

Cube

In geometry, a **cube**^[1] is a three-dimensional solid object bounded by sixsquare faces, facets or sides, with three meeting at eachvertex.

The cube is the onlyregular hexahedron and is one of the fivePlatonic solids. It has 6 faces, 12 edges, and 8 vertices.

The cube is also a square parallelepiped, an equilateral cuboid and a right rhombohedron. It is a regular square prism in three orientations, and a trigonal trapezohedron in four orientations.

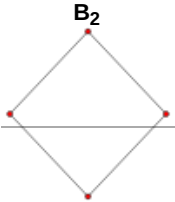
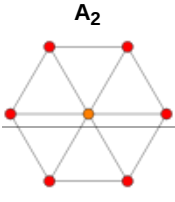
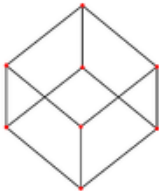
The cube is dual to the octahedron. It has cubical oroctahedral symmetry.

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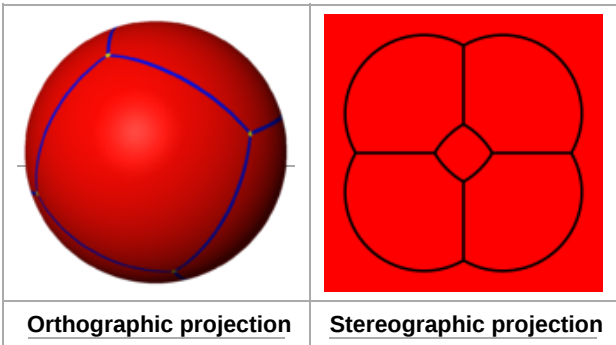
Orthogonal projections

The *cube* has four specialorthogonal projections centered, on a vertex, edges, face and normal to itsvertex figure. The first and third correspond to the A_2 and B_2 Coxeter planes.

Orthogonal projections		
Centered by	Face	Vertex
Coxeter planes	B_2	A_2
Projective symmetry	[4]	[6]
Tilted views		

Spherical tiling

The cube can also be represented as a spherical tiling, and projected onto the plane via a stereographic projection. This projection is conformal, preserving angles but not areas or lengths. Straight lines on the sphere are projected as circular arcs on the plane.



Cartesian coordinates

For a cube centered at the origin, with edges parallel to the axes and with an edge length of 2, the Cartesian coordinates of the vertices are

(±1, ±1, ±1)

while the interior consists of all points (x0, x1, x2) with -1 < xi < 1 for all i.

Equation in R3

In analytic geometry, a cube's surface with center (x0, y0, z0) and edge length of 2a is the locus of all points (x, y, z) such that

max{|x - x0|, |y - y0|, |z - z0|} = a.

Formulas

For a cube of edge length a:

surface area	6a²	volume	a³
face diagonal	√2a	space diagonal	√3a
radius of circumscribed sphere	√3/2 a	radius of sphere tangent to edges	a/√2
radius of inscribed sphere	a/2	angles between faces(in radians)	π/2

As the volume of a cube is the third power of its sides a × a × a, third powers are called cubes, by analogy with squares and second powers.

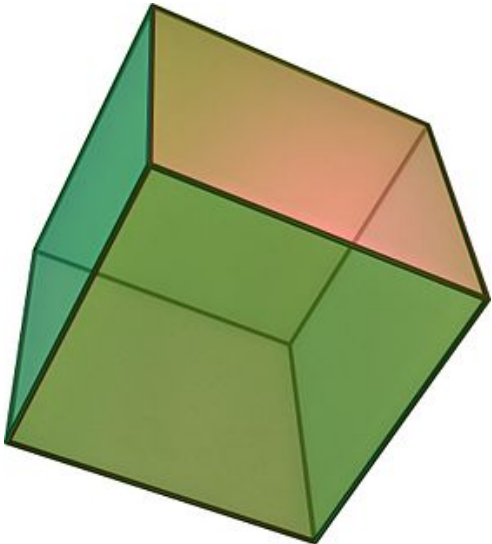
A cube has the largest volume among cuboids (rectangular boxes) with a given surface area. Also, a cube has the largest volume among cuboids with the same total linear size (length+width+height).

