

where a^2 is the area of the square base, b^2 is the area of the square top, and H is the distance from the base to the top. Find the volume of a truncated pyramid that has a base of 900 square meters, a top of 400 square meters, and a height H of 10 meters.

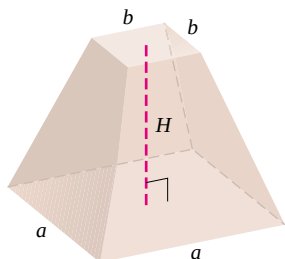


FIGURE FOR EXERCISE 87

88. Egyptian pyramid formula. Rewrite the formula of the previous exercise so that the denominator contains the number 3 only.

GETTING MORE INVOLVED



89. Discussion. On a test a student divided $3x^3 - 5x^2 - 3x + 7$ by $x - 3$ and got a quotient of $3x^2 + 4x$ and remainder $9x + 7$. Verify that the divisor times the quotient plus the remainder is equal to the dividend. Why was the student's answer incorrect?



90. Exploration. Use synthetic division to find the quotient when $x^5 - 1$ is divided by $x - 1$ and the quotient when $x^6 - 1$ is divided by $x - 1$. Observe the pattern in the first two quotients and then write the quotient for $x^9 - 1$ divided by $x - 1$ without dividing.

5.6

FACToring POLYNOMIALS

In this section

- Factoring Out the Greatest Common Factor (GCF)
- Factoring Out the Opposite of the GCF
- Factoring the Difference of Two Squares
- Factoring Perfect Square Trinomials
- Factoring a Difference or Sum of Two Cubes
- Factoring a Polynomial Completely
- Factoring by Substitution

In Section 5.5 you learned that a polynomial could be factored by using division: If we know one factor of a polynomial, then we can use it as a divisor to obtain the other factor, the quotient. However, this technique is not very practical because the division process can be somewhat tedious, and it is not easy to obtain a factor to use as the divisor. In this section and the next two sections we will develop better techniques for factoring polynomials. These techniques will be used for solving equations and problems in the last section of this chapter.

Factoring Out the Greatest Common Factor (GCF)

A natural number larger than 1 that has no factors other than itself and 1 is called a **prime number**. The numbers

$$2, 3, 5, 7, 11, 13, 17, 19, 23$$

are the first nine prime numbers. There are infinitely many prime numbers.

To factor a natural number **completely** means to write it as a product of prime numbers. In factoring 12 we might write $12 = 4 \cdot 3$. However, 12 is not factored completely as $4 \cdot 3$ because 4 is not a prime. To factor 12 completely, we write $12 = 2 \cdot 2 \cdot 3$ (or $2^2 \cdot 3$).

We use the distributive property to multiply a monomial and a binomial:

$$6x(2x - 1) = 12x^2 - 6x$$

If we start with $12x^2 - 6x$, we can use the distributive property to get

$$12x^2 - 6x = 6x(2x - 1).$$

We have **factored out** $6x$, which is a common factor of $12x^2$ and $-6x$. We could have factored out just 3 to get

$$12x^2 - 6x = 3(4x^2 - 2x),$$

but this would not be factoring out the *greatest* common factor. The **greatest common factor** (GCF) is a monomial that includes every number or variable that is a factor of all of the terms of the polynomial.

We can use the following strategy for finding the greatest common factor of a group of terms.

Strategy for Finding the Greatest Common Factor (GCF)

1. Factor each term completely.
2. Write a product using each factor that is common to all of the terms.
3. On each of these factors, use an exponent equal to the smallest exponent that appears on that factor in any of the terms.

EXAMPLE 1

The greatest common factor

Find the greatest common factor (GCF) for each group of terms.

a) $8x^2y$, $20xy^3$

b) $30a^2$, $45a^3b^2$, $75a^4b$

Solution

a) First factor each term completely:

$$8x^2y = 2^3x^2y$$

$$20xy^3 = 2^2 \cdot 5xy^3$$

The factors common to both terms are 2, x , and y . In the GCF we use the smallest exponent that appears on each factor in either of the terms. So the GCF is 2^2xy or $4xy$.

b) First factor each term completely:

$$30a^2 = 2 \cdot 3 \cdot 5a^2$$

$$45a^3b^2 = 3^2 \cdot 5a^3b^2$$

$$75a^4b = 3 \cdot 5^2a^4b$$

The GCF is $3 \cdot 5a^2$ or $15a^2$. ■

To factor out the GCF from a polynomial, find the GCF for the terms, then use the distributive property to factor it out.

