

Rational and irrational numbers

1

Fig 1.1 missing

How does the speed of a car affect its stopping distance in an emergency? Serious car accident scenes are often investigated to identify factors leading up to the crash. One measurement taken is the length of the skid marks which indicate the braking distance. From this and other information, such as the road's friction coefficient, the speed of a car before braking can be determined. If the formula used is $v = \sqrt{20d}$, where v is the speed in metres per second and d is the braking distance in metres, what would the speed of a car have been before braking if the skid mark measured 32.50 m in length?

For this scenario, the number you will obtain for the speed is an irrational number. In this chapter, you will find out the difference between rational and irrational numbers and learn to work with both.



Are you ready?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching **SkillSHEET**. Either click on the **SkillSHEET** icon next to the question on the *Maths Quest 10* CD-ROM or ask your teacher for a copy.

**1.1**

Simplifying fractions

1 Write each of the following fractions in simplest form.

a $\frac{39}{52}$

b $\frac{18}{72}$

c $\frac{27}{36}$

d $\frac{12}{10}$

**1.2**

Finding and converting to the lowest common denominator

2 Find the lowest common denominator of each of the following pairs of fractions.

a $\frac{5}{6}$ and $\frac{1}{3}$

b $\frac{3}{8}$ and $\frac{5}{12}$

c $\frac{7}{18}$ and $\frac{4}{27}$

d $\frac{1}{15}$ and $\frac{3}{20}$

**1.3**

Converting a mixed number into an improper fraction

3 Convert each of the following mixed numbers into an improper fraction.

a $2\frac{1}{3}$

b $3\frac{3}{4}$

c $5\frac{1}{10}$

d $4\frac{7}{8}$

**1.4**

Converting an improper fraction into a mixed number

4 Change each of the following improper fractions into a mixed number.

a $\frac{20}{9}$

b $\frac{21}{9}$

c $\frac{24}{10}$

d $\frac{17}{11}$

**1.5**

Converting a fraction into a decimal

5 Convert each of the following fractions into decimals.

a $\frac{3}{8}$

b $\frac{3}{16}$

c $\frac{8}{25}$

d $\frac{9}{40}$

**1.6**

Writing a recurring decimal in short form

6 Write each of the following decimals in recurring decimal form.

a 4.333...

b 5.428 571 428...

c 13.8383...

d 19.687 287 28...

**1.7**

Converting a terminating decimal into a fraction

7 Write each of the following decimals as fractions in simplest form.

a 0.6

b 0.75

c 0.125

d 0.025

**1.8**

Finding square roots, cube roots and other roots

8 Evaluate each of the following.

a $\sqrt{121}$

b $\sqrt[3]{27}$

c $\sqrt[5]{32}$

d $\sqrt[3]{1\,000\,000}$

**1.9**

Rounding to a given number of decimal places

9 Evaluate each of the following correct to 1 decimal place.

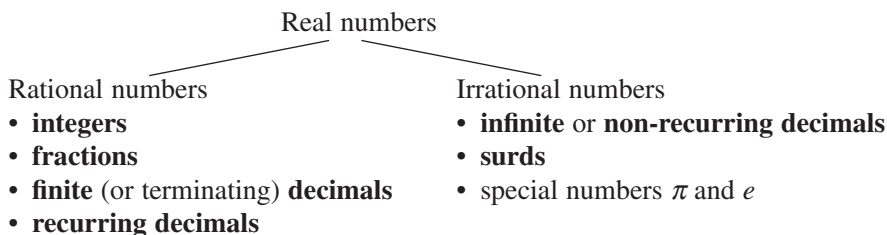
a $\sqrt{3}$

b $\sqrt{15}$

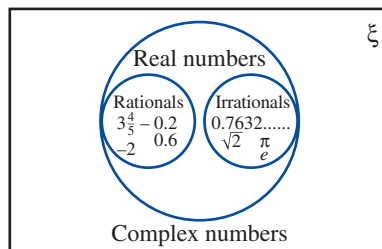
c $\sqrt{99}$

d $\sqrt{102}$

We use numbers such as integers, fractions and decimals every day. They form part of what is called the Real Number System. (There are numbers that do not fit into the Real Number System, called complex numbers, which you may come across in the future.) **Real numbers** can be divided into two categories — **rational numbers** and **irrational numbers**.



This can be represented using the following **Venn diagram**.



This chapter begins with a review of rational numbers such as fractions and recurring decimals. We then move on to consider irrational numbers, including surds. As you will see, rational numbers are those numbers which can be expressed as a **ratio** of two integers $\frac{a}{b}$ where $b \neq 0$ (that is, a rational number can be expressed as a fraction). Why are integers considered to be rational numbers?

Operations with fractions

From earlier years, you should be familiar with the main operations of using fractions. These include simplifying fractions, converting between mixed numbers and improper fractions and the four arithmetic operations.

Simplifying fractions

Fractional answers should always be expressed in simplest form. This is done by dividing both the **numerator** and the **denominator** by their highest common factor (HCF).

WORKED Example 1

Write $\frac{32}{44}$ in simplest form.

THINK

- 1 Write the fraction and divide both numerator and denominator by the HCF or highest common factor (4).
- 2 Write the answer.

WRITE

$$\begin{array}{r} \cancel{32}^8 \\ \cancel{44}^{11} \\ \hline = \frac{8}{11} \end{array}$$

Using the four operations with fractions

Addition and subtraction

1. When adding and subtracting fractions, write each fraction with the same denominator. This common denominator is the **lowest common multiple (LCM)** of all denominators in the question.
2. When adding mixed numbers, first change to improper fractions then follow step 1.
3. When subtracting mixed numbers, first change to improper fractions then follow step 1.

WORKED Example 2

Evaluate each of the following. **a** $\frac{3}{5} + \frac{5}{6}$ **b** $3\frac{1}{2} - 1\frac{4}{5}$

THINK

- a**
- 1 Write the expression.
 - 2 Write both fractions with the same denominator using equivalent fractions.
 - 3 Add the fractions by adding the numerators. Keep the denominator the same.
 - 4 Simplify by writing the answer as a mixed number.
- b**
- 1 Write the expression.
 - 2 Change each mixed number to an improper fraction.
 - 3 Write both fractions with the same denominator using equivalent fractions.
 - 4 Subtract the second fraction from the first.
 - 5 Simplify by writing the answer as a mixed number.

WRITE

$$\begin{aligned} \mathbf{a} \quad & \frac{3}{5} + \frac{5}{6} \\ &= \frac{18}{30} + \frac{25}{30} \\ &= \frac{43}{30} \\ &= 1\frac{13}{30} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad & 3\frac{1}{2} - 1\frac{4}{5} \\ &= \frac{7}{2} - \frac{9}{5} \\ &= \frac{35}{10} - \frac{18}{10} \\ &= \frac{17}{10} \\ &= 1\frac{7}{10} \end{aligned}$$

Multiplication and division

1. When multiplying fractions, cancel if appropriate, then multiply numerators and multiply denominators.
2. When dividing fractions, change the division sign to a multiplication sign, tip the second fraction upside down and follow the rules for multiplying fractions (multiply and tip).
3. Change mixed numbers to improper fractions before multiplying or dividing.

WORKED Example 3

Evaluate each of the following. **a** $\frac{3}{5} \times \frac{5}{6}$ **b** $2\frac{1}{3} \div \frac{3}{4}$

THINK

- a**
- 1 Write the expression.
 - 2 Cancel or divide numerators and denominators by the same number where applicable.
 - 3 Multiply the numerators together then the denominators together and simplify where applicable.

WRITE

$$\begin{aligned} \mathbf{a} \quad & \frac{3}{5} \times \frac{5}{6} \\ &= \frac{\cancel{3}^1}{1} \times \frac{\cancel{5}_1}{\cancel{6}_2} \\ &= \frac{1}{2} \end{aligned}$$

THINK

- b** ① Write the expression.
- ② Change any mixed numbers to improper fractions.
- ③ Change the division sign to a multiplication sign and tip the second fraction upside down (multiply and tip).
- ④ Multiply the numerators together and then multiply the denominators together.
- ⑤ Change the improper fraction to a mixed number.

WRITE

$$\begin{aligned} \text{b } 2\frac{1}{3} \div \frac{3}{4} \\ &= \frac{7}{3} \div \frac{3}{4} \\ &= \frac{7}{3} \times \frac{4}{3} \\ &= \frac{28}{9} \\ &= 3\frac{1}{9} \end{aligned}$$

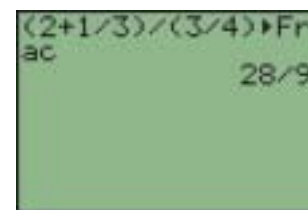
**GraphicsCalculator tip!****Fraction calculations**

As with any calculation involving fractions, if you wish to have an answer expressed as a fraction then each calculation needs to end by pressing **(MATH)**, selecting **1:Frac** and pressing **(ENTER)**.

The calculation for worked example 2(a) would be entered as $3 \div 5 + 5 \div 6$ then you would press **(MATH)**, select **1:Frac** and press **(ENTER)**.

When entering mixed numbers, it is necessary to use brackets. This allows the correct order of operations to occur.

The calculations for worked example 3(b) can be viewed in the screen shown. Note that the answers are given as improper fractions.



Fraction calculations

remember

1. To write fractions in simplest form, divide numerator and denominator by the HCF of both.
2. To add or subtract fractions, write each fraction with the same denominator first.
3. To add mixed numbers, change them to improper fractions first and then add.
4. To subtract mixed numbers, change them to improper fractions first and then subtract.
5. To multiply fractions, cancel if possible, then multiply the numerators together and then the denominators together. Simplify if appropriate.
6. To divide fractions, change the division sign to multiplication, tip the second fraction upside down then multiply and simplify if appropriate (multiply and tip).

EXERCISE

1A

Operations with fractions


WORKED Example 1
1 Write each of the following fractions in simplest form.

a $\frac{8}{12}$

b $\frac{6}{15}$

c $\frac{16}{20}$

d $\frac{16}{25}$

e $\frac{15}{27}$

f $\frac{16}{30}$

g $\frac{9}{54}$

h $\frac{10}{40}$

i $\frac{25}{45}$

j $\frac{56}{63}$

k $\frac{55}{132}$

l $\frac{36}{60}$


WORKED Example 2
2 Evaluate each of the following:

a $\frac{1}{4} + \frac{1}{3}$

b $\frac{1}{6} + \frac{2}{3}$

c $\frac{1}{2} + \frac{3}{4}$

d $\frac{2}{5} + \frac{7}{10}$

e $\frac{1}{2} - \frac{2}{9}$

f $\frac{5}{6} - \frac{7}{12}$

g $1\frac{1}{4} + \frac{4}{5}$

h $1\frac{5}{8} + \frac{2}{3}$

i $1\frac{3}{4} - \frac{8}{9}$

j $1\frac{1}{6} - \frac{5}{12}$

k $2\frac{1}{7} - 1\frac{2}{5}$

l $3\frac{2}{5} - 1\frac{3}{4}$


WORKED Example 3a
3 Evaluate each of the following:

a $\frac{2}{3} \times \frac{3}{4}$

b $\frac{2}{7} \times \frac{8}{9}$

c $\frac{3}{5} \times \frac{5}{6}$

d $\frac{3}{10} \times \frac{6}{11}$

e $\frac{5}{12} \times \frac{3}{4}$

f $\frac{7}{15} \times \frac{5}{8}$

g $1\frac{2}{5} \times \frac{4}{9}$

h $1\frac{7}{10} \times \frac{6}{7}$

i $\frac{5}{8} \times 2\frac{3}{4}$

j $2\frac{1}{2} \times 3\frac{5}{8}$

k $1\frac{1}{3} \times 2\frac{5}{8}$

l $1\frac{2}{7} \times 3\frac{1}{9}$


WORKED Example 3b
4 Evaluate each of the following:

a $\frac{1}{2} \div \frac{3}{5}$

b $\frac{4}{7} \div \frac{2}{3}$

c $\frac{5}{8} \div \frac{3}{4}$

d $\frac{11}{12} \div \frac{1}{3}$

e $\frac{7}{10} \div \frac{2}{5}$

f $\frac{15}{16} \div \frac{5}{8}$

g $1\frac{1}{4} \div \frac{2}{3}$

h $1\frac{3}{10} \div \frac{7}{10}$

i $\frac{5}{6} \div 1\frac{1}{3}$

j $2\frac{7}{8} \div 1\frac{4}{5}$

k $2\frac{11}{12} \div \frac{7}{9}$

l $3 \div 1\frac{4}{5}$

5 multiple choice
a $\frac{5}{8}$ is equal to:

A $\frac{15}{25}$

B $\frac{32}{40}$

C $\frac{60}{72}$

D $\frac{12}{18}$

E $\frac{30}{48}$

b $\frac{5}{7} + 1\frac{1}{3}$ is equal to:

A $\frac{9}{21}$

B $\frac{9}{10}$

C $1\frac{3}{5}$

D $2\frac{1}{21}$

E $1\frac{2}{3}$

c $\frac{3}{4} \div 1\frac{3}{5}$ is equal to:

A $2\frac{2}{15}$

B $1\frac{1}{5}$

C $\frac{15}{32}$

D $1\frac{17}{32}$

E $1\frac{1}{4}$

d If $\frac{1}{3}$ of a glass is filled with lemonade and $\frac{1}{2}$ with water, what fraction of the glass has no liquid?