Point in space

For a cube whose circumscribing sphere has radius R, and for a given point in its 3-dimensional space with distances di from the cube's eight vertices, we have^[2]

Σ8i=1 d_i^4 + 16R^4 / 9 = (Σ8i=1 d_i^2 + 2R^2 / 3)^2 .

Regular hexahedron



(Click here for rotating model)

Type	Platonic solid
Elements	F = 6, E = 12 V = 8 (χ = 2)
Faces by sides	6{4}
Conway notation	C
Schläfli symbols	{4,3}
	t{2,4} or {4}×{} tr{2,2} or {}×{}×{}
Face configuration	V3.3.3.3
Wythoff symbol	3 2 4
Coxeter diagram	⦿ ₄ —•—•
Symmetry	O _h , B ₃ , [4,3], (*432)
Rotation group	O, [4,3] ⁺ , (432)
References	U ₀₆ , C ₁₈ , W ₃
Properties	regular, convexzonohedron
Dihedral angle	90°
 4.4.4 (Vertex figure)	 Octahedron (dual polyhedron)

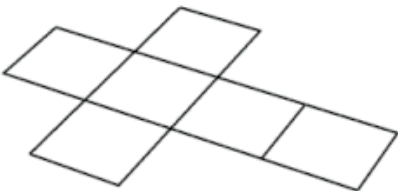
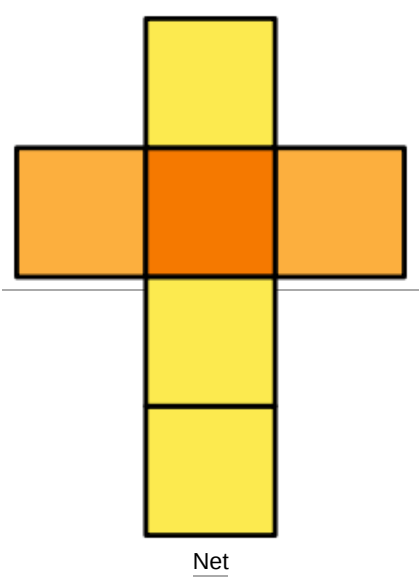
Doubling the cube

Doubling the cube, or the *Delian problem*, was the problem posed by ancient Greek mathematicians of using only a compass and straightedge to start with the length of the edge of a given cube and to construct the length of the edge of a cube with twice the volume of the original cube. They were unable to solve this problem, and in 1837 Pierre Wantzel proved it to be impossible because the cube root of 2 is not a constructible number.

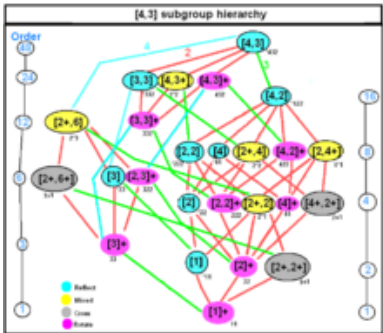
Uniform colorings and symmetry

The cube has three uniform colorings, named by the colors of the square faces around each vertex: 111, 112, 123.

The cube has three classes of symmetry, which can be represented by vertex-transitive coloring the faces. The highest octahedral symmetry O_h has all the faces the same color. The dihedral symmetry D_{4h} comes from the cube being a prism, with all four sides being the same color. The lowest symmetry D_{2h} is also a prismatic symmetry, with sides alternating colors, so there are three colors, paired by opposite sides. Each symmetry form has a different Wythoff symbol.