EXAMPLE 2

Factoring out the greatest common factor

Factor each polynomial by factoring out the GCF.

a) $5x^4 - 10x^3 + 15x^2$

b) $8xy^2 + 20x^2y$

c) $60x^5 + 24x^3 + 36x^2$

Solution

a) First factor each term completely:

$$5x^4 = 5x^4, \quad 10x^3 = 2 \cdot 5x^3, \quad 15x^2 = 3 \cdot 5x^2.$$

The GCF of the three terms is $5x^2$. Now factor $5x^2$ out of each term:

$$5x^4 - 10x^3 + 15x^2 = 5x^2(x^2 - 2x + 3)$$

b) The GCF for $8xy^2$ and $20x^2y$ is $4xy$:

$$8xy^2 + 20x^2y = 4xy(2y + 5x)$$

c) First factor each coefficient in $60x^5 + 24x^3 + 36x^2$:

$$60 = 2^2 \cdot 3 \cdot 5, \quad 24 = 2^3 \cdot 3, \quad 36 = 2^2 \cdot 3^2.$$

The GCF of the three terms is $2^2 \cdot 3x^2$ or $12x^2$:

$$60x^5 + 24x^3 + 36x^2 = 12x^2(5x^3 + 2x + 3)$$

In the next example the common factor in each term is a binomial. ■

study tip

Everyone has a different attention span. Start by studying 10 to 15 minutes at a time and then build up to longer periods over time. In your senior year you should be able to concentrate on one task for 30 to 45 minutes without a break. Be realistic. When you cannot remember what you have read and can no longer concentrate, take a break.

EXAMPLE 3 Factoring out a binomial

Factor.

a) $(x + 3)w + (x + 3)a$

b) $x(x - 9) - 4(x - 9)$

Solutiona) We treat $x + 3$ like a common monomial when factoring:

$$(x + 3)w + (x + 3)a = (x + 3)(w + a)$$

b) Factor out the common binomial $x - 9$:

$$x(x - 9) - 4(x - 9) = (x - 4)(x - 9)$$

Factoring Out the Opposite of the GCF

The GCF, the greatest common factor, for $-6x^2 - 4x$ is $2x$, but we can factor out either $2x$ or its opposite, $-2x$:

$$\begin{aligned} -6x^2 - 4x &= 2x(-3x - 2) \\ &= -2x(3x + 2) \end{aligned}$$

In Example 8 of this section it will be necessary to factor out the opposite of the GCF.

EXAMPLE 4 Factoring out the opposite of the GCF

Factor out the GCF, then factor out the opposite of the GCF.

a) $5x - 5y$

b) $-x^2 - 3$

c) $-x^3 + 3x^2 - 5x$

Solution

$$\begin{aligned} \text{a) } 5x - 5y &= 5(x - y) && \text{Factor out 5.} \\ &= -5(-x + y) && \text{Factor out -5.} \end{aligned}$$

$$\begin{aligned} \text{b) } -x^2 - 3 &= 1(-x^2 - 3) && \text{The GCF is 1.} \\ &= -1(x^2 + 3) && \text{Factor out -1.} \end{aligned}$$

$$\begin{aligned} \text{c) } -x^3 + 3x^2 - 5x &= x(-x^2 + 3x - 5) && \text{Factor out x.} \\ &= -x(x^2 - 3x + 5) && \text{Factor out -x.} \end{aligned}$$

Factoring the Difference of Two Squares

A first-degree polynomial in one variable, such as $3x - 5$, is called a linear polynomial. (The equation $3x - 5 = 0$ is a linear equation.)

Linear Polynomial

If a and b are real numbers with $a \neq 0$, then $ax + b$ is called a **linear polynomial**.

A second-degree polynomial such as $x^2 + 5x - 6$ is called a quadratic polynomial.