A $\frac{1}{2}$

B $\frac{5}{6}$

C $\frac{3}{5}$

D $\frac{2}{3}$

E $\frac{1}{6}$



- 6 Five hundred students attended the school athletics carnival. Three-fifths of them wore sunscreen without a hat and $\frac{1}{4}$ of them wore a hat but no sunscreen. If 10 students wore both a hat and sunscreen, how many students wore neither?



- 7 Phillip earns \$56 a week doing odd jobs. If he spends $\frac{5}{8}$ of his earnings on himself and saves $\frac{1}{5}$, how much does he have left to spend on other people?
- 8 A pizza had been divided into four equal pieces.
- i Bill came home with a friend and the two boys shared one piece. How much of the pizza was left?
 - ii Then Milly came in and ate $\frac{1}{3}$ of one of the remaining pieces. How much of the pizza did she eat and how much was left?
 - iii Later, Dad came home and ate $1\frac{1}{3}$ of the larger pieces which remained. How much did he eat and how much of the pizza was left?



- 1 Without using a calculator, find the value of:
 $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \dots - 98 + 99 - 100.$
- 2 Find a fraction that is greater than $\frac{5}{11}$ but less than $\frac{6}{13}$.
- 3 If this calculation continued forever, what would you expect the answer to be?
 $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots$
- 4 If this calculation continued forever, what would you expect the answer to be?
 $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \frac{1}{64} - \dots$

Finite and recurring decimals

The four basic operations when applied to decimals are very straightforward using a calculator. It is important that you are able to convert between the fractional and decimal forms of a rational number.

All fractions can be written as finite or recurring decimals. Finite (or terminating) decimals are exact and have not been rounded. Recurring decimals repeat the last decimal places over and over again. They are represented by a bar or dots placed over the repeating digit(s). Many calculators round the last digit on their screens, so recurring decimal patterns are sometimes difficult to recognise.

Converting between fractions and terminating decimals was covered in earlier years and can be revised by clicking on the SkillsSHEET icons here or in exercise 1B.

Converting a fraction to a recurring decimal requires you to recognise the recurring pattern when it appears.



WORKED Example 4

Express each of the following fractions as a recurring decimal.

a $\frac{7}{12}$

b $\frac{3}{7}$

THINK

- a** ① Write the fraction.
- ② Divide the numerator by the denominator until a recurring pattern emerges.
- ③ Write the answer.

- b** ① Write the fraction.
- ② Divide the numerator by the denominator until a recurring pattern emerges.
- ③ Write the answer.

WRITE

a $\frac{7}{12}$

$$\begin{array}{r} 0.58333 \\ 12 \overline{) 7.00000} \end{array}$$

$$\frac{7}{12} = 0.58\dot{3}$$

b $\frac{3}{7}$

$$\begin{array}{r} 0.4285714285714 \\ 7 \overline{) 3.00000000000} \end{array}$$

$$\frac{3}{7} = 0.\overline{428571}$$

If asked to convert a fraction to a decimal without a specific number of decimal places or significant figures required, work until a pattern emerges or a finite answer is found. Some recurring patterns will quickly become obvious.

To convert recurring decimals to fractions requires some algebraic skills.

WORKED Example 5

Convert each of the following to a fraction in simplest form.

a $0.\overline{63}$

THINK

- a** ① Write the recurring decimal and its expanded form.
- ② Let x equal the expanded form and call it equation [1].
- ③ Multiply both sides of equation [1] by 100 because there are two repeating digits and call the new equation [2].
- ④ Subtract [1] from [2] in order to eliminate the recurring part of the decimal.
- ⑤ Solve the equation and write the answer in simplest form.

b $0.6\dot{3}$

WRITE

a $0.\overline{63} = 0.636\ 363\ \dots$

Let $x = 0.636\ 363\ \dots$ [1]

[1] $\times 100$: $100x = 63.636\ 363\ \dots$ [2]

[2] $-$ [1]: $100x - x = 63.636\ 363\ \dots$
 $-0.636\ 363\ \dots$

$99x = 63$

$x = \frac{63}{99}$

$x = \frac{7}{11}$

- b** ① Write the recurring decimal and its expanded form.
- ② Let x equal the expanded form and call it [1].
- ③ Multiply both sides of equation [1] by 10 because there is one repeating digit and call the new equation [2].
- ④ Subtract [1] from [2] in order to eliminate the recurring part of the decimal.
- ⑤ Solve the equation.
- ⑥ Simplify where appropriate. (Multiply numerator and denominator by 10 to obtain whole numbers.)

b $0.6\dot{3} = 0.633\ 333\ 3\ \dots$

Let $x = 0.633\ 333\ \dots$ [1]

$10x = 6.333\ 33\ \dots$ [2]

[2] $-$ [1]: $10x - x = 6.333\ 33\ \dots$
 $-0.6333\ 33\ \dots$

$9x = 5.7$

$x = \frac{5.7}{9}$

$x = \frac{57}{90}$

$x = \frac{19}{30}$

Similarly, for three repeating digits, multiply by 1000; for four repeating digits, multiply by 10 000; and so on. It is possible to do this using other multiples of 10.

Can you see why recurring decimals are considered to be rational numbers?

remember

1. To convert a fraction to a decimal, divide the numerator by the denominator.
2. To write a recurring decimal, place a dot or line segment over all recurring digits.
3. Rational numbers are those numbers that can be written as a fraction with integers in both numerator and denominator. (The denominator cannot be zero.) They include: integers, fractions, finite and recurring decimals.

History of mathematics

SRINIVASA RAMANUJAN (1887–1920)



During his life ...

The Sherlock Holmes stories are written.

X-rays are discovered.

The Wright brothers build their aircraft.

World War I is fought.

Srinivasa Ramanujan was an Indian mathematician. He was born in Madras into a very poor family. Although he was a self-taught mathematical genius, Ramanujan failed to graduate from college and the best job he could find was as a clerk. Fortunately some of the people he worked with noticed his amazing abilities — he had discovered

more than 100 theorems including results on elliptic integrals and analytic number theory.

Ramanujan was persuaded to send his theorems to Cambridge University in England for evaluation. Godfrey Hardy, a fellow of Trinity College who assessed the work, was very impressed. He organised a scholarship that enabled Ramanujan to come to Cambridge in 1914. The notebooks which Ramanujan brought with him to Cambridge displayed an obvious lack of formal training in mathematics and showed that he was unaware of many of the findings of other mathematicians. Remarkably he seemed to achieve many of his results by intuition.

While at Cambridge, Ramanujan published many papers, some in conjunction with Godfrey Hardy. He worked in several areas of mathematics including number theory, elliptic functions, continued fractions and prime numbers. Palindromes were also of interest to him. A palindrome reads the same backwards as forwards, such as 12321 or abcba. He was elected a fellow of Trinity in 1918 but poor health forced him to return to India. Ramanujan died of tuberculosis at the age of 32.

Questions

1. What had Ramanujan discovered before he went to Cambridge?
2. Name four areas of mathematics that Ramanujan worked in.
3. How old was he when he died?
4. Challenge: Ramanujan found a formula for π as below. Use a calculator or computer to see what value you get for this irrational number.

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!(1103 + 26\,390n)}{(n!)^4(396^{4n})}$$

EXERCISE 1B

Finite and recurring decimals

1 Express each of the following fractions as a finite decimal.

a $\frac{3}{4}$

b $\frac{2}{5}$

c $\frac{9}{10}$

d $\frac{5}{8}$

e $\frac{33}{50}$

f $\frac{11}{40}$

g $\frac{73}{80}$

h $\frac{5}{16}$

i $\frac{13}{25}$

j $\frac{9}{20}$

k $\frac{57}{100}$

l $\frac{2}{25}$

2 Write each of the following as an exact recurring decimal.

a $0.333\ 3\ \dots$

b $0.166\ 66\ \dots$

c $0.323\ 232\ \dots$

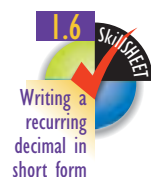
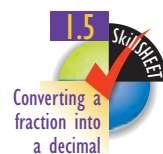
d $0.785\ 55\ \dots$

e $0.594\ 594\ 594\ \dots$

f $0.125\ 125\ 151\ 51\ \dots$

g $0.375\ 463\ 75\ \dots$

h $0.814\ 358\ 14\ \dots$



WORKED
Example

4

3 Express each of the following fractions as a recurring decimal.

a $\frac{2}{3}$

b $\frac{3}{11}$

c $\frac{8}{9}$

d $\frac{5}{18}$

e $\frac{5}{6}$

f $\frac{1}{7}$

g $\frac{11}{12}$

h $\frac{1}{15}$

i $\frac{10}{11}$

j $\frac{7}{24}$

k $\frac{17}{30}$

l $\frac{7}{27}$

4 **multiple choice**

a $\frac{31}{10\ 000}$ is equal to:

A 0.031

B 0.0031

C 0.000 31

D 0.003

E $0.\overline{31}$

b $\frac{67}{99}$ is equal to:

A 0.676

B $0.\overline{676}$

C 0.67

D $0.\overline{67}$

E $0.67\dot{6}$

c $\frac{9}{14}$ is equal to:

A 0.642 857 142

B $0.\overline{642\ 857\ 1}$

C $0.6\overline{42\ 857\ 1}$

D 0.642 857 1

E $0.642\ 857\ 1$

d $\frac{10}{81}$ is equal to:

A 0.123 456 79

B $0.\overline{123\ 456\ 78}$

C $0.1\overline{23\ 456\ 78}$

D $0.\overline{123\ 456\ 79}$

E $0.123\ 456\ 790$

e $0.\overline{185}$ is equal to:

A $\frac{37}{200}$

B $\frac{2}{11}$

C $\frac{26}{135}$

D $\frac{5}{27}$

E $\frac{167}{900}$

5 Convert each of the following to a fraction in simplest form.

a 0.8

b 0.3

c 0.14

d 0.67

e 0.95

f 0.75

g 0.12

h 0.875

i 0.675

j 0.357

k 0.884

l 0.3625

WORKED
Example

5

6 Convert each of the following to a fraction in simplest form.

a $0.\dot{5}$

b $0.\dot{6}$

c $0.\overline{84}$

d $0.\overline{71}$

e $0.4\dot{6}$

f $0.\overline{18}$

g $0.1\dot{8}$

h $0.2\dot{7}$

i $0.36\dot{3}$

j $0.\overline{382}$

k $0.\overline{616}$

l $0.\overline{725}$

7 multiple choice

a 0.58 is equal to:

A $\frac{5}{8}$

B $\frac{58}{99}$

C $\frac{29}{50}$

D $\frac{58}{10}$

E $\frac{43}{90}$

b 0.0625 is equal to:

A $\frac{1}{16}$

B $\frac{5}{8}$

C $\frac{3}{50}$

D $\frac{625}{999}$

E $\frac{7}{11}$

c $0.\dot{3}2$ is equal to:

A $\frac{32}{99}$

B $\frac{29}{99}$

C $\frac{29}{90}$

D $\frac{16}{45}$

E $\frac{8}{25}$

d $0.\overline{90}$ is equal to:

A $\frac{9}{10}$

B $\frac{9}{11}$

C $\frac{91}{99}$

D $\frac{10}{11}$

E $\frac{44}{45}$

10 QUICK QUESTIONS 1

1 Simplify $\frac{64}{88}$.

2 Evaluate $1\frac{7}{8} + 2\frac{3}{7}$.

3 Evaluate $\frac{3}{8} \times \frac{2}{9} \times \frac{4}{5}$.

4 Evaluate $1\frac{11}{21} \div 2\frac{2}{7}$.

5 Evaluate $2\frac{1}{2} - \frac{3}{8} \times 1\frac{7}{9}$.

6 Write $\frac{13}{40}$ as a finite decimal.

7 Write $\frac{1}{6}$ as a recurring decimal.

8 Write 0.625 as a fraction in simplest form.

9 Write $0.\dot{7}$ as a fraction in simplest form.

10 Write $0.2\overline{56}$ as a fraction in simplest form.

Irrational numbers

Irrational numbers are those which cannot be expressed as fractions. These include

- (i) non-recurring, infinite decimals
- (ii) the special numbers π and e
- (iii) surds or roots of numbers that do not have a finite, exact answer; for example, $\sqrt{5}$ and $\sqrt[3]{6}$.

