



Square Roots and the Pythagorean Theorem

4.8 OBJECTIVES

1. Find the square root of a perfect square
2. Use the Pythagorean theorem to find the length of a missing side of a right triangle
3. Approximate the square root of a number

Some numbers can be written as the product of two identical factors, for example,

$$9 = 3 \times 3$$

Either factor is called a **square root** of the number. The symbol $\sqrt{\quad}$ (called a **radical sign**) is used to indicate a square root. Thus $\sqrt{9} = 3$ because $3 \times 3 = 9$.

Example 1

Finding the Square Root

NOTE To use the $\sqrt{\quad}$ key with a scientific calculator, first enter the 49, then press the key. With a graphing calculator, press the key first, then enter the 49 and a closing parenthesis.

Find the square root of 49 and of 16.

(a) $\sqrt{49} = 7$ Because $7 \times 7 = 49$

(b) $\sqrt{16} = 4$ Because $4 \times 4 = 16$



CHECK YOURSELF 1

Find the square root of each of the following.

(a) $\sqrt{121}$

(b) $\sqrt{36}$

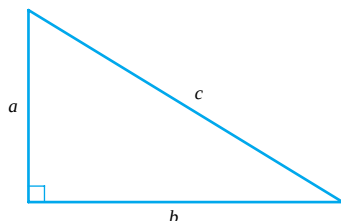
The most frequently used theorem in geometry is undoubtedly the Pythagorean theorem. In this section you will use that theorem. You will also learn a little about the history of the theorem. It is a theorem that applies only to right triangles.

The side opposite the right angle of a right triangle is called the **hypotenuse**.

Example 2

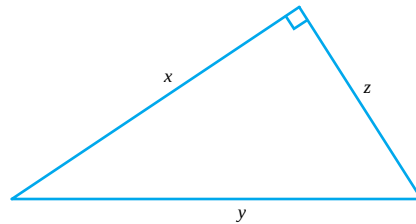
Identifying the Hypotenuse

In the following right triangle, the side labeled c is the hypotenuse.



**CHECK YOURSELF 2**

Which side represents the hypotenuse of the given right triangle?



The numbers 3, 4, and 5 have a special relationship. Together they are called a **perfect triple**, which means that when you square all three numbers, the sum of the smaller squares equals the squared value of the larger number.

Example 3**Identifying Perfect Triples**

Show that each of the following is a perfect triple.

(a) 3, 4, and 5

$$3^2 = 9, \quad 4^2 = 16, \quad 5^2 = 25$$

and $9 + 16 = 25$, so we can say that $3^2 + 4^2 = 5^2$.

(b) 7, 24, and 25

$$7^2 = 49, \quad 24^2 = 576, \quad 25^2 = 625$$

and $49 + 576 = 625$, so we can say that $7^2 + 24^2 = 25^2$.

**CHECK YOURSELF 3**

Show that each of the following is a perfect triple.

(a) 5, 12, and 13

(b) 6, 8, and 10

All the triples that you have seen, and many more, were known by the Babylonians more than 4000 years ago. Stone tablets that had dozens of perfect triples carved into them have been found. The basis of the Pythagorean theorem was understood long before the time of Pythagoras (ca. 540 B.C.). The Babylonians not only understood perfect triples but also knew how triples related to a right triangle.

Rules and Properties: The Pythagorean Theorem (Version 1)

If the lengths of the three sides of a right triangle are all integers, they will form a perfect triple, with the hypotenuse as the longest side.

There are two other forms in which the Pythagorean theorem is regularly presented. It is important that you see the connection between the three forms.

Rules and Properties: The Pythagorean Theorem (Version 2)

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

NOTE This is the version that you will refer to in your algebra classes.

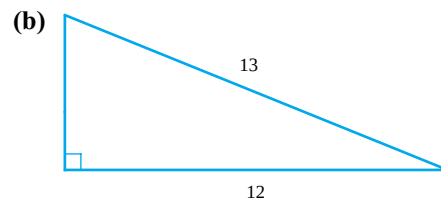
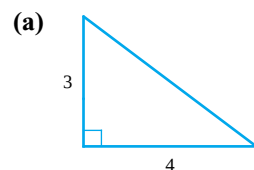
Rules and Properties: The Pythagorean Theorem (Version 3)

Given a right triangle with sides a and b and hypotenuse c , it is always true that

$$c^2 = a^2 + b^2$$

Example 4**Finding the Length of a Leg of a Right Triangle**

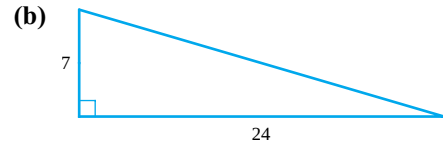
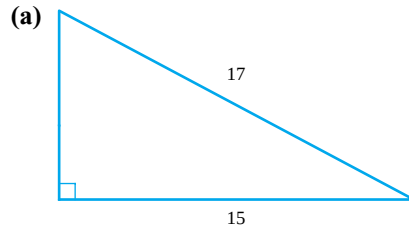
Find the missing integer length for each right triangle.



