, but $\sqrt{30}$ is closer to $\underline{5}$. $\frac{30-25}{36-25} = \frac{5}{11}$ Therefore, $\sqrt{30}$ is about $5\frac{5}{11}$ or <u>5.5</u> (calculator check: <u>5.5</u>) 5. $\sqrt{64} < \sqrt{77} < \sqrt{81}$, but $\sqrt{77}$ is closer to $\sqrt{81}$. $8 < \sqrt{77} < 9$, but $\sqrt{77}$ is closer to $\underline{9}$. $\frac{77-64}{81-64} = \frac{13}{17}$ Therefore, $\sqrt{77}$ is about $8\frac{13}{17}$ or <u>8.8</u> (calculator check: <u>8.8</u>)				

21.2 Squares and Square Roots

SP14 More Square Roots Estimates		Number	Between square roots of perfect squares:	Between two integers:	About (fraction):	About (decimal):	Calculator check (to the nearest tenth)
	1.	$\sqrt{5}$	$\sqrt{4}$ and $\sqrt{9}$	2 & 3	$2\frac{1}{5}$	2.2	2.2
	2.	$\sqrt{500}$	$\sqrt{484}$ and $\sqrt{529}$	22 & 23	$22\frac{16}{45}$	22.4	22.4
	3.	$\sqrt{220}$	$\sqrt{196}$ and $\sqrt{225}$	14 & 15	$14\frac{24}{29}$	14.8	14.8
	4.	$\sqrt{78}$	$\sqrt{64}$ and $\sqrt{81}$	8 & 9	$8\frac{14}{17}$	8.8	8.8
	5.	$\sqrt{20}$	$\sqrt{16}$ and $\sqrt{25}$	4 & 5	$4\frac{4}{9}$	4.4	4.5
	6.	$\sqrt{303}$	$\sqrt{289}$ and $\sqrt{324}$	17 & 18	$17\frac{2}{5}$	17.4	17.4
	7.	$\sqrt{267}$	$\sqrt{256}$ and $\sqrt{289}$	16 & 17	$16\frac{1}{3}$	16.3	16.3
	8.	Estimates are generally accurate to the nearest tenth. Then method is useful if a calculator is not available. Estimation develops number sense.					

STUDENT PAGES

SQUARES AND SQUARE ROOTS

Ready (Summary)

We will find squares and square roots of numbers, and approximate square roots that are not perfect squares.

Set (Goals)

- Understand geometrically and numerically the connection between squaring a number and finding the square root of a number.
- Approximate a square root by locating it between two consecutive integers.
- Use fractions and decimals to approximate square roots.

Go (Warmup)

Draw several squares of different sizes on the grid paper. Record the side length and area for each.

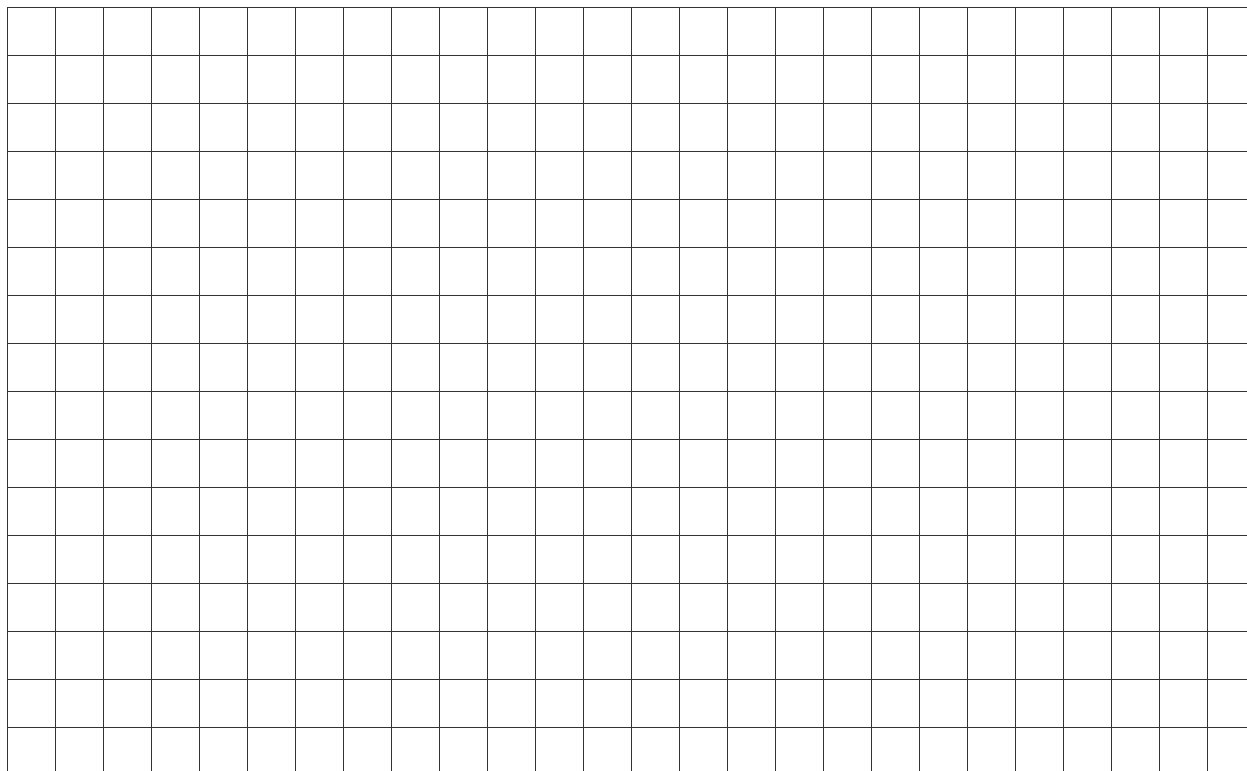


TABLE OF SQUARES

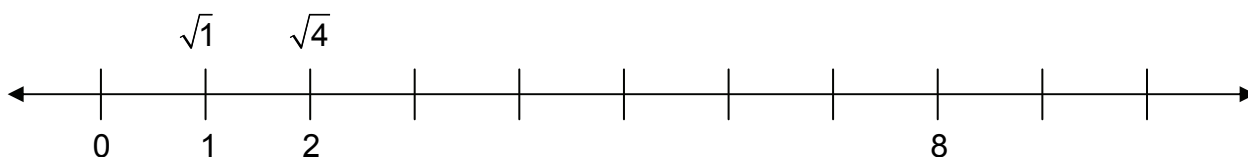
Complete the table.