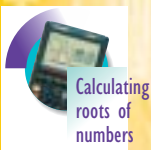
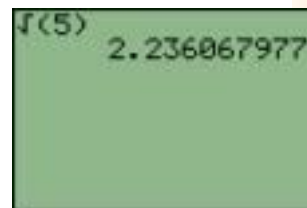
A surd is an exact answer but the calculator answer is an approximation because it has been rounded.



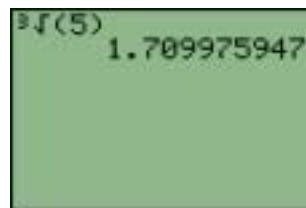
GraphicsCalculator tip!

Calculating roots of numbers

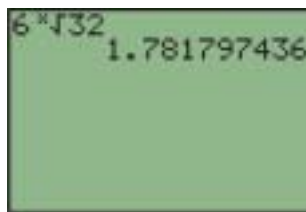
1. To find the square root of 5 on your graphics calculator press (2nd) [$\sqrt{}$], enter the number concerned (in this case, 5), close the brackets by pressing () (this is optional) and press (ENTER).



2. To find the cube root of 5 we need to use the **MATH** function. Press **(MATH)**, select **4: $\sqrt[3]{}$** , press **(5)**, close the brackets by pressing **()** (this is optional) and then **(ENTER)**.



3. To find higher order roots we again use the **MATH** function. To find the 6th root of 32, first enter **(6)** then press **(MATH)**, select **5: $\sqrt[x]{}$** , press **(32)** and then **(ENTER)**.



WORKED Example 6

State whether each of the following numbers is a surd or not.

a $\sqrt{3}$

b $\sqrt{0.49}$

c $\sqrt[3]{8}$

d $\sqrt[5]{15}$

THINK

- a** ① Write the number. Consider square roots which can be evaluated:
 $\sqrt{1} = 1$ and $\sqrt{4} = 2$.
 ② Check on a calculator if necessary then state whether the number is a surd or not.
- b** ① Write the number. Consider whether the number is a perfect square or not.
 ② Check with a calculator if necessary and then write the exact answer if there is one.
- c** Write the number. Consider whether the cube root can be found by cubing small numbers and write the exact answer if there is one. $1 \times 1 \times 1 = 1$; $2 \times 2 \times 2 = 8$
- d** Write the number. Consider whether the 5th root can be found and write the exact answer if there is one. $1^5 = 1$ is too small; $2^5 = 32$ is too big.

WRITE

a $\sqrt{3}$ is a surd.

b $\sqrt{0.49} = 0.7$ so $\sqrt{0.49}$ is not a surd.

c $\sqrt[3]{8} = 2$ so $\sqrt[3]{8}$ is not a surd.

d $\sqrt[5]{15}$ is a surd.

Rational approximations for surds

When an infinite decimal number is rounded, the answer is not exact, but it is very close to the actual value of the number. It is called a rational approximation because once it is rounded it becomes finite and is therefore rational.

WORKED Example 7

Find the value of $\sqrt{56}$ correct to 2 decimal places.

THINK

- 1 Write the surd and use a calculator to find the answer.
- 2 Round the answer to 2 decimal places by checking the 3rd decimal place.

WRITE

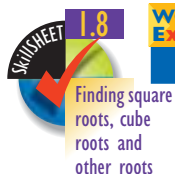
$$\sqrt{56} \approx 7.483\,314\,774$$

$$= 7.48 \text{ (2 decimal places)}$$

Exact answers are the most accurate and should be used in all working. Irrational numbers that are rounded are close approximations to their true values and should be used in the final answer only when asked for.

remember

1. Irrational numbers are those which cannot be expressed as fractions. These include:
 - (i) non-recurring, infinite decimals
 - (ii) the special numbers, π and e
 - (iii) surds.
2. A surd is an exact value. π and e are also exact values.
3. Rounded decimal answers to surd questions are only rational approximations.

EXERCISE**1C****Irrational numbers****WORKED Example 6**

- 1 State whether each of the following numbers is a surd or not.

a $\sqrt{7}$

b $\sqrt{100}$

c $\sqrt{9}$

d $\sqrt{74}$

e $\sqrt[5]{64}$

f $\sqrt[6]{216}$

g $\sqrt[3]{-1}$

h $\sqrt[4]{2401}$

i $\sqrt[7]{-2354}$

j 6

k $\sqrt[7]{8}$

l $\sqrt[16]{\frac{1}{25}}$

2 multiple choice

- a Which of the following is a surd?

A $\sqrt{28.09}$

B π

C $\sqrt{48.84}$

D 0.9875

E $\sqrt{0}$

- b Which of the following is not a surd?

A $\sqrt{65}$

B $\sqrt{56}$

C $\sqrt{46}$

D $\sqrt{64}$

E $\sqrt{101}$

- c Which of the following is a surd?

A $\sqrt{4.48}$

B 0.83

C $\sqrt{4.84}$

D $0.8\dot{3}$

E $\sqrt[4]{\frac{1}{4}}$

- d Which of the following is not a surd?

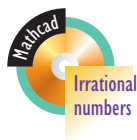
A $\sqrt{5.44}$

B $\sqrt[3]{82.511}$

C $\sqrt[4]{108.8844}$

D $\sqrt{0.9}$

E $\sqrt[5]{143.489\,07}$



e Which of the following numbers is irrational?

- A** a square root of a negative number
B a recurring decimal
C a fraction with a negative denominator
D a surd
E a finite decimal

3 Classify each of the following numbers as either rational or irrational.

- a** 5 **b** $\sqrt{5}$ **c** $\frac{1}{5}$ **d** 0.55
e $\sqrt{16}$ **f** 4.124 242 4 ... **g** $7\frac{4}{9}$ **h** $\sqrt[3]{8}$
i 5.0129 **j** $\sqrt[4]{15}$ **k** -60 **l** 2.714 365 ...

**WORKED
Example**

7

4 Find the value of each of the following, correct to 3 decimal places.

- a** $\sqrt{67}$ **b** $\sqrt{82}$ **c** $\sqrt{147}$ **d** $\sqrt{5.22}$
e $\sqrt{6.9}$ **f** $\sqrt{0.754}$ **g** $\sqrt{2534}$ **h** $\sqrt{1962}$
i $\sqrt{607.774}$ **j** $\sqrt{8935.0725}$ **k** $\sqrt{12.065}$ **l** $\sqrt{355.169}$

5 Find approximate answers to each of the following surds, rounded to 4 significant figures.

- a** $\sqrt[3]{23}$ **b** $\sqrt[3]{-895}$ **c** $\sqrt[5]{1048}$ **d** $\sqrt[6]{45\,867}$ **e** $\sqrt[4]{654.8}$
f $\sqrt[8]{1.5}$ **g** $\sqrt[3]{2.8856}$ **h** $\sqrt[9]{-54\,988}$ **i** $\sqrt[5]{-84.848\,484}$ **j** $\sqrt[4]{0.7882}$

6 Calculate each of the following, correct to the nearest whole number.

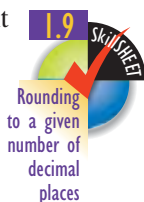
- a** $\sqrt{546}$ **b** $\sqrt[4]{54\,637}$ **c** $\sqrt[5]{697\,643}$ **d** $\sqrt[3]{-2116}$ **e** $\sqrt[7]{8\,564\,943}$

7 **multiple choice**

- a** $\sqrt{43.403}$ correct to 4 decimal places is:
A 6.5881 **B** 6.5880 **C** 6.5889 **D** 6.5888 **E** 6.589
b $\sqrt{65 - 55} + 25$, rounded to 3 decimal places is:
A 5.916 **B** -21.938 **C** 28.162 **D** 25.646 **E** 15
c $\sqrt{56} \times \sqrt{68 - 42} \div 8$ rounded to 2 decimal places is:
A 59.42 **B** 61.67 **C** 494.02 **D** 59.28 **E** 61.66
d $\sqrt[3]{456} \times \sqrt[5]{-456} - \sqrt[4]{456}$ rounded to the nearest whole number is:
A 5 **B** 22 **C** 31 **D** -22 **E** -31
e $\frac{\sqrt{56.6 + 65.5}}{\sqrt{56.6} - \sqrt{65.5}}$ rounded to 3 decimal places is:
A 49.583 **B** -19.389 **C** -6.624 **D** -27.402 **E** 5.236

8 Calculate each of the following, correct to 2 decimal places:

- a** $\sqrt{67} + \sqrt{54 \times 43}$ **b** $\sqrt[3]{768} - \sqrt{564} + \sqrt[4]{684}$
c $\sqrt{8.3 - 5.7} \times \sqrt{8.3 - 5.7}$ **d** $\sqrt{5.86 \div 8.64} \div \sqrt[3]{4.23}$
e $\frac{6.7 \times \sqrt{4.9}}{\sqrt{6.7} \div 4.9}$ **f** $\frac{\sqrt{58.8 - 21.7}}{\sqrt{58.8} - \sqrt{21.7}}$



- 9 Rali's solution to the equation $3x = 13$ is $x = 4.33$, while Tig writes his answer as $x = 4\frac{1}{3}$. When Rali is marked wrong and Tig marked right by their teacher, Rali complains.

a Do you think the teacher is right or wrong?

The teacher then asks the two students to compare the decimal and fractional parts of the answer.

b Write Rali's decimal remainder as a fraction.

c Find the difference between the two fractions.

d Multiply Rali's fraction by 120 000 and multiply Tig's fraction by 120 000.

e Find the difference between the two answers.

f Compare the difference between the two fractions from part **c** and the difference between the two amounts in part **d**. Comment.

- 10 Takako is building a corner cupboard to go in her bedroom and she wants it to be 10 cm along each wall.

a Use Pythagoras' theorem to find the exact length of timber required to complete the triangle.

b Find a rational approximation for the length, rounding your answer to the nearest millimetre.

- 11 Phillip uses a ladder which is 5 metres long to reach his bedroom window. He cannot put the foot of the ladder in the garden bed, which is 1 metre wide. If the ladder just reaches the window, how high above the ground is Phillip's window?

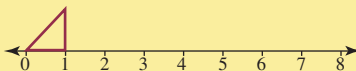


THINKING

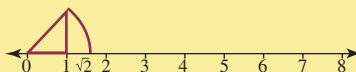
Plotting irrational numbers on the number line

We know that it is possible to find the exact square root of some numbers, but not others. For example, we can find $\sqrt{4}$ exactly but not $\sqrt{3}$ or $\sqrt{5}$. Our calculator can find a decimal approximation of these, but because they cannot be found exactly they are called irrational numbers. There is a method, however, of showing their exact location on a number line.

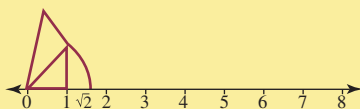
- 1 Using graph paper draw a right-angled triangle with two equal sides of length 1 cm as shown below.



- 2 Using Pythagoras' theorem, the length of the hypotenuse of this triangle is $\sqrt{2}$ units. Use a pair of compasses to make an arc that will show the location of $\sqrt{2}$ on the number line.



- 3 Draw another right triangle using the hypotenuse of the first triangle as one side and make the other side 1 cm in length.