Net of cube



Octahedral symmetrytree

Name	Regular hexahedron	Square prism		Rectangular cuboid	Rhombic prism	Trigonal trapezohedron
<u>Coxeter diagram</u>						
<u>Schläfli symbol</u>	{4,3}	{4}×{ } rr{4,2}	s ₂ {2,4}	{ } ³ tr{2,2}	{ }×2{ }	
<u>Wythoff symbol</u>	3 4 2	4 2 2		2 2 2		
<u>Symmetry</u>	O _h [4,3] (*432)	D _{4h} [4,2] (*422)	D _{2d} [4,2 ⁺] (2*2)	D _{2h} [2,2] (*222)		D _{3d} [6,2 ⁺] (2*3)
<u>Symmetry order</u>	24	16	8	8		12
<u>Image (uniform coloring)</u>	 (111)	 (112)	 (112)	 (123)	 (112)	 (111), (112)

Geometric relations

A cube has eleven nets (one shown above): that is, there are eleven ways to flatten a hollow cube by cutting seven edges.^[3] To color the cube so that no two adjacent faces have the same color, one would need at least three colors.

The cube is the cell of the only regular tiling of three-dimensional Euclidean space. It is also unique among the Platonic solids in having faces with an even number of sides and, consequently, it is the only member of that group that is azonohedron (every face has point symmetry).

The cube can be cut into six identical square pyramids. If these square pyramids are then attached to the faces of a second cube, a rhombic dodecahedron is obtained (with pairs of coplanar triangles combined into rhombic faces).

Other dimensions

The analogue of a cube in four-dimensional Euclidean space has a special name—a tesseract or hypercube. More properly, a hypercube (or *n*-dimensional cube or simply *n*-cube) is the analogue of the cube in *n*-dimensional Euclidean space and a tesseract is the order 4 hypercube. A hypercube is also called a *measure polytope*.

There are analogues of the cube in lower dimensions too: a point in dimension 0, a segment in one dimension and a square in two dimensions.

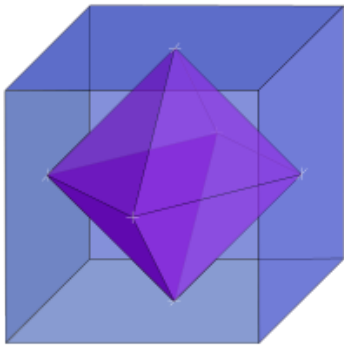
Related polyhedra

The quotient of the cube by the antipodal map yields a projective polyhedron the hemicube.

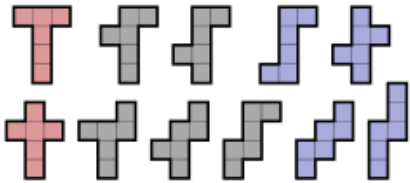
If the original cube has edge length 1, its dual polyhedron (an octahedron) has edge length $\sqrt{2}/2$.

The cube is a special case in various classes of general polyhedra:

Name	Equal edge-lengths?	Equal angles?	Right angles?
Cube	Yes	Yes	Yes
<u>Rhombohedron</u>	Yes	Yes	No
<u>Cuboid</u>	No	Yes	Yes
<u>Parallelepiped</u>	No	Yes	No
<u>quadrilaterally faced hexahedron</u>	No	No	No



The dual of a cube is an octahedron, seen here with vertices at the center of the cube's square faces.



The 11 nets of the cube.



These familiar six-sided dice are cube-shaped.

The vertices of a cube can be grouped into two groups of four, each forming a regular tetrahedron; more generally this is referred to as a demicube. These two together form a regular compound, the stella octangula. The intersection of the two forms a regular octahedron. The symmetries of a regular tetrahedron correspond to those of a cube which map each tetrahedron to itself; the other symmetries of the cube map the two to each other.

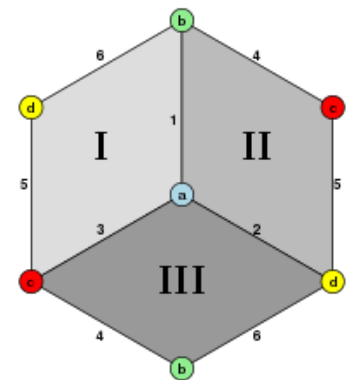
One such regular tetrahedron has a volume of $\frac{1}{3}$ of that of the cube. The remaining space consists of four equal irregular tetrahedra with a volume of $\frac{1}{6}$ of that of the cube, each.