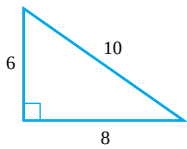
- (a) A perfect triple will be formed if the hypotenuse is 5 units long, creating the triple 3, 4, 5. Note that $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.
- (b) The triple must be 5, 12, 13, which makes the missing length 5 units. Here, $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

**CHECK YOURSELF 4**

Find the integer length of the unlabeled side for each right triangle.

**Example 5****Using the Pythagorean Theorem**

NOTE The triangle has sides 6, 8, and 10.



If the lengths of two sides of a right triangle are 6 and 8, find the length of the hypotenuse.

$$c^2 = a^2 + b^2 \quad \text{The value of the hypotenuse is found from the Pythagorean theorem with } a = 6 \text{ and } b = 8.$$

$$c^2 = (6)^2 + (8)^2 = 36 + 64 = 100$$

$$c = \sqrt{100} = 10 \quad \text{The length of the hypotenuse is 10 (because } 10^2 = 100\text{)}$$

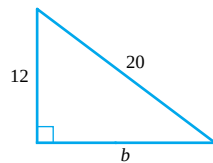
**CHECK YOURSELF 5**

Find the hypotenuse of a right triangle whose sides measure 9 and 12.

In some right triangles, the lengths of the hypotenuse and one side are given and we are asked to find the length of the missing side.

Example 6**Using the Pythagorean Theorem**

Find the missing length.



$$a^2 + b^2 = c^2 \quad \text{Use the Pythagorean theorem with } a = 12 \text{ and } c = 20.$$

$$(12)^2 + b^2 = (20)^2$$

$$144 + b^2 = 400$$

$$b^2 = 400 - 144 = 256$$

$$b = \sqrt{256} = 16 \quad \text{The missing side is 16.}$$

**CHECK YOURSELF 6**

Find the missing length for a right triangle with one leg measuring 8 centimeters (cm) and the hypotenuse measuring 10 cm.

Not every square root is a whole number. In fact, there are only 10 whole-number square roots for the numbers from 1 to 100. They are the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100. However, we can approximate square roots that are not whole numbers. For example, we know that the square root of 12 is not a whole number. We also know that its value must lie somewhere between the square root of 9 ($\sqrt{9} = 3$) and the square root of 16 ($\sqrt{16} = 4$). That is, $\sqrt{12}$ is between 3 and 4.

Example 7**Approximating Square Roots**

Approximate $\sqrt{29}$.

The $\sqrt{25} = 5$ and the $\sqrt{36} = 6$, so the $\sqrt{29}$ must be between 5 and 6.

**CHECK YOURSELF 7**

$\sqrt{19}$ is between which of the following?

(a) 4 and 5

(b) 5 and 6

(c) 6 and 7

A scientific calculator can be used to evaluate expressions that contain square roots, as Example 8 illustrates.

Example 8**Evaluating Expressions Using a Calculator**

Use a scientific calculator to approximate the value of each expression. Round your answer to the nearest hundredth.

(a) $\sqrt{177}$ Using the calculator, you find $\sqrt{177} = 13.3041 \dots$ To the nearest hundredth, $\sqrt{177} = 13.30$.

(b) $4(\sqrt{93})$ Be certain that you enter the entire expression into the calculator. Then round the answer. Here, $4(\sqrt{93}) = 38.5746 \dots$ To the nearest hundredth, $4(\sqrt{93}) = 38.57$.

**CHECK YOURSELF 8**

Use a scientific calculator to approximate the value of each expression. Round your answer to the nearest hundredth.

(a) $\sqrt{357}$

(b) $7(\sqrt{71})$

CHECK YOURSELF ANSWERS

1. (a) 11; (b) 6 2. Side y 3. (a) $5^2 + 12^2 = 25 + 144 = 169$, $13^2 = 169$,
so $5^2 + 12^2 = 13^2$; (b) $6^2 + 8^2 = 36 + 64 = 100$, $10^2 = 100$ so $6^2 + 8^2 = 10^2$
4. (a) 8; (b) 25 5. 15 6. 6 cm 7. (a) 4 and 5
8. (a) 18.89; (b) 58.98



Exercises

In exercises 1 to 4, find the square root.