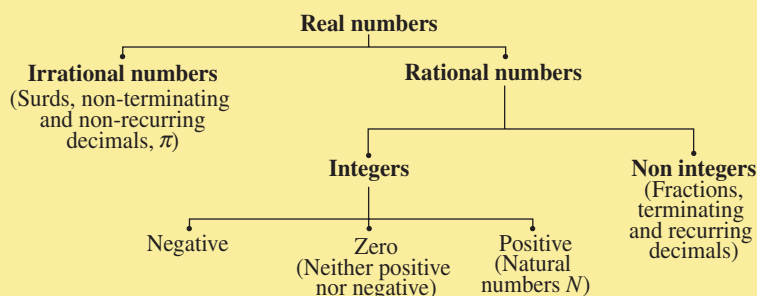
- 4 The hypotenuse of this triangle will have a length of $\sqrt{3}$ units. Draw an arc to find the location of $\sqrt{3}$ on the number line.
- 5 Repeat steps 3 and 4 to draw triangles that will have sides of length $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$ units, etc.



THINKING

Where do I belong?

The real number system can be divided into two distinct sets — rational numbers and irrational numbers. Rational numbers are those which can be written as the ratio of two integers. Irrational numbers cannot be written as the ratio of two integers. Included in this set are surds, non-terminating and non-recurring decimals, and symbols such as π . The rational numbers can be divided into the subsets of integers and non-integers. Further division of the integer set gives subsets of negative integers, positive integers (natural numbers) and zero. The relationship between these sets is illustrated in the chart below.



A Venn diagram can also be used to show the relationship between the sets. Consider the diagram at right.

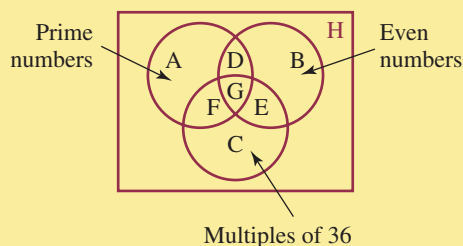
The circle labelled 'Prime numbers' contains all the prime numbers.

The circle labelled 'Even numbers' contains all the even numbers.

The circle labelled 'Multiples of 36' contains all the multiples of 36.

The region A contains all the prime numbers that are neither even nor multiples of 36.

The region D contains all the prime numbers that are even.



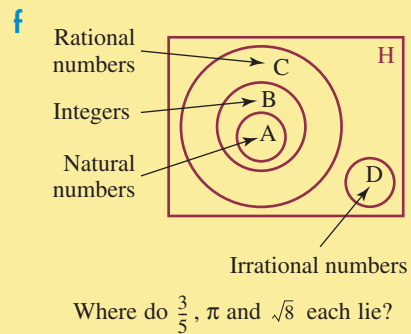
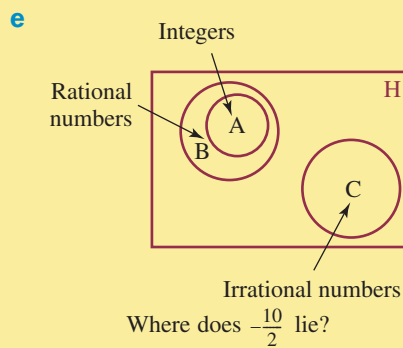
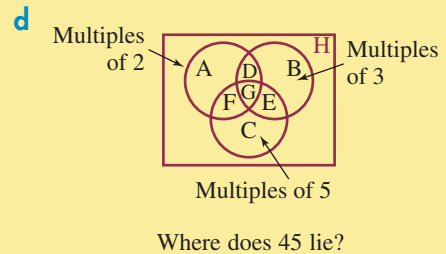
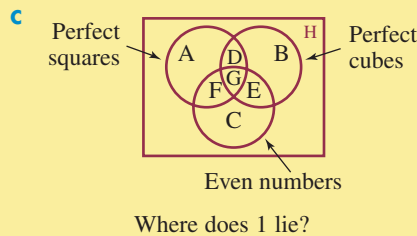
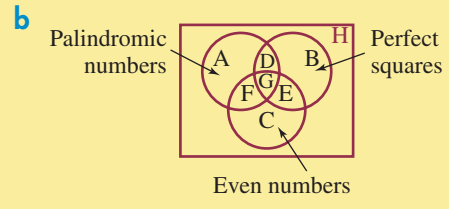
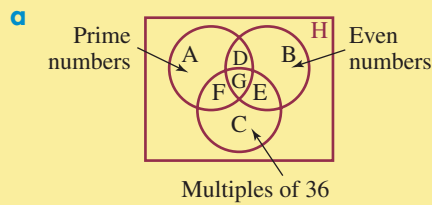
(Continued)

The region G contains all the prime numbers that are even and multiples of 36.
The region H contains all the numbers that are not prime, not even and non-multiples of 36.

Similar reasoning would define all the other regions.

The size of the circle or the size of the overlapping region does not represent the number of entries in the region.

- 1 Consider each of the following Venn diagrams and indicate the region in which the specified number would lie.



- 2 Sets can be classed as discrete or continuous. Discrete sets are those that have discrete elements; that is, the elements can assume countable values. Continuous sets are those whose elements are continuous; that is, the elements can assume all possible values in a given interval. Classify the following sets as discrete or continuous.

a {Natural numbers}

b {Integers}

c {Rational numbers}

d {Irrational numbers}

- 3** A finite set is one with a fixed countable number of elements (even though this number may be very large). An infinite set contains an infinite number of elements. Classify the following sets as finite or infinite.

a {Positive integers}

b $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi\right\}$

c {The first ten multiples of 8}

d {Irrational numbers greater than 10}

- 4** Consider the following elements from the set of real numbers:

$\left\{\sqrt{2}, \frac{2}{3}, 0.875, 6\frac{4}{7}, 2\pi, 4\sqrt{3}, 0.43528\dots, \frac{11}{13}, 0.\bar{4}, \frac{\pi}{4}, 6, 0, -\sqrt{144}, -6, \sqrt{6}\right\}.$

a Indicate the set in which each element belongs.

b Place the elements in increasing order of magnitude.

- 5** State whether the following combination of sets is possible and give an example of each.

a Discrete and finite

b Discrete and infinite

c Continuous and finite

d Continuous and infinite



COMMUNICATION

The Golden Ratio

Some shapes appear more pleasing to the eye than others. Artists and designers take advantage of these proportions in paintings, buildings and a variety of objects. One of the most pleasing proportions is that of the Golden Ratio which is seen in many rectangular shapes. Let us construct a Golden Rectangle and investigate its properties.

- 1** Draw a 2 cm square ABCD.
- 2** Mark the midpoint of AB as the point E. Join EC. Triangle BCE is right-angled. What are the exact lengths of EB, BC and CE? Leave your answer in surd form, if necessary.
- 3** Use a pair of compasses with centre at E and radius EC to draw an arc cutting AB extended at the point F.
- 4** Complete the rectangle AFGD.
- 5** Write down the exact lengths (in surd form) of the line segments AF and BF.
- 6** Calculate the ratios AF : AD and GF : BF, correct to 2 decimal places.
- 7** The rectangles AFGD and BFGC are Golden Rectangles. What is the ratio of the longer to the shorter side in each case? This is known as the Golden Ratio.



- 1 Find three numbers, w , x and y , none of which are perfect squares or zero and that make the following relationship true.

$$\sqrt{w} + \sqrt{x} = \sqrt{y}$$

- 2 a If $13^2 = 169$, $133^2 = 17\,689$ and $1333^2 = 1\,776\,889$, write down the answer to $13\,333^2$ without using a calculator or computer.
 b If $19^2 = 361$, $199^2 = 39\,601$ and $1999^2 = 3\,996\,001$, write down the answer to $19\,999^2$ without using a calculator or computer.
 c Can you find another number between 13 and 19 where a similar pattern can be used?

Simplifying surds

Some surds, like some fractions, can be reduced to simplest form.

Only square roots will be considered in this section.

Consider: $\sqrt{36} = 6$

Now, $36 = 9 \times 4$, so we could say:

$$\sqrt{9 \times 4} = 6$$

Taking $\sqrt{9}$ and $\sqrt{4}$ separately:

$$\sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$$

If both $\sqrt{9 \times 4} = 6$ and $\sqrt{9} \times \sqrt{4} = 6$, then $\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4}$.

This property can be stated as: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and can be used to simplify surds.

$$\sqrt{8} = \sqrt{4 \times 2}$$

$$= \sqrt{4} \times \sqrt{2}$$

$$= 2 \times \sqrt{2} \text{ which can be written as } 2\sqrt{2}.$$

A surd can be simplified by dividing it into two square roots, one of which is the highest perfect square that will divide evenly into the original number.

WORKED Example 8

Simplify each of the following.

a $\sqrt{40}$

THINK

- a 1 Write the surd and divide it into two parts, one being the highest perfect square that will divide into the surd.
 2 Write in simplest form by taking the square root of the perfect square.

b $\sqrt{72}$

WRITE

$$\begin{aligned} \text{a } \sqrt{40} &= \sqrt{4 \times 10} \\ &= \sqrt{4} \times \sqrt{10} \\ &= 2\sqrt{10} \end{aligned}$$

THINK

- b** ① Write the surd and divide it into two parts, one being the highest perfect square that will divide into the surd.
- ② Write in simplest form by taking the square root of the perfect square.

WRITE

$$\begin{aligned}
 \text{b } \sqrt{72} &= \sqrt{36 \times 2} \\
 &= \sqrt{36} \times \sqrt{2} \\
 &= 6\sqrt{2}
 \end{aligned}$$

If a smaller perfect square is chosen the first time, the surd can be simplified in more than one step.

$$\begin{aligned}
 \sqrt{72} &= \sqrt{4} \times \sqrt{18} \\
 &= 2\sqrt{18} \\
 &= 2 \times \sqrt{9} \times \sqrt{2} \\
 &= 2 \times 3\sqrt{2} \\
 &= 6\sqrt{2}
 \end{aligned}$$

This is the same answer as found in worked example 8(b) but an extra step is included. When dividing surds into two parts, it is critical that one is a perfect square.

For example, $\sqrt{72} = \sqrt{24} \times \sqrt{3}$ is of no use because an exact square root cannot be found for either part of the answer.

WORKED Example 9

Simplify $6\sqrt{20}$.

THINK

- ① Write the expression and then divide the surd into two parts, where one square root is a perfect square.
- ② Evaluate the part which is a perfect square.
- ③ Multiply the whole numbers and write the answer in simplest form.

WRITE

$$\begin{aligned}
 6\sqrt{20} &= 6 \times \sqrt{4 \times 5} \\
 &= 6 \times \sqrt{4} \times \sqrt{5} \\
 &= 6 \times 2\sqrt{5} \\
 &= 12\sqrt{5}
 \end{aligned}$$

Sometimes it is necessary to change a simplified surd to a whole surd. The reverse process is applied here where the rational part is squared before being placed back under the square root sign.

WORKED Example 10

Write $5\sqrt{3}$ in the form $\sqrt{a}$.

THINK

- ① Write the expression as a product of an integer and a surd.

WRITE

$$5\sqrt{3} = 5 \times \sqrt{3}$$

Continued over page 

THINK

- 2 Square the whole number part, then express the whole number as a square root.
- 3 Write the simplified surd and express it as the product of 2 square roots, one of which is the square root in step 2.
- 4 Multiply the square roots to give a single surd.

WRITE

$$= \sqrt{5^2} \times \sqrt{3}$$

$$= \sqrt{25} \times \sqrt{3}$$

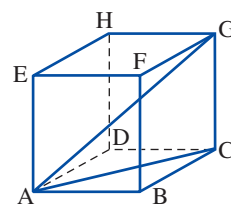
$$= \sqrt{25 \times 3}$$

$$= \sqrt{75}$$

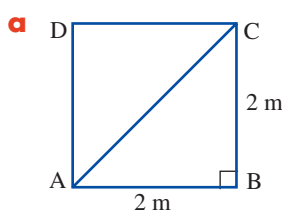
WORKED Example 11

Ms Jennings plans to have a climbing frame that is in the shape of a large cube with sides 2 metres long built in the school playground.

- a Find the length of material required to join the opposite vertices of the face which is on the ground.
- b Find the exact length of material required to strengthen the frame by joining a vertex on the ground to the vertex which is in the air and which is furthest away.
- c Find an approximate answer rounded to the nearest cm.

**THINK**

- a 1 Draw a diagram of the face, mark in the diagonal, the appropriate measurements and label the vertices.