Quadratic Polynomial

If a , b , and c are real numbers with $a \neq 0$, then $ax^2 + bx + c$ is called a **quadratic polynomial**.

helpful hint

The prefix “quad” means four. So why is a polynomial of three terms called quadratic? Perhaps it is because a quadratic polynomial can often be factored into a product of two binomials.

One of the main goals of this chapter is to write a quadratic polynomial (when possible) as a product of linear factors.

Consider the quadratic polynomial $x^2 - 25$. We recognize that $x^2 - 25$ is a difference of two squares, $x^2 - 5^2$. We recall that the product of a sum and a difference is a difference of two squares: $(a + b)(a - b) = a^2 - b^2$. If we reverse this special product rule, we get a rule for factoring the difference of two squares.

Factoring the Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b)$$

The difference of two squares factors as the product of a sum and a difference. To factor $x^2 - 25$, we replace a by x and b by 5 to get

$$x^2 - 25 = (x + 5)(x - 5).$$

This equation expresses a quadratic polynomial as a product of two linear factors.

EXAMPLE 5

Factoring the difference of two squares

Factor each polynomial.

a) $y^2 - 36$

b) $9x^2 - 1$

c) $4x^2 - y^2$

Solution

Each of these binomials is a difference of two squares. Each binomial factors into a product of a sum and a difference.

a) $y^2 - 36 = (y + 6)(y - 6)$ We could also write $(y - 6)(y + 6)$ because the factors can be written in any order.

b) $9x^2 - 1 = (3x + 1)(3x - 1)$

c) $4x^2 - y^2 = (2x + y)(2x - y)$ ■

helpful hint

Using the power of a power rule, we can see that any even power is a perfect square:

$$x^{2n} = (x^n)^2$$

Factoring Perfect Square Trinomials

The trinomial that results from squaring a binomial is called a **perfect square trinomial**. We can reverse the rules from Section 5.4 for the square of a sum or a difference to get rules for factoring.

Factoring Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Consider the polynomial $x^2 + 6x + 9$. If we recognize that

$$x^2 + 6x + 9 = x^2 + 2 \cdot x \cdot 3 + 3^2,$$

then we can see that it is a perfect square trinomial. It fits the rule if $a = x$ and $b = 3$:

$$x^2 + 6x + 9 = (x + 3)^2$$

Perfect square trinomials can be identified by using the following strategy.

Strategy for Identifying Perfect Square Trinomials

A trinomial is a perfect square trinomial if

1. the first and last terms are of the form a^2 and b^2 ,
2. the middle term is 2 or -2 times the product of a and b .

We use this strategy in the next example.

EXAMPLE 6 Factoring perfect square trinomials

Factor each polynomial.

- a) $x^2 - 8x + 16$ b) $a^2 + 14a + 49$ c) $4x^2 + 12x + 9$

Solution

- a) Because the first term is x^2 , the last is 4^2 , and $-2(x)(4)$ is equal to the middle term $-8x$, the trinomial $x^2 - 8x + 16$ is a perfect square trinomial:

$$x^2 - 8x + 16 = (x - 4)^2$$

- b) Because $49 = 7^2$ and $14a = 2(a)(7)$, we have a perfect square trinomial:

$$a^2 + 14a + 49 = (a + 7)^2$$

- c) Because $4x^2 = (2x)^2$, $9 = 3^2$, and the middle term $12x$ is equal to $2(2x)(3)$, the trinomial $4x^2 + 12x + 9$ is a perfect square trinomial:

$$4x^2 + 12x + 9 = (2x + 3)^2$$

Factoring a Difference or a Sum of Two Cubes

In Example 6 of Section 5.5 we divided $a^3 - b^3$ by $a - b$ to get the quotient $a^2 + ab + b^2$ and no remainder. So $a - b$ is a factor of $a^3 - b^3$, a difference of two cubes. If you divide $a^3 + b^3$ by $a + b$, you will get the quotient $a^2 - ab + b^2$ and no remainder. Try it. So $a + b$ is a factor of $a^3 + b^3$, a sum of two cubes. These results give us two more factoring rules.

Factoring a Difference or a Sum of Two Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

EXAMPLE 7 Factoring a difference or a sum of two cubes

Factor each polynomial.