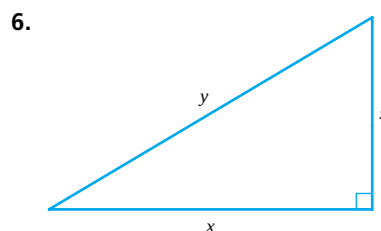
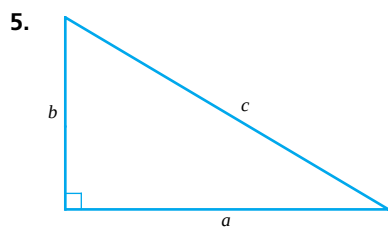
1. $\sqrt{64}$

2. $\sqrt{121}$

3. $\sqrt{169}$

4. $\sqrt{196}$

Identify the hypotenuse of the given triangles by giving its letter.



For exercises 7 to 12, identify which numbers are perfect triples.

7. 3, 4, 5

8. 4, 5, 6

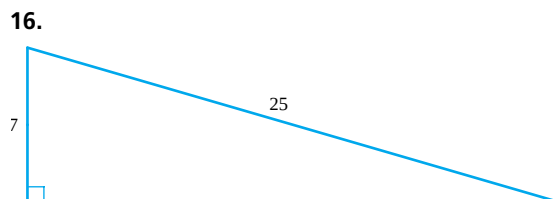
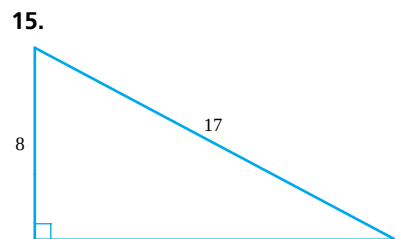
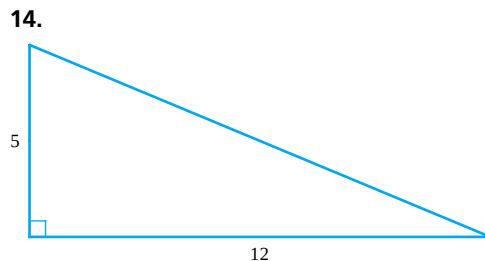
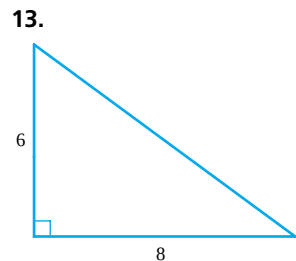
9. 7, 12, 13

10. 5, 12, 13

11. 8, 15, 17

12. 9, 12, 15

For exercises 13 to 16, find the missing length for each right triangle.



Name _____

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ANSWERS

1. _____

2. _____

3. _____

4. _____

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9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

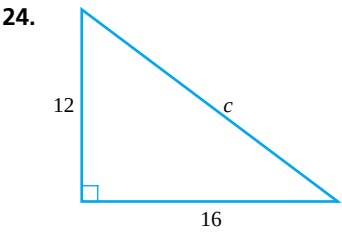
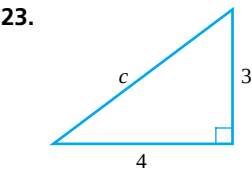
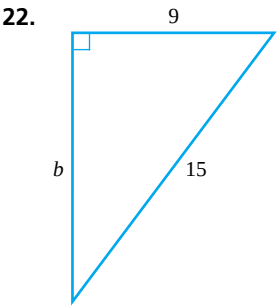
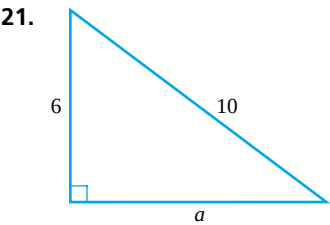
ANSWERS

17. _____
18. _____
19. _____
20. _____
21. _____
22. _____
23. _____
24. _____
25. _____

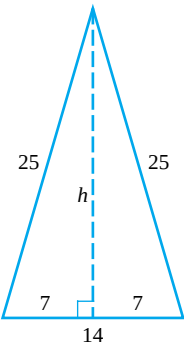
Select the correct approximation for each of the following.

17. Is $\sqrt{23}$ between (a) 3 and 4, (b) 4 and 5, or (c) 5 and 6?
18. Is $\sqrt{15}$ between (a) 1 and 2, (b) 2 and 3, or (c) 3 and 4?
19. Is $\sqrt{44}$ between (a) 6 and 7, (b) 7 and 8, or (c) 8 and 9?
20. Is $\sqrt{31}$ between (a) 3 and 4, (b) 4 and 5, or (c) 5 and 6?

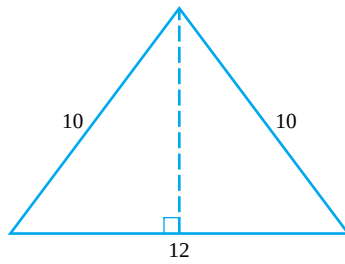
In exercises 21 to 24, find the perimeter of each triangle shown. (*Hint*: First find the missing side.)