WRITE

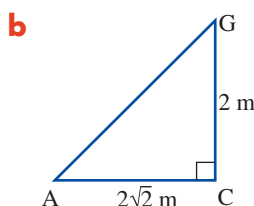
- 2 Use Pythagoras' theorem to find the length of the diagonal.

$$\begin{aligned}\overline{AC}^2 &= \overline{AB}^2 + \overline{BC}^2 \\ &= 2^2 + 2^2 \\ &= 8 \\ \overline{AC} &= \sqrt{8} \\ &= 2\sqrt{2}\end{aligned}$$

- 3 Answer the question in a sentence.

$2\sqrt{2}$ metres of material is required.

- b 1 Draw a diagram of the triangle required, label the vertices and mark in the appropriate measurements.



THINK

- 2 Use Pythagoras' theorem to find the length of the diagonal.

- 3 Simplify the surd.

- 4 Write your answer in a sentence.

- c Round the answer to 2 decimal places.

WRITE

$$\begin{aligned}\overline{AG}^2 &= \overline{CG}^2 + \overline{AC}^2 \\ &= 2^2 + (2\sqrt{2})^2 \\ &= 12\end{aligned}$$

$$\overline{AG} = \sqrt{12}$$

$$= 2\sqrt{3}$$

The length of material required is $2\sqrt{3}$ metres.

- c The approximate length to the nearest cm is 3.50 metres.

remember

1. To simplify a surd, divide it into two square roots, one of which is a perfect square.
2. Not all surds can be simplified.
3. $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
4. Some perfect squares to learn are: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 ...

EXERCISE 1D**Simplifying surds****WORKED Example 8**

- 1 Simplify each of the following.

a $\sqrt{20}$

b $\sqrt{8}$

c $\sqrt{18}$

d $\sqrt{49}$

e $\sqrt{30}$

f $\sqrt{50}$

g $\sqrt{28}$

h $\sqrt{108}$

i $\sqrt{288}$

j $\sqrt{48}$

k $\sqrt{500}$

l $\sqrt{162}$

m $\sqrt{52}$

n $\sqrt{55}$

o $\sqrt{84}$

p $\sqrt{98}$

q $\sqrt{363}$

r $\sqrt{343}$

s $\sqrt{78}$

t $\sqrt{160}$

WORKED Example 9

- 2 Simplify each of the following.

a $2\sqrt{8}$

b $5\sqrt{27}$

c $6\sqrt{64}$

d $7\sqrt{50}$

e $10\sqrt{24}$

f $5\sqrt{12}$

g $4\sqrt{42}$

h $12\sqrt{72}$

i $9\sqrt{45}$

j $12\sqrt{242}$

WORKED Example 10

- 3 Write each of the following in the form $\sqrt{a}$.

a $2\sqrt{3}$

b $5\sqrt{7}$

c $6\sqrt{3}$

d $4\sqrt{5}$

e $8\sqrt{6}$

f $3\sqrt{10}$

g $4\sqrt{2}$

h $12\sqrt{5}$

i $10\sqrt{6}$

j $13\sqrt{2}$



4 multiple choice

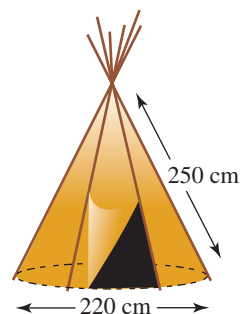
- a $\sqrt{1000}$ is equal to:
 A 31.6228 B $50\sqrt{2}$ C $50\sqrt{10}$
 D $10\sqrt{10}$ E $100\sqrt{10}$
- b $\sqrt{80}$ in simplest form is equal to:
 A $4\sqrt{5}$ B $2\sqrt{20}$ C $8\sqrt{10}$
 D $5\sqrt{16}$ E 10
- c Which of the following surds is in simplest form?
 A $\sqrt{60}$ B $\sqrt{147}$ C $\sqrt{105}$
 D $\sqrt{117}$ E $\sqrt{432}$
- d Which of the following surds is not in simplest form?
 A $\sqrt{102}$ B $\sqrt{110}$ C $\sqrt{116}$
 D $\sqrt{118}$ E $\sqrt{122}$
- e $6\sqrt{5}$ is equal to:
 A $\sqrt{900}$ B $\sqrt{30}$ C $\sqrt{150}$
 D $\sqrt{180}$ E 13.42
- f Which one of the following is not equal to the rest?
 A $\sqrt{128}$ B $2\sqrt{32}$ C $8\sqrt{2}$
 D $4\sqrt{8}$ E $64\sqrt{2}$
- g Which one of the following is not equal to the rest?
 A $4\sqrt{4}$ B $2\sqrt{16}$ C 8
 D 16 E $\sqrt{64}$
- h $5\sqrt{48}$ is equal to:
 A $80\sqrt{3}$ B $20\sqrt{3}$ C $9\sqrt{3}$
 D $21\sqrt{3}$ E $15\sqrt{16}$

5 Challenge: Reduce each of the following to simplest form.

- a $\sqrt{675}$ b $\sqrt{1805}$ c $\sqrt{1792}$ d $\sqrt{578}$
 e $\sqrt{a^2c}$ f $\sqrt{bd^4}$ g $\sqrt{h^2jk^2}$ h $\sqrt{f^3}$

**WORKED
Example**
 11

- 6 A large die with sides measuring 3 metres is to be placed in front of the casino at Crib Point. The die is placed on one of its vertices with the opposite vertex directly above it.
 a Find the length of the diagonal of one of the faces.
 b Find the exact height of the die.
 c Find the difference between the height of the die and the height of a 12-metre wall directly behind it. Approximate the answer to 3 decimal places.
- 7 A tent in the shape of a tepee is being used as a cubby house. The diameter of the base is 220 cm and the slant height is 250 cm.
 a How high is the tepee? Write the answer in simplest surd form.
 b Find an approximation for the height of the tepee in centimetres, rounding the answer to the nearest centimetre.



Career profile

PETER RICHARDSON — Analyst Programmer



Qualifications:

Bachelor of Applied Science (Computer Science and Software Engineering)

I entered this field as a change of career and find it to be interesting and diverse.

I use basic mathematical skills throughout the day to calculate screen heights and check whether all necessary fields and labels will fit. More advanced mathematics, such as working with formulas and other secondary school mathematics, are used in Excel spreadsheets for statistics and data manipulation.

During a typical day, all my work is done on computer, usually using a software package to write code in Java or Cobol. I create screens for use by clients, and the supporting code to ensure screens react as expected.

Questions

1. What computer language does Peter use to write code?
2. Name one aspect of Peter's job.
3. Find out what courses are available to become an analyst programmer.



COMMUNICATION

Braking distances

At the start of the chapter, a formula was given to calculate the speed of a car before the brakes are applied to bring it to a stop in an emergency. The formula given was $v = \sqrt{20d}$ where v is the speed in m/s and d is the braking distance in m.

- 1 What is the speed of a car before braking if the braking distance is 32.50 m?
- 2 Explain why your answer to part 1 is an irrational number.
- 3 State your answer to part 1 as an exact irrational number in simplest form and as a rational approximation.
- 4 Convert the speed from m/s to km/h.
- 5 Calculate the speed of a car before braking if the braking distance is 31.25 m.
- 6 Is your answer to part 5 rational or irrational?
- 7 State your answer to part 5 in km/h. Is this number rational or irrational?

(Continued)

The effect of speed

Research using data from actual road crashes has estimated the relative risk for cars travelling at or above 60 km/h becoming involved in a casualty crash (a car crash in which people are killed or hospitalised). It was found that the risk doubled for every 5 km/h above 60 km/h. So a car travelling at 65 km/h was twice as likely to be involved in a casualty crash as one travelling at 60, while the risk for a car travelling at 70 km/h was four times as great.

We will consider two elements which affect the distance travelled by a car after the driver has perceived danger — the reaction time of the driver and the braking distance of the car.

Let's consider the total distance travelled to bring cars travelling at different speeds to a stop after the driver first perceives danger. Assume a reaction time of 1.5 seconds. (This means that the car continues to travel at the same speed for 1.5 s until the brakes are applied.)

- 8 Complete the following table. (Remember to convert speed in km/h to m/s before substituting into a formula to find the distance in m.)

Speed		Distance travelled to bring a car to a complete stop (metres)		
km/h	m/s	Reaction distance	Braking distance	Total stopping distance
60				
65				
70				

- 9 Compare the difference between the total stopping distance travelled at each of the given speeds.
- 10 Give an example to explain how the difference between these stopping distances could literally mean the difference between life and death.
- 11 What other factors could affect the stopping distance of a car?

Addition and subtraction of surds

Operations with surds have the same rules as operations in algebra.

- Like surds are those which contain the same surd when written in simplest form.
- Like surds can be added or subtracted after they have been written in simplest form.

WORKED Example 12

Simplify each of the following. **a** $6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5}$ **b** $3\sqrt{2} - 5 + 4\sqrt{2} + 9$

THINK

- a** 1 Write the expression.
- 2 All surds are in simplest form, so collect like surds.

WRITE

$$\begin{aligned} \mathbf{a} \quad & 6\sqrt{3} + 2\sqrt{3} + 4\sqrt{5} - 5\sqrt{5} \\ & = 8\sqrt{3} - \sqrt{5} \end{aligned}$$

THINK

- b** ① Write the expression.
 ② All surds are in simplest form, so collect like terms.

WRITE

$$\begin{aligned} \mathbf{b} \quad & 3\sqrt{2} - 5 + 4\sqrt{2} + 9 \\ & 3\sqrt{2} + 4\sqrt{2} - 5 + 9 \\ & = 7\sqrt{2} + 4 \end{aligned}$$

We need to check that all surds are fully simplified before we can be sure whether or not they can be added or subtracted as like terms.

WORKED Example 13

Simplify $5\sqrt{75} - 6\sqrt{12} + 2\sqrt{8} + 4\sqrt{3}$.

THINK

- ① Write the expression.
 ② Simplify all surds.
 ③ Collect like surds.

WRITE

$$\begin{aligned} & 5\sqrt{75} - 6\sqrt{12} + 2\sqrt{8} + 4\sqrt{3} \\ & = (5 \times \sqrt{25} \times \sqrt{3}) - (6 \times \sqrt{4} \times \sqrt{3}) + (2 \times \sqrt{4} \times \sqrt{2}) + 4\sqrt{3} \\ & = 25\sqrt{3} - 12\sqrt{3} + 4\sqrt{2} + 4\sqrt{3} \\ & = 17\sqrt{3} + 4\sqrt{2} \end{aligned}$$

remember

- Only like surds can be added or subtracted.
- All surds must be written in simplest form before adding or subtracting.