The rectified cube is the cuboctahedron. If smaller corners are cut off we get a polyhedron with six octagonal faces and eight triangular ones. In particular we can get regular octagons (truncated cube). The rhombicuboctahedron is obtained by cutting off both corners and edgesto the correct amount.

A cube can be inscribed in a dodecahedron so that each vertex of the cube is a vertex of the dodecahedron and each edge is a diagonal of one of the dodecahedron's faces; taking all such cubes gives rise to the regular compound of five cubes.

If two opposite corners of a cube are truncated at the depth of the three vertices directly connected to them, an irregular octahedron is obtained. Eight of these irregular octahedra can be attached to the triangular faces of a regular octahedron to obtain the cuboctahedron.

The cube is topologically related to a series of spherical polyhedra and tilings with order 3 vertex figures



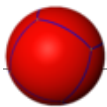
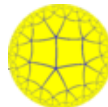
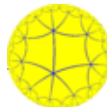
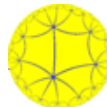
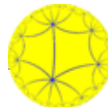
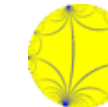
The hemicube is the 2-to-1 quotient of the cube.

*n32 symmetry mutation of regular tilings: {4,3}											
Spherical				Euclidean	Compact hyperb.		Paraco.	Noncompact hyperbolic			
											
{2,3}	{3,3}	{4,3}	{5,3}	{6,3}	{7,3}	{8,3}	{∞,3}	{12i,3}	{9i,3}	{6i,3}	{3i,3}

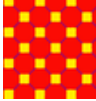
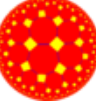
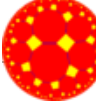
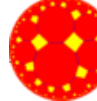
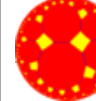
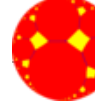
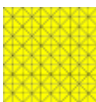
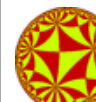
The cuboctahedron is one of a family of uniform polyhedra related to the cube and regular octahedron.

Uniform octahedral polyhedra										
Symmetry: [4,3], (*432)							[4,3] ⁺ (432)	[1 ⁺ ,4,3] = [3,3] (*332)	[3 ⁺ ,4] (3*2)	
{4,3}	t{4,3}	r{4,3} r{3 ^{1,1} }	t{3,4} t{3 ^{1,1} }	{3,4} {3 ^{1,1} }	rr{4,3} s ₂ {3,4}	tr{4,3}	sr{4,3}	h{4,3} {3,3}	h ₂ {4,3} t{3,3}	s{3,4} s{3 ^{1,1} }
										
										
										
Duals to uniform polyhedra										
V4 ³	V3.8 ²	V(3.4) ²	V4.6 ²	V3 ⁴	V3.4 ³	V4.6.8	V3 ⁴ .4	V3 ³	V3.6 ²	V3 ⁵
										
										
										

The cube is topologically related as a part of sequence of regular tilings, extending into the hyperbolic plane {4,p}, p=3,4,5...

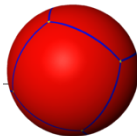
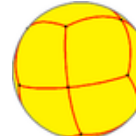
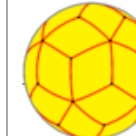
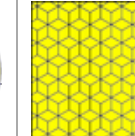
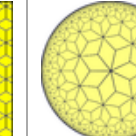
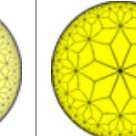
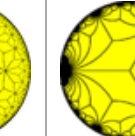
<i>*n</i> 42 symmetry mutation of regular tilings: {4 <i>n</i> }						
Spherical	Euclidean	Compact hyperbolic				Paracompact
 {4,3}	 {4,4}	 {4,5}	 {4,6}	 {4,7}	 {4,8}...	 {4,∞}
						

With dihedral symmetry, Dih₄, the cube is topologically related in a series of uniform polyhedra and tilings 4.2*n*.2*n*, extending into the hyperbolic plane:

<i>*n</i> 42 symmetry mutation of truncated tilings: 4.2.2 <i>n</i>								
Symmetry <i>*n</i> 42 [<i>n</i> ,4]	Spherical		Euclidean	Compact hyperbolic				Paracomp.
	*242 [2,4]	*342 [3,4]	*442 [4,4]	*542 [5,4]	*642 [6,4]	*742 [7,4]	*842 [8,4]...	*∞42 [∞,4]
Truncated figures								
Config.	4.4.4	4.6.6	4.8.8	4.10.10	4.12.12	4.14.14	4.16.16	4.∞.∞
n-kis figures								
Config.	V4.4.4	V4.6.6	V4.8.8	V4.10.10	V4.12.12	V4.14.14	V4.16.16	V4.∞.∞

All these figures have octahedral symmetry.