- a) $x^3 - 8$ b) $y^3 + 1$ c) $8z^3 - 27$

Solution

- a) Because $8 = 2^3$, we can use the formula for factoring the difference of two cubes. In the formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, let $a = x$ and $b = 2$:

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$$

- b) $y^3 + 1 = y^3 + 1^3$

$$= (y + 1)(y^2 - y + 1)$$

Recognize a sum of two cubes.

Let $a = y$ and $b = 1$ in the formula for the sum of two cubes.

$$\begin{aligned}\text{c) } 8z^3 - 27 &= (2z)^3 - 3^3 && \text{Recognize a difference of two cubes.} \\ &= (2z - 3)(4z^2 + 6z + 9) && \text{Let } a = 2z \text{ and } b = 3 \text{ in the formula} \\ &&& \text{for a difference of two cubes.}\end{aligned}$$

Factoring a Polynomial Completely

Polynomials that cannot be factored are called **prime polynomials**. Because binomials such as $x + 5$, $a - 6$, and $3x + 1$ cannot be factored, they are prime polynomials. A polynomial is **factored completely** when it is written as a product of prime polynomials. To factor completely, always factor out the GCF (or its opposite) first. Then continue to factor until all of the factors are prime.

EXAMPLE 8

Factoring completely

Factor each polynomial completely.

$$\text{a) } 5x^2 - 20 \qquad \text{b) } 3a^3 - 30a^2 + 75a \qquad \text{c) } -2b^4 + 16b$$

Solution

$$\begin{aligned}\text{a) } 5x^2 - 20 &= 5(x^2 - 4) && \text{Greatest common factor} \\ &= 5(x - 2)(x + 2) && \text{Difference of two squares} \\ \text{b) } 3a^3 - 30a^2 + 75a &= 3a(a^2 - 10a + 25) && \text{Greatest common factor} \\ &= 3a(a - 5)^2 && \text{Perfect square trinomial} \\ \text{c) } -2b^4 + 16b &= -2b(b^3 - 8) && \text{Factor out } -2b \text{ to make} \\ &&& \text{the next step easier.} \\ &= -2b(b - 2)(b^2 + 2b + 4) && \text{Difference of two cubes}\end{aligned}$$

Factoring by Substitution

So far, the polynomials that we have factored, without common factors, have all been of degree 2 or 3. Some polynomials of higher degree can be factored by substituting a single variable for a variable with a higher power. After factoring, we replace the single variable by the higher-power variable. This method is called **substitution**.

EXAMPLE 9

Factoring by substitution

Factor each polynomial.

$$\text{a) } x^4 - 9 \qquad \text{b) } y^8 - 14y^4 + 49$$

Solution

a) We recognize $x^4 - 9$ as a difference of two squares in which $x^4 = (x^2)^2$ and $9 = 3^2$. If we let $w = x^2$, then $w^2 = x^4$. So we can replace x^4 by w^2 and factor:

$$\begin{aligned}x^4 - 9 &= w^2 - 9 && \text{Replace } x^4 \text{ by } w^2. \\ &= (w + 3)(w - 3) && \text{Difference of two squares} \\ &= (x^2 + 3)(x^2 - 3) && \text{Replace } w \text{ by } x^2.\end{aligned}$$

b) We recognize $y^8 - 14y^4 + 49$ as a perfect square trinomial in which $y^8 = (y^4)^2$ and $49 = 7^2$. We let $w = y^4$ and $w^2 = y^8$:

$$\begin{aligned}y^8 - 14y^4 + 49 &= w^2 - 14w + 49 && \text{Replace } y^4 \text{ by } w \text{ and } y^8 \text{ by } w^2. \\ &= (w - 7)^2 && \text{Perfect square trinomial} \\ &= (y^4 - 7)^2 && \text{Replace } w \text{ by } y^4.\end{aligned}$$

helpful hint

It is not actually necessary to perform the substitution step. If you can recognize that

$$x^4 - 9 = (x^2 - 3)(x^2 + 3)$$

then skip the substitution.

CAUTION The polynomials that we factor by substitution must contain just the right powers of the variable. We can factor $y^8 - 14y^4 + 49$ because $(y^4)^2 = y^8$, but we cannot factor $y^7 - 14y^4 + 49$ by substitution.