EXERCISE 1E**Addition and subtraction of surds****WORKED Example 12**

- ① Simplify each of the following.

a $6\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

c $-3\sqrt{3} - 7\sqrt{3} + 4\sqrt{3}$

e $10\sqrt{11} - 6\sqrt{11} + \sqrt{11}$

g $4\sqrt{2} + 6\sqrt{2} + 5\sqrt{3} + 2\sqrt{3}$

i $5\sqrt{10} + 2\sqrt{3} + 3\sqrt{10} + 5\sqrt{3}$

k $6\sqrt{6} + \sqrt{2} - 4\sqrt{6} - \sqrt{2}$

m $10\sqrt{7} - 4 - 2\sqrt{7} - 7$

o $\sqrt{13} + 4\sqrt{7} - 2\sqrt{13} - 3\sqrt{7}$

q $5\sqrt{2} + \sqrt{7} - 3\sqrt{7} - 4\sqrt{7}$

b $4\sqrt{5} - 6\sqrt{5} - 2\sqrt{5}$

d $-9\sqrt{6} + 6\sqrt{6} + 3\sqrt{6}$

f $\sqrt{7} + \sqrt{7}$

h $10\sqrt{5} - 2\sqrt{5} + 8\sqrt{6} - 7\sqrt{6}$

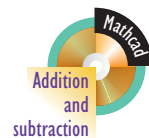
j $12\sqrt{2} - 3\sqrt{5} + 4\sqrt{2} - 8\sqrt{5}$

l $16\sqrt{5} + 8 + 7 - 11\sqrt{5}$

n $6 + 2\sqrt{2} + 5 - 3\sqrt{2}$

p $8\sqrt{6} - 4\sqrt{3} + 2\sqrt{6} - 7\sqrt{6}$

r $1 + \sqrt{5} - \sqrt{5} + 1$



**WORKED
Example****13****2** Simplify each of the following.

a $\sqrt{8} + \sqrt{18} - \sqrt{32}$

c $-\sqrt{12} + \sqrt{75} - \sqrt{192}$

e $\sqrt{24} + \sqrt{180} + \sqrt{54}$

g $2\sqrt{24} + 3\sqrt{20} - 7\sqrt{8}$

i $6\sqrt{44} + 4\sqrt{120} - \sqrt{99} - 3\sqrt{270}$

k $\sqrt{98} + 3\sqrt{147} - 8\sqrt{18} + 6\sqrt{192}$

m $5\sqrt{81} - 4\sqrt{162} + 6\sqrt{16} - \sqrt{450}$

b $\sqrt{45} - \sqrt{80} + \sqrt{5}$

d $\sqrt{7} + \sqrt{28} - \sqrt{343}$

f $\sqrt{12} + \sqrt{20} - \sqrt{125}$

h $3\sqrt{45} + 2\sqrt{12} + 5\sqrt{80} + 3\sqrt{108}$

j $2\sqrt{32} - 5\sqrt{45} - 4\sqrt{180} + 10\sqrt{8}$

l $2\sqrt{250} + 5\sqrt{200} - \sqrt{128} + 4\sqrt{40}$

n $\sqrt{108} + \sqrt{125} - 3\sqrt{8} + 9\sqrt{80}$

3 multiple choice**a** $\sqrt{2} + 6\sqrt{3} - 5\sqrt{2} - 4\sqrt{3}$ is equal to:

A $-5\sqrt{2} + 2\sqrt{3}$

B $-3\sqrt{2} + 23$

C $6\sqrt{2} + 2\sqrt{3}$

D $-4\sqrt{2} + 2\sqrt{3}$

E -3

b $6 - 5\sqrt{6} + 4\sqrt{6} - 8$ is equal to:

A $-2 - \sqrt{6}$

B $14 - \sqrt{6}$

C $-2 + \sqrt{6}$

D $-2 - 9\sqrt{6}$

E $14 + \sqrt{6}$

c $4\sqrt{8} - 6\sqrt{12} - 7\sqrt{18} + 2\sqrt{27}$ is equal to:

A $-7\sqrt{5}$

B $29\sqrt{2} - 18\sqrt{3}$

C $-13\sqrt{2} - 6\sqrt{3}$

D $-13\sqrt{2} + 6\sqrt{3}$

E cannot be simplified

d $2\sqrt{20} + 5\sqrt{24} - \sqrt{54} + 5\sqrt{45}$ is equal to:

A $19\sqrt{5} + 7\sqrt{6}$

B $9\sqrt{5} - 7\sqrt{6}$

C $-11\sqrt{5} + 7\sqrt{6}$

D $-11\sqrt{5} - 7\sqrt{6}$

E $12\sqrt{35}$

4 Elizabeth wants narrow wooden frames for three different-sized photographs, the smallest frame measuring 2×2 cm, the second 3×3 cm and the largest 4×6 cm. If each frame is made up of four pieces of timber to go around the edge of the photograph and one diagonal support, how much timber is needed to make the three frames? Give your answer in simplest surd form.

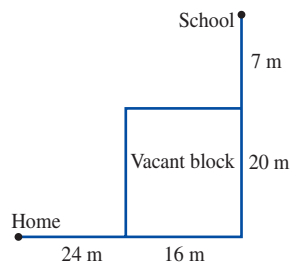
5 Harry and William walk to school each day. If the ground is not wet and boggy they can cut across a vacant block, otherwise they must stay on the paths.

a Find the distance that they walk when it is wet and they follow the path.

b Find the distance that they walk on a fine day when they follow the shortest path across the vacant block. Give your answer in simplest surd form.

c Exactly how much further do they walk when it is a wet day?

d Approximately how much further do they walk when it is a wet day?





When 3 people fell in the water, why did only 2 of them get their hair wet?

The answer to each question and the letter beside it give the puzzle answer code.

$$A = 3\sqrt{5} + \sqrt{5}$$

$$=$$

$$B = 5\sqrt{2} + 3\sqrt{2}$$

$$=$$

$$C = \sqrt{3} + \sqrt{3}$$

$$=$$

$$D = 2\sqrt{6} + 3\sqrt{6}$$

$$=$$

$$E = 2\sqrt{7} + \sqrt{7}$$

$$=$$

$$A = \sqrt{8} + 3\sqrt{2}$$

$$=$$

$$F = 3\sqrt{3} + \sqrt{12}$$

$$=$$

$$E = 6\sqrt{7} - 4\sqrt{7}$$

$$=$$

$$H = 3\sqrt{6} - 2\sqrt{6}$$

$$=$$

$$L = 7\sqrt{5} - 5\sqrt{5}$$

$$=$$

$$E = 10\sqrt{3} - 4\sqrt{3}$$

$$=$$

$$D = \sqrt{18} - 2\sqrt{2}$$

$$=$$

$$M = 7\sqrt{6} - \sqrt{54}$$

$$=$$

$$N = \sqrt{45} - \sqrt{20}$$

$$=$$

$$O = 6\sqrt{7} - \sqrt{28}$$

$$=$$

$$A = 5\sqrt{2} + 3\sqrt{3} - \sqrt{2}$$

$$=$$

$$S = \sqrt{5} + 3\sqrt{5} - \sqrt{3}$$

$$=$$

$$T = \sqrt{8} + \sqrt{18} - \sqrt{2}$$

$$=$$

$$U = \sqrt{12} - \sqrt{32} + 6\sqrt{2}$$

$$=$$

$$E = \sqrt{50} + \sqrt{27} - 5\sqrt{2}$$

$$=$$

$$W = \sqrt{3} + \sqrt{4} + \sqrt{5}$$

$$=$$

$$A = \sqrt{48} - 2\sqrt{3} + \sqrt{20}$$

$$=$$

$$B = \sqrt{108} - 5\sqrt{3}$$

$$=$$

$$D = \sqrt{2} + 2\sqrt{5} + 3\sqrt{2} - \sqrt{5}$$

$$=$$

$$E = \sqrt{200} - \sqrt{147}$$

$$=$$

$$H = 5 - 2\sqrt{2} + \sqrt{9}$$

$$=$$

$$O = \sqrt{75} + 4\sqrt{5} + \sqrt{12}$$

$$=$$

$$S = 8\sqrt{5} - \sqrt{45} - \sqrt{20}$$

$$=$$

$$E = \sqrt{150} + 2\sqrt{6} - \sqrt{96}$$

$$=$$

$$\sqrt{3}$$

$$3\sqrt{6}$$

$$2\sqrt{3}$$

$$4\sqrt{2} + 3\sqrt{3}$$

$$2\sqrt{2} + 2\sqrt{3}$$

$$4\sqrt{5} - \sqrt{3}$$

$$3\sqrt{7}$$

$$7\sqrt{3} + 4\sqrt{5}$$

$$\sqrt{5}$$

$$6\sqrt{3}$$

$$4\sqrt{7}$$

$$5\sqrt{3}$$

$$4\sqrt{2}$$

$$8 - 2\sqrt{2}$$

$$2\sqrt{7}$$

$$4\sqrt{6}$$

$$2 + \sqrt{3} + \sqrt{5}$$

$$4\sqrt{5}$$

$$3\sqrt{5}$$

$$8\sqrt{2}$$

$$2\sqrt{3} + 2\sqrt{5}$$

$$2\sqrt{5}$$

$$4\sqrt{2} + \sqrt{5}$$

$$\sqrt{6}$$

$$10\sqrt{2} - 7\sqrt{3}$$

$$5\sqrt{2}$$

$$5\sqrt{6}$$

$$3\sqrt{3}$$

$$\sqrt{2}$$

Multiplication and division of surds

Surds can be multiplied and divided in the same way as **pronumerals** are in algebra.

The multiplication rule, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, was used in the form $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ when simplifying surds.

This rule can be extended to: $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$.

The division rule is $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$.

An example of this is: $\frac{\sqrt{36}}{\sqrt{9}} = \frac{6}{3}$ while $\sqrt{\frac{36}{9}} = \sqrt{4}$
 $= 2$ $= 2$
 so $\frac{\sqrt{36}}{\sqrt{9}} = \sqrt{\frac{36}{9}}$

All answers should be written in simplest form.

WORKED Example 14

Simplify each of the following.

a $\sqrt{3} \times \sqrt{6}$

b $\sqrt{7} \times \sqrt{7}$

c $-4\sqrt{5} \times 7\sqrt{6}$

THINK

- a** ① Write the expression and multiply the surds.
 ② Simplify if appropriate.
- b** ① Write the expression and multiply the surds.
 ② Simplify if appropriate.
 (Note that $\sqrt{a} \times \sqrt{a} = a$, so the answer could have been found in one step.)

WRITE

a $\sqrt{3} \times \sqrt{6} = \sqrt{18}$

$$= \sqrt{9} \times \sqrt{2}$$

$$= 3\sqrt{2}$$

b $\sqrt{7} \times \sqrt{7} = \sqrt{49}$

$$= 7$$

- c** ① Write the expression, multiply whole numbers and multiply the surds.
 ② Simplify if appropriate.

c $-4\sqrt{5} \times 7\sqrt{6} = -4 \times 7 \times \sqrt{5} \times \sqrt{6}$
 $= -28\sqrt{30}$

When dividing surds, it is easier if both the numerator and denominator are simplified before dividing. If this is done we can then simplify the fraction formed by the rational and irrational parts separately.

WORKED Example 15Simplify each of the following. **a** $\frac{\sqrt{40}}{\sqrt{2}}$

b $\frac{\sqrt{60}}{2}$

c $\frac{16\sqrt{15}}{24\sqrt{75}}$

THINK

- a** 1 Write the expression and simplify the numerator.
- 2 Write the surds under the one square root sign and divide.
- b** 1 Write the expression and simplify the numerator.
- 2 Divide numerator and denominator by 2, which is the common factor.
- c** 1 Write the expression and simplify the denominator. (The numerator is already fully simplified.)
- 2 Simplify the fraction formed by the rational and irrational parts separately.

WRITE

$$\begin{aligned}\mathbf{a} \quad \frac{\sqrt{40}}{\sqrt{2}} &= \frac{2\sqrt{10}}{\sqrt{2}} \\ &= 2\sqrt{\frac{10}{2}} \\ &= 2\sqrt{5}\end{aligned}$$

$$\begin{aligned}\mathbf{b} \quad \frac{\sqrt{60}}{2} &= \frac{2\sqrt{15}}{2} \\ &= \sqrt{15}\end{aligned}$$

$$\begin{aligned}\mathbf{c} \quad \frac{16\sqrt{15}}{24\sqrt{75}} &= \frac{16\sqrt{15}}{120\sqrt{3}} \\ &= \frac{16\sqrt{15}}{24 \times 5\sqrt{3}} \\ &= \frac{2}{15} \times \sqrt{\frac{15}{3}} \\ &= \frac{2\sqrt{5}}{15}\end{aligned}$$

A mixed number under a square root sign must be changed to an improper fraction and then simplified.

WORKED Example 16Simplify $\sqrt{3\frac{1}{2}}$.**THINK**

- 1 Write the expression.
- 2 Change the mixed number to an improper fraction. Neither the numerator nor the denominator are perfect squares so both the numerator and denominator are written as surds.

WRITE

$$\begin{aligned}\sqrt{3\frac{1}{2}} \\ &= \sqrt{\frac{7}{2}} \\ &= \frac{\sqrt{7}}{\sqrt{2}}\end{aligned}$$

The same algebraic rules apply to surds when expanding brackets. Each term inside the brackets is multiplied by the term immediately outside the brackets.

WORKED Example 17

Expand each of the following, simplifying where appropriate.

a $\sqrt{7}(5 - \sqrt{2})$

THINK

- a**
- 1 Write the expression.
 - 2 Remove the brackets by multiplying the surd outside the brackets by each term inside the brackets.
- b**
- 1 Write the expression.
 - 2 Remove the brackets by multiplying the term outside the brackets by each term inside the brackets.
 - 3 Simplify as appropriate.

b $5\sqrt{3}(\sqrt{3} + 2\sqrt{6})$

WRITE

a $\sqrt{7}(5 - \sqrt{2})$
 $= 5\sqrt{7} - \sqrt{14}$

b $5\sqrt{3}(\sqrt{3} + 2\sqrt{6})$
 $= 5\sqrt{9} + 10\sqrt{18}$
 $= (5 \times 3) + (10 \times \sqrt{9} \times \sqrt{2})$
 $= 15 + (10 \times 3 \times \sqrt{2})$
 $= 15 + 30\sqrt{2}$

Binomial expansions are completed by multiplying the first term from the first bracket with the entire second bracket, then multiplying the second term from the first bracket by the entire second bracket.