The cube is a part of a sequence of rhombic polyhedra and tilings with [*n*,3] Coxeter group symmetry. The cube can be seen as a rhombic hexahedron where the rhombi are squares.

Symmetry mutations of dual quasiregular tilings $V(3.n)^2$							
<i>*n</i> 32	Spherical			Euclidean	Hyperbolic		
	*332	*432	*532	*632	*732	*832...	*∞32
Tiling							
Conf.	$V(3.3)^2$	$V(3.4)^2$	$V(3.5)^2$	$V(3.6)^2$	$V(3.7)^2$	$V(3.8)^2$	$V(3.∞)^2$

The cube is a square prism.

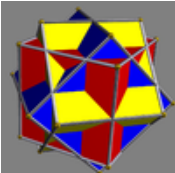
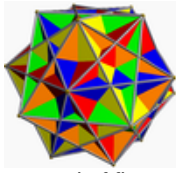
Family of uniform prisms

Polyhedron											
Coxeter											
Tiling											
Config.	2.4.4	3.4.4	4.4.4	5.4.4	6.4.4	7.4.4	8.4.4	9.4.4	10.4.4	11.4.4	12.4.4

As a trigonal trapezohedron the cube is related to the hexagonal dihedral symmetry family

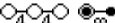
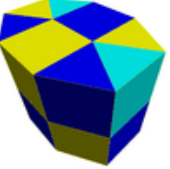
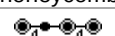
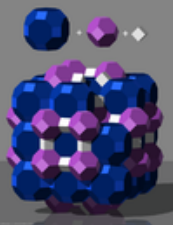
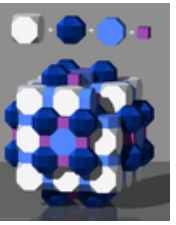
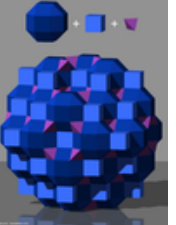
Uniform hexagonal dihedral spherical polyhedra								
Symmetry: [6,2], (*622)						[6,2] ⁺ , (622)	[6,2 ⁺], (2*3)	
								
								
{6,2}	t{6,2}	r{6,2}	t{2,6}	{2,6}	rr{6,2}	tr{6,2}	sr{6,2}	s{2,6}
Duals to uniforms								
								
<u>V6²</u>	<u>V12²</u>	<u>V6²</u>	<u>V4.4.6</u>	<u>V2⁶</u>	<u>V4.4.6</u>	<u>V4.4.12</u>	<u>V3.3.3.6</u>	<u>V3.3.3.3</u>

Regular and uniform compounds of cubes

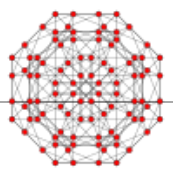
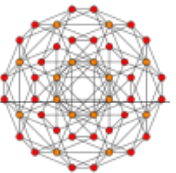
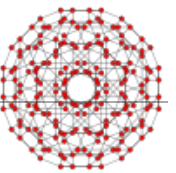
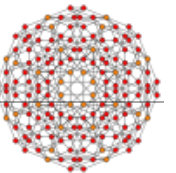
	
Compound of three cubes	Compound of five cubes

In uniform honeycombs and polychora

It is an element of 9 of 28convex uniform honeycombs

Cubic honeycomb 	Truncated square prismatic honeycomb 	Snub square prismatic honeycomb 	Elongated triangular prismatic honeycomb 	Gyroelongated triangular prismatic honeycomb
				
