

Square number

A **square number**, sometimes also called a **perfect square**, is the result of an integer multiplied by itself. 1, 4, 9, 16 and 25 are the first five square numbers. In a formula, the square of a number *n* is denoted *n*² (exponentiation), usually pronounced as "*n* squared". The name square number comes from the name of the shape; see below.

Square numbers are non-negative. Another way of saying that a (non-negative) number is a square number, is that its square root is again an integer. For example, $\sqrt{9} = 3$, so 9 is a square number

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Examples

The squares (sequence A000290 in the OEIS) smaller than 60² are:

0 ² = <u>0</u>	10 ² = <u>100</u>	20 ² = 400	30 ² = 900
1 ² = <u>1</u>	11 ² = 121	21 ² = 441	31 ² = 961
2 ² = <u>4</u>	12 ² = 144	22 ² = 484	32 ² = 1024
3 ² = <u>9</u>	13 ² = 169	23 ² = 529	33 ² = 1089
4 ² = <u>16</u>	14 ² = 196	24 ² = 576	34 ² = 1156
5 ² = <u>25</u>	15 ² = 225	25 ² = 625	35 ² = 1225
6 ² = 36	16 ² = 256	26 ² = 676	36 ² = 1296
7 ² = 49	17 ² = 289	27 ² = 729	37 ² = 1369
8 ² = 64	18 ² = 324	28 ² = 784	38 ² = 1444
9 ² = 81	19 ² = 361	29 ² = 841	39 ² = 1521

40 ² = 1600	50 ² = 2500
41 ² = 1681	51 ² = 2601
42 ² = 1764	52 ² = 2704
43 ² = 1849	53 ² = 2809
44 ² = 1936	54 ² = 2916
45 ² = 2025	55 ² = 3025
46 ² = 2116	56 ² = 3136

$$47^2 = 2209$$

$$48^2 = 2304$$

$$49^2 = 2401$$

$$57^2 = 3249$$

$$58^2 = 3364$$

$$59^2 = 3481$$

There are infinitely many square numbers, as there are infinitely many natural numbers

Properties

The number m is a square number if and only if one can compose a square of m equal (lesser) squares:

$$m = 1^2 = 1$$



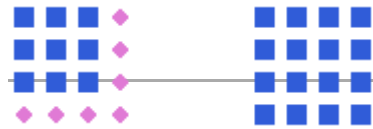
$$m = 2^2 = 4$$



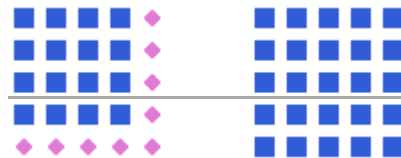
$$m = 3^2 = 9$$



$$m = 4^2 = 16$$



$$m = 5^2 = 25$$



Note: White gaps between squares serve only to improve visual perception. There must be no gaps between actual squares.

A square with side length n has area n^2 .

The expression for the n th square number is n^2 . This is also equal to the sum of the first n odd numbers as can be seen in the above pictures, where a square results from the previous one by adding an odd number of points (shown in magenta). The formula follows:

$$n^2 = \sum_{k=1}^n (2k - 1).$$

So for example, $5^2 = 25 = 1 + 3 + 5 + 7 + 9$.

A square number can end only with digits 0, 1, 4, 6, 9, or 25 in base 10, as follows:

1. If the last digit of a number is 0, its square ends in an even number of 0s (so at least 00) and the digits preceding the ending 0s must also form a square.
2. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.
3. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.
4. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.
5. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be **odd**.
6. If the last digit of a number is 5, its square ends in 25 and the preceding digits must be 0, 2, 06, or 56.

A square number cannot be a perfect number.

All fourth powers, sixth powers, eighth powers and so on are perfect squares.

Special cases

- If the number is of the form $10n+5$ where m represents the preceding digits, its square is $100n^2+25$ where $n = m \times (m + 1)$ and represents digits before 25. For example the square of 65 can be calculated by $6 \times (6 + 1) = 42$ which makes the square equal to 4225.
- If the number is of the form $10m+0$ where m represents the preceding digits, its square is $100n^2$ where $n = m^2$. For example the square of 70 is 4900.
- If the number has two digits and is of the form $10m+n$ where m represents the units digit, its square is $AABB$ where $AA = 25 + m$ and $BB = m^2$. Example: To calculate the square of 57, $25 + 7 = 32$ and $7^2 = 49$, which means $57^2 = 3249$.

Odd and even square numbers

Squares of even numbers are even (and in fact divisible by 4), since $(2n)^2 = 4n^2$.

Squares of odd numbers are odd, since $(2n + 1)^2 = 4(n^2 + n) + 1$.

It follows that square roots of even square numbers are even, and square roots of odd square numbers are odd.

As all even square numbers are divisible by 4, the even numbers of the form $4n + 2$ are not square numbers.

As all odd square numbers are of the form $4n + 1$, the odd numbers of the form $4n + 3$ are not square numbers.

Squares of odd numbers are of the form $8n + 1$, since $(2n + 1)^2 = 4n(n + 1) + 1$ and $n(n + 1)$ is an even number

Notes

References

- Eric W. Weisstein, *Square Number* at [MathWorld](#).

Further reading

- Conway, J. H. and Guy, R. K. *The Book of Numbers* New York: Springer-Verlag, pp. 30–32, 1996. ISBN 0-387-97993-X

Other websites

- [Learn Square Numbers](#) Practice square numbers up to 144 with this children's multiplication game
- Dario Alpern, [Sum of squares](#) A Java applet to decompose a natural number into a sum of up to four squares.
- [Fibonacci and Square Numbers](#) at [Convergence](#)
- [The first 1,000,000 perfect squares](#) Includes a program for generating perfect squares up to 10^5 .

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