WORKED Example 18

Expand $(\sqrt{2} + \sqrt{6})(2\sqrt{3} - \sqrt{6})$.

THINK

- 1 Write the expression.
- 2 Multiply each term in the first bracket by each term in the second bracket.
- 3 Simplify surds.

WRITE

$(\sqrt{2} + \sqrt{6})(2\sqrt{3} - \sqrt{6})$

$= \sqrt{2} \times 2\sqrt{3} + \sqrt{2} \times -\sqrt{6} + \sqrt{6} \times 2\sqrt{3} + \sqrt{6} \times -\sqrt{6}$
 $= 2\sqrt{6} - \sqrt{12} + 2\sqrt{18} - \sqrt{36}$
 $= 2\sqrt{6} - (\sqrt{4} \times \sqrt{3}) + (2 \times \sqrt{9} \times \sqrt{2}) - 6$
 $= 2\sqrt{6} - 2\sqrt{3} + (2 \times 3 \times \sqrt{2}) - 6$
 $= 2\sqrt{6} - 2\sqrt{3} + 6\sqrt{2} - 6$

remember

1. To multiply and divide surds, use the following rules.

(i) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ (ii) $c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$ (iii) $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

2. Leave answers in simplest surd form.

3. To remove a bracket containing surds, multiply each term outside the bracket by each term inside the bracket.

4. To expand two brackets containing surds, multiply each term in the first bracket by each term in the second bracket.

EXERCISE

1F

Multiplication and division of surds

WORKED
Example

14

1 Simplify each of the following.

a $5 \times \sqrt{5}$

b $\sqrt{5} \times \sqrt{5}$

c $-\sqrt{5} \times \sqrt{5}$

d $\sqrt{5} \times \sqrt{7}$

e $\sqrt{6} \times -\sqrt{11}$

f $\sqrt{32} \times \sqrt{2}$

g $\sqrt{25} \times -\sqrt{4}$

h $\sqrt{30} \times \sqrt{2}$

i $\sqrt{7} \times \sqrt{8}$

j $\sqrt{12} \times \sqrt{6}$

k $-\sqrt{90} \times -\sqrt{5}$

l $3\sqrt{2} \times 4\sqrt{2}$

m $-5\sqrt{5} \times 6\sqrt{5}$

n $3\sqrt{10} \times 2\sqrt{8}$

o $7\sqrt{3} \times -4\sqrt{12}$

p $2\sqrt{3} \times \sqrt{6}$

q $-10\sqrt{5} \times -5\sqrt{125}$

r $3\sqrt{8} \times 6\sqrt{9}$

s $8\sqrt{16} \times 10\sqrt{50}$

t $\sqrt{7} \times 4\sqrt{49}$

u $-2\sqrt{5} \times -3\sqrt{2} \times \sqrt{6}$

2 multiple choice

a $2\sqrt{6} \times 5\sqrt{4} \times 6\sqrt{6}$ is equal to:

A $13\sqrt{12}$

B $60\sqrt{12}$

C 132

D 156

E 720

b $-3\sqrt{8} \times -4\sqrt{6}$ is equal to:

A $-7\sqrt{48}$

B $-12\sqrt{48}$

C $48\sqrt{3}$

D $-48\sqrt{3}$

E $4\sqrt{3}$

c $6\sqrt{5} + 4\sqrt{5} \times 2\sqrt{5}$ is equal to:

A $6\sqrt{5} + 40$

B $6\sqrt{5} + 30$

C $14\sqrt{5}$

D $100\sqrt{5}$

E 500

WORKED
Example

15

3 Simplify each of the following.

a $\frac{\sqrt{6}}{\sqrt{2}}$

b $\frac{\sqrt{10}}{\sqrt{5}}$

c $\frac{\sqrt{20}}{\sqrt{4}}$

d $\frac{\sqrt{32}}{\sqrt{16}}$

e $\frac{\sqrt{75}}{\sqrt{5}}$

f $\frac{\sqrt{30}}{\sqrt{10}}$

g $\frac{4\sqrt{5}}{4}$

h $\frac{4\sqrt{5}}{\sqrt{5}}$

i $\frac{-6\sqrt{10}}{3\sqrt{2}}$

j $\frac{18\sqrt{18}}{2\sqrt{6}}$

k $\frac{-24\sqrt{6}}{6\sqrt{12}}$

l $\frac{5\sqrt{6}}{10\sqrt{3}}$

m $\frac{15\sqrt{15}}{20\sqrt{45}}$

n $\frac{3\sqrt{200}}{2\sqrt{2}}$

o $\frac{16\sqrt{125}}{-10\sqrt{5}}$

p $\frac{6}{6\sqrt{6}}$

q $\frac{-14\sqrt{49}}{-10\sqrt{81}}$

r $\frac{5\sqrt{3} \times 3\sqrt{3}}{2\sqrt{2} \times 8\sqrt{2}}$

s $\frac{2\sqrt{5} \times 3\sqrt{6}}{4\sqrt{10} \times 2\sqrt{3}}$

t $\frac{2\sqrt{2} \times \sqrt{5} \times 6\sqrt{2}}{5\sqrt{8} \times 2\sqrt{5}}$

4 multiple choice

a $\frac{-\sqrt{75}}{3}$ is equal to:

A -5

B $\frac{-5\sqrt{3}}{3}$

C 5

D $\frac{-25\sqrt{3}}{3}$

E -25

b $\frac{10\sqrt{12}}{20\sqrt{2}}$ is equal to:

- A** $2\sqrt{6}$ **B** $\frac{2}{\sqrt{6}}$ **C** $\frac{\sqrt{6}}{2}$ **D** 3 **E** $\frac{1}{3}$

c $\frac{6\sqrt{20} \times 4\sqrt{2}}{16\sqrt{3} \times 2\sqrt{10}}$ is equal to:

- A** $\frac{4\sqrt{3}}{3}$ **B** $\frac{3\sqrt{3}}{4}$ **C** $\frac{3}{2\sqrt{3}}$ **D** $\frac{4}{2\sqrt{3}}$ **E** $\frac{1}{4}$

d $\frac{8\sqrt{6} + 6\sqrt{10}}{2\sqrt{2}}$ is equal to:

- A** $6\sqrt{3} + 4\sqrt{5}$ **B** $\frac{4}{\sqrt{3}} + \frac{3}{\sqrt{5}}$ **C** $4\sqrt{3} + 6\sqrt{10}$ **D** $4\sqrt{3} + 3\sqrt{5}$ **E** $\frac{28}{\sqrt{2}}$

WORKED Example 16

5 Simplify each of the following.

- a** $\sqrt{2\frac{7}{9}}$ **b** $\sqrt{1\frac{13}{36}}$ **c** $\sqrt{2\frac{1}{4}}$ **d** $\sqrt{3\frac{1}{16}}$

WORKED Example 17

6 Expand each of the following, simplifying where appropriate.

- a** $3(\sqrt{2} + \sqrt{5})$ **b** $5(\sqrt{6} - \sqrt{2})$ **c** $6(\sqrt{5} + \sqrt{11})$
d $8(\sqrt{2} + 3)$ **e** $4(\sqrt{7} - 5)$ **f** $2(5 - \sqrt{2})$
g $7(6 + \sqrt{7})$ **h** $\sqrt{3}(\sqrt{2} + \sqrt{5})$ **i** $\sqrt{10}(\sqrt{2} + 2)$
j $\sqrt{14}(\sqrt{3} - 8)$ **k** $\sqrt{5}(\sqrt{5} + 2)$ **l** $\sqrt{6}(\sqrt{6} - 5)$
m $\sqrt{8}(\sqrt{2} + \sqrt{8})$ **n** $6\sqrt{5}(2\sqrt{5} - 3)$ **o** $2\sqrt{7}(3\sqrt{8} + 4\sqrt{5})$
p $3\sqrt{5}(2\sqrt{20} - 5\sqrt{5})$ **q** $5\sqrt{2}(5\sqrt{2} - 3)$ **r** $4\sqrt{3}(2\sqrt{2} - 5\sqrt{3})$

WORKED Example 18

7 Expand each of the following.

- a** $(\sqrt{5} + \sqrt{3})(2\sqrt{2} - \sqrt{3})$ **b** $(\sqrt{7} + \sqrt{2})(3\sqrt{5} - \sqrt{2})$ **c** $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$
d $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$ **e** $(2\sqrt{2} + \sqrt{5})(3\sqrt{2} - \sqrt{5})$ **f** $(3\sqrt{2} + \sqrt{3})(5\sqrt{2} - \sqrt{3})$
g $(\sqrt{5} - \sqrt{3})^2$ **h** $(\sqrt{2} + \sqrt{3})^2$ **i** $(2\sqrt{6} - 3\sqrt{2})^2$

8 A tray, 24 cm by 28 cm, is used for cooking biscuits. Square biscuits, measuring 4 cm by 4 cm are placed on the tray.

- a** What is the greatest number of biscuits that would fit on the tray if it was not necessary to allow for expansion in the cooking?
b If each biscuit had a strip of green mint placed along its diagonal, how much mint would be required for each biscuit? Give an exact answer in simplest surd form.
c How many centimetres of mint would be necessary for all the biscuits to be decorated in this way?
d If the dimensions of the tray were $12\sqrt{6}$ cm and $14\sqrt{3}$ cm, find the area of the tray in simplest surd form.
e Use approximations for the lengths of the sides of the tray to find how many of the 4×4 biscuits would fit on the new tray.



- 9 The material in the front face of the roof of a house has to be replaced. The face is triangular in shape.



- a If the vertical height is half the width of the base and the slant length is 6 metres, find the exact vertical height of this part of the roof.
b Find the exact area of the front face of the roof.



Recurring surds

Consider the expression $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \dots}}}}$. We will call this a recurring surd. Although $\sqrt{6}$ is irrational, this recurring surd actually has a rational answer. To find it we form a quadratic equation.

- 1 Find an expression for x^2 .
- 2 In your expression for x^2 , you should be able to find the original expression for x . Substitute the pronumeral x for this expression.
- 3 You should now be able to form a quadratic equation to solve. You will get two solutions but you need consider only the positive solution.
- 4 Now use the same method to find the value of $x = \sqrt{6 - \sqrt{6 - \sqrt{6 - \sqrt{6 - \dots}}}}$.
- 5 Evaluate the following recurring surds.

a $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}}$ b $x = \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \dots}}}}$

c $x = \sqrt{12 - \sqrt{12 - \sqrt{12 - \sqrt{12 - \dots}}}}$ d $x = \sqrt{20 - \sqrt{20 - \sqrt{20 - \sqrt{20 - \dots}}}}$

- 6 Try writing a few recurring surds of your own. Some will not have a rational answer. Can you find the condition for a recurring surd to have a rational answer?

10 QUICK QUESTIONS 2

- 1 Express $2\frac{1}{4}$ as a finite decimal.
- 2 Express $\frac{5}{11}$ as a recurring decimal.
- 3 Convert $0.\overline{63}$ to a simple fraction.
- 4 Which of the following is irrational? $\sqrt{81}$, $\sqrt{99}$, $\sqrt{169}$
- 5 Calculate $\sqrt{16.44}$ correct to 2 decimal places.
- 6 Evaluate $\sqrt{72} \times \sqrt{2} \div \sqrt{36}$.
- 7 Simplify $\sqrt{90}$.
- 8 Simplify $5\sqrt{2} + \sqrt{8} + 3\sqrt{18}$.
- 9 Simplify $4\sqrt{5} \times \sqrt{40}$.
- 10 Simplify $\frac{2\sqrt{6}}{\sqrt{72}}$.

Writing surd fractions with a rational denominator

$\frac{1}{\sqrt{2}}$ is a fraction with a surd in the denominator. If we multiply $\frac{1}{\sqrt{2}}$ by 1, its value will remain unchanged. If the numerator and the denominator are both multiplied by the same number, the value of the fraction stays the same because we are multiplying by 1.