Cantellated cubic honeycomb 	Cantitruncated cubic honeycomb 	Runcitruncated cubic honeycomb 	Runcinated alternated cubic honeycomb 	
				

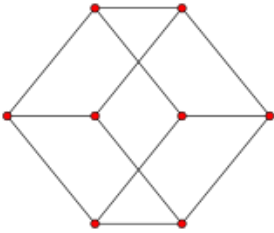
It is also an element of five fourdimensional uniform polychora

Tesseract 	Cantellated 16-cell 	Runcinated tesseract 	Cantitruncated 16-cell 	Runcitruncated 16-cell 
				

Cubical graph

The skeleton of the cube (the vertices and edges) form a graph, with 8 vertices, and 12 edges. It is a special case of thehypercube graph^[4] It is one of 5 Platonic graphs, each a skeleton of itsPlatonic solid

An extension is the three dimensionalk-ary Hamming graph, which for *k* = 2 is the cube graph. Graphs of this sort occur in the theory ofparallel processingin computers.

Cubical graph	
	
Named after	Q ₃
Vertices	8
Edges	12
Radius	3
Diameter	3
Girth	4
Automorphisms	48
Chromatic number	2
Properties	Hamiltonian, regular, symmetric, distance-regular, distance-transitive, 3-vertex-connected, planar graph

See also

- Tesseract
- Trapezohedron

Miscellaneous cubes

- Cube (film)
- Diamond cubic
- Lövhcim cube of emotion
- Cube of Heymans
- Necker Cube
- OLAP cube
- Prince Rupert's cube
- Rubik's Cube
- The Cube (game show)
- Unit cube
- Yoshimoto Cube
- Kaaba

References

- English *cube* from Old French < Latincubus < Greek κύβος (*kubos*) meaning "a cube, a die, vertebra". In turn from PIE **keu(b)-*, "to bend, turn".
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<http://forumgeom.fau.edu/FG2016volume16/FG201627.pdf>
- Weisstein, Eric W. "Cube" (<http://mathworld.wolfram.com/Cube.html>)*MathWorld*.
- Weisstein, Eric W. "Cubical graph" (<http://mathworld.wolfram.com/CubicalGraph.html>)*MathWorld*.

External links

- Weisstein, Eric W. "Cube". *MathWorld*.
- Cube: Interactive Polyhedron Model
- Volume of a cube, with interactive animation
- Cube (Robert Webb's site)

Fundamental convex regular and uniform polytopes in dimensions 2–10					
Family	A_n	B_n	$I_2(p) / D_n$	$E_6 / E_7 / E_8 / F_4 / G_2$	H_n
Regular polygon	Triangle	Square	p-gon	Hexagon	Pentagon
Uniform polyhedron	Tetrahedron	Octahedron • Cube	Demicube		Dodecahedron • Icosahedron
Uniform 4-polytope	5-cell	16-cell • Tesseract	Demitesseract	24-cell	120-cell • 600-cell
Uniform 5-polytope	5-simplex	5-orthoplex • 5-cube	5-demicube		
Uniform 6-polytope	6-simplex	6-orthoplex • 6-cube	6-demicube	$1_{22} \cdot 2_{21}$	
Uniform 7-polytope	7-simplex	7-orthoplex • 7-cube	7-demicube	$1_{32} \cdot 2_{31} \cdot 3_{21}$	
Uniform 8-polytope	8-simplex	8-orthoplex • 8-cube	8-demicube	$1_{42} \cdot 2_{41} \cdot 4_{21}$	
Uniform 9-polytope	9-simplex	9-orthoplex • 9-cube	9-demicube		
Uniform 10-polytope	10-simplex	10-orthoplex • 10-cube	10-demicube		
Uniform <i>n</i>-polytope	<i>n</i> -simplex	<i>n</i> -orthoplex • <i>n</i> -cube	<i>n</i> -demicube	$1_{k2} \cdot 2_{k1} \cdot k_{21}$	<i>n</i> -pentagonal polytope
Topics: <u>Polytope families</u> • <u>Regular polytope</u> • <u>List of regular polytopes and compounds</u>					

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