In the next example we use substitution to factor polynomials that have variables as exponents.

EXAMPLE 10 Polynomials with variable exponents

Factor completely. The variables used in the exponents represent positive integers.

a) $x^{2m} - y^2$

b) $z^{2n+1} - 6z^{n+1} + 9z$

Solution

a) Notice that $x^{2m} = (x^m)^2$. So if we let $w = x^m$, then $w^2 = x^{2m}$:

$$\begin{aligned} x^{2m} - y^2 &= w^2 - y^2 && \text{Substitution} \\ &= (w + y)(w - y) && \text{Difference of two squares} \\ &= (x^m + y)(x^m - y) && \text{Replace } w \text{ by } x^m. \end{aligned}$$

b) First factor out the common factor z :

$$\begin{aligned} z^{2n+1} - 6z^{n+1} + 9z &= z(z^{2n} - 6z^n + 9) \\ &= z(a^2 - 6a + 9) && \text{Let } a = z^n. \\ &= z(a - 3)^2 && \text{Perfect square trinomial} \\ &= z(z^n - 3)^2 && \text{Replace } a \text{ by } z^n. \end{aligned}$$

WARM - U P S

True or false? Explain your answer.

- For the polynomial $3x^2y - 6xy^2$ we can factor out either $3xy$ or $-3xy$.
- The greatest common factor for the polynomial $8a^3 - 15b^2$ is 1.
- $2x - 4 = -2(2 - x)$ for any value of x .
- $x^2 - 16 = (x - 4)(x + 4)$ for any value of x .
- The polynomial $x^2 + 6x + 36$ is a perfect square trinomial.
- The polynomial $y^2 + 16$ is a perfect square trinomial.
- $9x^2 + 21x + 49 = (3x + 7)^2$ for any value of x .
- The polynomial $x + 1$ is a factor of $x^3 + 1$.
- $x^3 - 27 = (x - 3)(x^2 + 6x + 9)$ for any value of x .
- $x^3 - 8 = (x - 2)^3$ for any value of x .

5.6 EXERCISES

Reading and Writing After reading this section, write out the answers to these questions. Use complete sentences.

1. What is a prime number?

2. When is a natural number factored completely?

3. What is the greatest common factor for the terms of a polynomial?

4. What are the two ways to factor out the greatest common factor?

5. What is a linear polynomial?
6. What is a quadratic polynomial?
7. What is a prime polynomial?
8. When is a polynomial factored completely?

Find the greatest common factor for each group of terms. See Example 1.

9. 48, 36x
10. $42a$, $28a^2$
11. $9wx$, $21wy$, $15xy$
12. $70x^2$, $84x$, $42x^3$
13. $24x^2y$, $42xy^2$, $66xy^3$
14. $60a^2b^5$, $140a^9b^2$, $40a^3b^6$

Factor out the greatest common factor in each expression. See Examples 2 and 3.

15. $x^3 - 5x$
16. $10x^2 - 20y^3$
17. $48wx + 36wy$
18. $42wz + 28wa$
19. $2x^3 - 4x^2 + 6x$
20. $6x^3 - 12x^2 + 18x$
21. $36a^3b^6 - 24a^4b^2 + 60a^5b^3$
22. $44x^8y^6z - 110x^6y^9z^2$
23. $(x - 6)a + (x - 6)b$
24. $(y - 4)3 + (y - 4)b$
25. $(y - 1)^2y + (y - 1)^2z$
26. $(w - 2)^2 \cdot w + (w - 2)^2 \cdot 3$

Factor out the greatest common factor, then factor out the opposite of the greatest common factor. See Example 4.

27. $2x - 2y$
28. $-3x + 6$
29. $6x^2 - 3x$
30. $10x^2 + 5x$
31. $-w^3 + 3w^2$
32. $-2w^4 + 6w^3$
33. $-a^3 + a^2 - 7a$
34. $-2a^4 - 4a^3 + 6a^2$

Factor each polynomial. See Example 5.

35. $x^2 - 100$
36. $81 - y^2$
37. $4y^2 - 49$
38. $16b^2 - 1$

39. $9x^2 - 25a^2$
40. $121a^2 - b^2$
41. $144w^2z^2 - 1$
42. $x^2y^2 - 9c^2$

Factor each polynomial. See Example 6.