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The value of the fraction has not changed but the denominator is now rational.

WORKED Example 19

Express each of the following fractions in simplest form with a rational denominator.

a $\frac{1}{\sqrt{5}}$

THINK

- a** 1 Write the fraction.
- 2 Multiply the numerator and the denominator by the surd in the denominator.

b $\frac{7\sqrt{2}}{4\sqrt{7}}$

WRITE

$$\begin{aligned} \text{a } & \frac{1}{\sqrt{5}} \\ &= \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{\sqrt{5}}{\sqrt{25}} \end{aligned}$$

THINK

- 3** Simplify.
- b 1** Write the fraction.
- 2** Multiply the numerator and the denominator by the surd in the denominator and simplify.
- 3** Simplify by cancelling.

WRITE

$$\begin{aligned}
 &= \frac{\sqrt{5}}{5} \\
 \mathbf{b} \quad &\frac{7\sqrt{2}}{4\sqrt{7}} \\
 &= \frac{7\sqrt{2}}{4\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\
 &= \frac{7\sqrt{14}}{4\sqrt{49}} \\
 &= \frac{7\sqrt{14}}{4 \times 7} \\
 &= \frac{7\sqrt{14}}{20} \\
 &= \frac{\sqrt{14}}{4}
 \end{aligned}$$

If there is a binomial denominator (two terms) such as $(3 + \sqrt{2})$ then the fraction can be written with a rational denominator by multiplying numerator and denominator by the same expression with the opposite sign. That is, $(3 - \sqrt{2})$ because:

$$\begin{aligned}
 (3 + \sqrt{2})(3 - \sqrt{2}) &= 9 - 3\sqrt{2} + 3\sqrt{2} - 2 \\
 &= 9 - 2 \\
 &= 7
 \end{aligned}$$

Using the difference of two squares rule removes the surd.

WORKED Example 20

Express $\frac{5}{2 + \sqrt{3}}$ in simplest form with a rational denominator.

THINK

- 1** Write the fraction.
- 2** Multiply both numerator and denominator by $(2 - \sqrt{3})$.

WRITE

$$\begin{aligned}
 &\frac{5}{2 + \sqrt{3}} \\
 &= \frac{5}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \\
 &= \frac{5(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})} \\
 &= \frac{5(2 - \sqrt{3})}{4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}}
 \end{aligned}$$

Continued over page 

THINK

- 3 Expand the denominator.
- 4 Simplify if applicable.

WRITE

$$= \frac{5(2 - \sqrt{3})}{4 - 3}$$

$$= 5(2 - \sqrt{3})$$

remember

To express fractions in simplest form with a rational denominator:

1. If the fraction has a single surd in the denominator, multiply both numerator and denominator by the surd.
2. If the fraction has an integer multiplied by a surd in the denominator, multiply both numerator and denominator by the surd only.
3. Simplify the denominator before rationalising.
4. If the fraction's denominator is the sum of 2 terms, multiply numerator and denominator by the difference of the 2 terms.
5. If the fraction's denominator is the difference of 2 terms, multiply numerator and denominator by the sum of the 2 terms.

EXERCISE 1G**Writing surd fractions with a rational denominator****WORKED Example 19a**

- 1 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{1}{\sqrt{3}}$

b $\frac{1}{\sqrt{5}}$

c $\frac{1}{\sqrt{6}}$

d $\frac{1}{\sqrt{7}}$

e $\frac{2}{\sqrt{10}}$

f $\frac{5}{\sqrt{5}}$

g $\frac{3}{\sqrt{15}}$

h $\frac{6}{\sqrt{30}}$

- 2 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{\sqrt{3}}{\sqrt{5}}$

b $\frac{\sqrt{5}}{\sqrt{6}}$

c $\frac{\sqrt{2}}{\sqrt{3}}$

d $\frac{\sqrt{6}}{\sqrt{10}}$

e $\frac{\sqrt{8}}{\sqrt{3}}$

f $\frac{\sqrt{12}}{\sqrt{7}}$

g $\frac{\sqrt{18}}{\sqrt{5}}$

h $\frac{\sqrt{3}}{\sqrt{2}}$

i $\frac{5\sqrt{6}}{\sqrt{5}}$

j $\frac{2\sqrt{3}}{\sqrt{2}}$

k $\frac{3\sqrt{5}}{\sqrt{6}}$

l $\frac{5\sqrt{7}}{\sqrt{10}}$

WORKED Example 19b

- 3 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{6\sqrt{5}}{7\sqrt{3}}$

b $\frac{14\sqrt{6}}{3\sqrt{7}}$

c $\frac{4\sqrt{3}}{5\sqrt{2}}$

d $\frac{5\sqrt{2}}{4\sqrt{10}}$



4 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{2}{\sqrt{8}}$

b $\frac{4}{\sqrt{12}}$

c $\frac{3}{\sqrt{18}}$

d $\frac{5\sqrt{3}}{\sqrt{20}}$

5 Find half of each of the following fractions by first expressing each one with a rational denominator.

a $\frac{24}{\sqrt{32}}$

b $\frac{20}{\sqrt{50}}$

**WORKED
Example**
20

6 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{5}{2 - \sqrt{3}}$

b $\frac{2}{1 + \sqrt{2}}$

c $\frac{4}{\sqrt{5} + 2}$

d $\frac{6}{3 - \sqrt{7}}$

e $\frac{3\sqrt{3}}{\sqrt{5} - \sqrt{2}}$

f $\frac{2\sqrt{5}}{\sqrt{5} + \sqrt{3}}$

g $\frac{5\sqrt{2}}{\sqrt{7} - \sqrt{2}}$

h $\frac{6\sqrt{6}}{3\sqrt{6} - 5\sqrt{2}}$



Applying the difference of two squares rule to expressions with surds



summary

Copy the sentences below. Fill in the gaps by choosing the correct word or expression from the word list that follows.

- 1 To express a fraction as a finite _____, divide the numerator by the denominator.
- 2 To express a fraction as a recurring decimal, divide the numerator by the denominator and write the decimal with _____ signs over the recurring decimal pattern.
- 3 A _____ number is one that can be expressed as a fraction.
- 4 Finite and _____ decimals are rational.
- 5 To express a recurring decimal as a fraction, eliminate the repeating decimal digits by multiplying by an appropriate _____ of 10, then subtract the original decimal and write the remainder as a fraction.
- 6 Numbers that cannot be expressed as _____ are irrational.
- 7 Any roots of numbers that do not have finite answers are called _____ and are irrational.
- 8 When calculating surds on the calculator, the resultant answer is only an _____.
- 9 Some surds can be simplified by dividing the original surd into the product of two other surds, one of which is a _____ square which can be calculated exactly.
- 10 Surds which do not have a perfect square _____ cannot be simplified.
- 11 Only _____ surds can be added or subtracted.
- 12 Surds can be _____ and divided.

WORD LIST

repeater
like
factor

decimal
multiple
surds

multiplied
fractions
perfect

rational
recurring
approximation

CHAPTER review

- 1 Evaluate the following. **a** $\frac{1}{4} + \frac{1}{3}$ **b** $\frac{1}{4} - \frac{1}{3}$ **c** $\frac{1}{4} \times \frac{1}{3}$ **d** $\frac{1}{4} \div \frac{1}{3}$
- 2 Two-fifths of students at Farnham High catch a bus to school, $\frac{3}{8}$ walk to school and the rest come by car or bike. If there are 560 students at the school, how many come by car or bike?
- 3 Express each of the following as a decimal number, giving exact answers.

a $\frac{2}{25}$ **b** $\frac{13}{16}$ **c** $\frac{2}{7}$ **d** $\frac{5}{9}$
- 4 **multiple choice**

a $\frac{11}{14}$ as a recurring decimal is:
A 0.785 714 285 **B** $0.\overline{785\,714\,2}$
C $0.\overline{785\,714\,2}$ **D** 0.785 71 (to 5 d.p.)
E cannot be written as a recurring decimal

b $0.\overline{30}$ is equal to:
A $\frac{3}{10}$ **B** $\frac{1}{3}$ **C** $\frac{11}{30}$ **D** $\frac{3}{11}$ **E** $\frac{10}{33}$
- 5 Convert each of the following to a fraction in simplest form.

a 0.8 **b** $0.\dot{8}$ **c** 0.83 **d** $0.8\dot{3}$ **e** $0.\overline{83}$
- 6 Explain why $\sqrt{15}$ is a surd and $\sqrt{16}$ is not a surd.
- 7 Calculate each of the following, rounding the answer to 1 decimal place.

a $\sqrt{62}$ **b** $\sqrt{72} + \sqrt{27}$ **c** $\frac{7 - \sqrt{7}}{\sqrt{7} + 7}$ **d** $\frac{\sqrt{6} \times \sqrt{5}}{\sqrt{6} - \sqrt{5}}$
- 8 A vertical flagpole is supported by a wire attached from the top of the pole to the horizontal ground, 4 m from the base of the pole. If the flagpole is 9 m tall, what is the length of the supporting wire?
- 9 Simplify each of the following.

a $\sqrt{99}$ **b** $\sqrt{175}$ **c** $6\sqrt{32}$ **d** $4\sqrt{90}$
- 10 **multiple choice**

$\sqrt{96}$ written in simplest form is:
A $4\sqrt{6}$ **B** $2\sqrt{24}$ **C** $8\sqrt{12}$ **D** $16\sqrt{6}$ **E** $12\sqrt{3}$
- 11 Express each of the following in the form $\sqrt{a}$.

a $5\sqrt{6}$ **b** $6\sqrt{5}$ **c** $11\sqrt{5}$ **d** $3\sqrt{2}$



1A

1A

1B

1B

1B

1C

1C

1C

1D

1D

1D

1E

12 Simplify each of the following.

a $\sqrt{6} + 3\sqrt{7} - 4\sqrt{7} + 3\sqrt{6}$

b $\sqrt{12} + \sqrt{243} - \sqrt{108}$

c $5\sqrt{28} + 2\sqrt{45} - 4\sqrt{112} + 3\sqrt{80}$

1E

13 multiple choice

 $\sqrt{27} + \sqrt{50} - \sqrt{72} + \sqrt{300}$ is equal to:

A $30\sqrt{3} - 30\sqrt{2}$

B $13\sqrt{3} + 11\sqrt{2}$

C $13\sqrt{3} + \sqrt{2}$

D $13\sqrt{3} - \sqrt{2}$

E $\sqrt{305}$

1F

14 Find the perimeter of the following in simplest surd form:

a a square of side length $(3 + \sqrt{2})$ cm

b a rectangle 20 cm by $(8 + \sqrt{5})$ cm.

1F

15 Simplify each of the following.

a $\sqrt{5} \times \sqrt{10}$

b $4\sqrt{3} \times 6\sqrt{7}$

c $\sqrt{13} \times \sqrt{13}$

d $\frac{-16\sqrt{12}}{8\sqrt{2}}$

e $\frac{35\sqrt{32}}{20\sqrt{8}}$

f $\frac{2\sqrt{5} \times 6\sqrt{6}}{4\sqrt{3} \times 3\sqrt{12}}$

1F

16 Expand and simplify each of the following.

a $6\sqrt{5}(2\sqrt{5} + 3\sqrt{20})$

b $(4\sqrt{3} - 5)^2$

1F

17 multiple choice

 $\frac{15\sqrt{48}}{20\sqrt{6}}$ written in simplest form is:

A $\frac{3\sqrt{8}}{4}$

B $\frac{4\sqrt{8}}{3}$

C $\frac{3\sqrt{2}}{2}$

D $\frac{4\sqrt{2}}{3}$

E 6

1G

18 multiple choice

 $\frac{2}{\sqrt{5}}$ written with a rational denominator in simplest form is:

A $\frac{2}{5}$

B $\frac{2\sqrt{5}}{5}$

C $\frac{5}{2}$

D $\frac{\sqrt{5}}{2}$

E $\frac{\sqrt{5}}{5}$

1G

19 Express each of the following fractions in simplest form with a rational denominator.

a $\frac{1}{2\sqrt{7}}$

b $\frac{5\sqrt{2}}{2\sqrt{3}}$

c $\frac{1}{\sqrt{5} + 2}$

d $\frac{\sqrt{6}}{2\sqrt{5} - 3\sqrt{2}}$