$1^2 = \underline{\hspace{2cm}}$	$2^2 = \underline{\hspace{2cm}}$	$3^2 = \underline{\hspace{2cm}}$	$4^2 = \underline{\hspace{2cm}}$	$5^2 = \underline{\hspace{2cm}}$
$6^2 = \underline{\hspace{2cm}}$	$7^2 = \underline{\hspace{2cm}}$	$8^2 = \underline{\hspace{2cm}}$	$9^2 = \underline{\hspace{2cm}}$	$10^2 = \underline{\hspace{2cm}}$
$11^2 = \underline{\hspace{2cm}}$	$12^2 = \underline{\hspace{2cm}}$	$13^2 = \underline{\hspace{2cm}}$	$14^2 = \underline{\hspace{2cm}}$	$15^2 = \underline{\hspace{2cm}}$
$16^2 = \underline{\hspace{2cm}}$	$17^2 = \underline{\hspace{2cm}}$	$18^2 = \underline{\hspace{2cm}}$	$19^2 = \underline{\hspace{2cm}}$	$20^2 = \underline{\hspace{2cm}}$
$21^2 = \underline{\hspace{2cm}}$	$22^2 = \underline{\hspace{2cm}}$	$23^2 = \underline{\hspace{2cm}}$	$24^2 = \underline{\hspace{2cm}}$	$25^2 = \underline{\hspace{2cm}}$

ESTIMATING SQUARE ROOTS

1. Locate the following numbers on the number line below:

$$\sqrt{0} \quad \sqrt{1} \quad \sqrt{4} \quad \sqrt{9} \quad \sqrt{16} \quad \sqrt{25} \quad \sqrt{36} \quad \sqrt{49} \quad \sqrt{64} \quad \sqrt{81} \quad \sqrt{100}$$



2. Use the values on your number line to estimate the location of $\sqrt{27}$.

$$\sqrt{25} < \sqrt{27} < \sqrt{36}, \text{ but } \sqrt{27} \text{ is closer to } \underline{\hspace{2cm}}.$$

$$5 < \sqrt{27} < \underline{\hspace{2cm}}, \text{ but } \sqrt{27} \text{ is closer to } \underline{\hspace{2cm}}.$$

Estimate the decimal part of $\sqrt{27}$ as a fraction. $\frac{27-25}{36-25} = \frac{\boxed{}}{\boxed{}}$

Therefore, $\sqrt{27}$ is about $5\frac{\boxed{}}{\boxed{}}$. (calculator check: $\underline{\hspace{2cm}}$)

3. Use the values on your number line to estimate the location of $\sqrt{40}$.

$$\underline{\hspace{2cm}} < \sqrt{40} < \underline{\hspace{2cm}}, \text{ but } \sqrt{40} \text{ is closer to } \underline{\hspace{2cm}}.$$

$$\underline{\hspace{2cm}} < \sqrt{40} < \underline{\hspace{2cm}}, \text{ but } \sqrt{40} \text{ is closer to } \underline{\hspace{2cm}}.$$

Estimate the decimal part of $\sqrt{40}$ as a fraction. $\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$

Therefore, $\sqrt{40}$ is about $\underline{\hspace{2cm}}$. (calculator check: $\underline{\hspace{2cm}}$)

ESTIMATING SQUARE ROOTS (continued)

4. Use the values on your number line to estimate the location of $\sqrt{30}$.

_____ $< \sqrt{30} <$ _____, but $\sqrt{30}$ is closer to _____.

_____ $< \sqrt{30} <$ _____, but $\sqrt{30}$ is closer to _____.

Estimate the decimal part of $\sqrt{30}$ as a fraction.

	=	

Therefore, $\sqrt{30}$ is about _____. (calculator check: _____)

5. Use the values on your number line to estimate the location of $\sqrt{77}$.

_____ $< \sqrt{77} <$ _____, but $\sqrt{77}$ is closer to _____.

_____ $< \sqrt{77} <$ _____, but $\sqrt{77}$ is closer to _____.

Estimate the decimal part of $\sqrt{77}$ as a fraction.

	=	

Therefore, $\sqrt{77}$ is about _____. (calculator check: _____)

MORE SQUARE ROOT ESTIMATES

Use fractions and decimals to approximate each square root.

	A Number	B Between square roots of perfect squares:	C Between 2 consecutive integers:	D About (fraction):	E About (decimal):	F Calculator check (to nearest tenth)
1.	$\sqrt{5}$	$\sqrt{4}$ and $\sqrt{9}$	2 and 3	$2\frac{1}{5}$	2.2	2.2
2.	$\sqrt{500}$					
3.	$\sqrt{220}$					
4.	$\sqrt{78}$					
5.	$\sqrt{20}$					
6.	$\sqrt{303}$					
7.	$\sqrt{267}$					

8. Compare your square root estimates in columns C and D to the rounded answer in column E. What are some advantages and disadvantages of each?