25. Find the altitude, h , of the isosceles triangle shown.



26. Find the altitude of the isosceles triangle shown.



26. _____

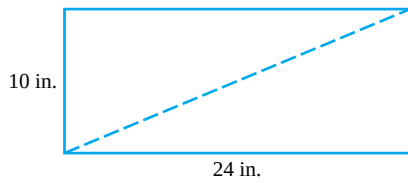
27. _____

28. _____

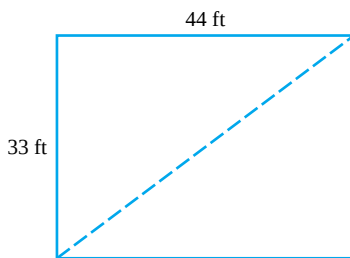
29. _____

In exercises 27 and 28, find the length of the diagonal of each rectangle.

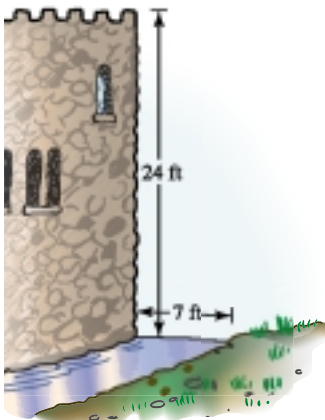
27.



28.

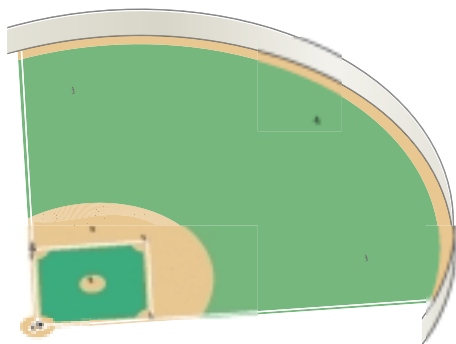


29. A castle wall, 24 feet high, is surrounded by a moat 7 feet across. Will a 26-foot ladder, placed at the edge of the moat, be long enough to reach the top of the wall?



30.

30. A baseball diamond is the shape of a square that has sides of length 90 feet. Find the distance from home plate to second base.



Answers

1. 8 3. 13 5. c 7. Yes 9. No 11. Yes 13. 10
 15. 15 17. b 19. a 21. 24 23. 12 25. 24
 27. 26 in. 29. Yes



Using Your Calculator to Find Square Roots

To find a square root on your scientific calculator, you use the square root key. On some calculators, you simply enter the number, then press the square root key. With others, you must use the second function on the $\boxed{x^2}$ (or $\boxed{y^x}$) key and specify the root you wish to find.

Example 1

Finding a Square Root Using the Calculator

Find the square root of 256.

$$256 \sqrt{}$$

Display $\boxed{16}$

or

$$256 \boxed{2nd} \boxed{\sqrt[y^x]{}} 2 \boxed{=}$$

Display $\boxed{16}$ The "2" is entered for the 2nd (square) root.



CHECK YOURSELF 1

Find the square root of 361.

As we saw in the previous section, not every square root is a whole number. Your calculator can help give you the *approximate* square root of any number.

Example 2

Finding an Approximate Square Root

Approximate the square root of 29. Round your answer to the nearest tenth.

Enter

$$29 \sqrt{}$$

Your calculator display will read something like this:

Display $\boxed{5.385164807}$

This is an *approximation* of the square root. It is rounded to the nearest billionth place. The calculator cannot display the exact answer because there is no end to the sequence of

digits (and also no pattern.) If the square root of a whole number is not another whole number, then the answer has an infinite number of digits.

To find the approximate square root, we round to the nearest tenth. Our approximation for the square root of 29 is 5.4.



CHECK YOURSELF 2

Approximate the square root of 19. Round your answer to the nearest tenth.

CHECK YOURSELF ANSWERS

1. 19 2. 4.4



Calculator Exercises

Use your calculator to find the square root of each of the following.

1. 64

2. 144

3. 289

4. 1024

5. 1849

6. 784

7. 8649

8. 5329

9. 3844

10. 3364

Use your calculator to approximate the following square roots. Round to the nearest tenth.

11. $\sqrt{23}$

12. $\sqrt{31}$

13. $\sqrt{51}$

14. $\sqrt{42}$

15. $\sqrt{134}$

16. $\sqrt{251}$

Name _____

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ANSWERS

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

Answers

1. 8 3. 17 5. 43 7. 93 9. 62 11. 4.8 13. 7.1
15. 11.6