43. $x^2 - 20x + 100$
44. $y^2 + 10y + 25$
45. $4m^2 - 4m + 1$
46. $9t^2 + 30t + 25$
47. $w^2 - 2wt + t^2$
48. $4r^2 + 20rt + 25t^2$

Factor. See Example 7.

49. $a^3 - 1$
50. $w^3 + 1$
51. $w^3 + 27$
52. $x^3 - 64$
53. $8x^3 - 1$
54. $27x^3 + 1$
55. $a^3 + 8$
56. $m^3 - 8$

Factor each polynomial completely. See Example 8.

57. $2x^2 - 8$
58. $3x^3 - 27x$
59. $x^3 + 10x^2 + 25x$
60. $5a^4m - 45a^2m$
61. $4x^2 + 4x + 1$
62. $ax^2 - 8ax + 16a$
63. $(x + 3)x + (x + 3)7$
64. $(x - 2)x - (x - 2)5$
65. $6y^2 + 3y$
66. $4y^2 - y$
67. $4x^2 - 20x + 25$
68. $a^3x^3 - 6a^2x^2 + 9ax$
69. $2m^4 - 2mn^3$
70. $5x^3y^2 - y^5$
71. $(2x - 3)x - (2x - 3)2$
72. $(2x + 1)x + (2x + 1)3$
73. $9a^3 - aw^2$
74. $2bn^2 - 4b^2n + 2b^3$
75. $-5a^2 + 30a - 45$
76. $-2x^2 + 50$
77. $16 - 54x^3$
78. $27x^2y - 64x^2y^4$
79. $-3y^3 - 18y^2 - 27y$
80. $-2m^2n - 8mn - 8n$
81. $-7a^2b^2 + 7$
82. $-17a^2 - 17a$

Factor each polynomial completely. See Example 9.

83. $x^{10} - 9$

84. $y^8 - 4$

85. $z^{12} - 6z^6 + 9$

86. $a^6 + 10a^3 + 25$

87. $2x^7 + 8x^4 + 8x$

88. $x^{13} - 6x^7 + 9x$

89. $4x^5 + 4x^3 + x$

90. $18x^6 + 24x^3 + 8$

91. $x^6 - 8$

92. $y^6 - 27$

93. $2x^9 + 16$

94. $x^{13} + x$

Factor each polynomial completely. The variables used as exponents represent positive integers. See Example 10.

95. $a^{2n} - 1$

96. $b^{4n} - 9$

97. $a^{2r} + 6a^r + 9$

98. $u^{6n} - 4u^{3n} + 4$

99. $x^{3m} - 8$

100. $y^{3n} + 1$

101. $a^{3m} - b^3$

102. $r^{3m} + 8t^3$

103. $k^{2w+1} - 10k^{w+1} + 25k$

104. $4a^{2t+1} + 4a^{t+1} + a$

105. $uv^{6k} - 2u^2v^{4k} + u^3v^{2k}$

106. $u^{3m}v^n + 2u^{2m}v^{2n} + u^m v^{3n}$

Replace k in each trinomial by a number that makes the trinomial a perfect square trinomial.

107. $x^2 + 6x + k$

108. $y^2 - 8y + k$

109. $4a^2 - ka + 25$

110. $9u^2 + kuv + 49v^2$

111. $km^2 - 24m + 9$

112. $kz^2 + 40z + 16$

113. $81y^2 - 180y + k$

114. $36a^2 + 60a + k$

Solve each problem.

115. **Volume of a bird cage.** A company makes rectangular shaped bird cages with height b inches and square bottoms. The volume of these cages is given by the function

$$V = b^3 - 6b^2 + 9b.$$

- What is the length of a side of the square bottom?
- Use the function to find the volume of a cage with a height of 18 inches.

- c) Use the accompanying graph to estimate the height of a cage for which the volume is 20,000 cubic inches.

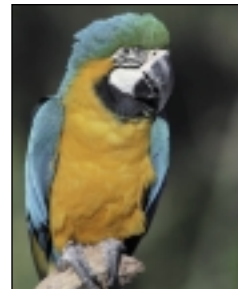
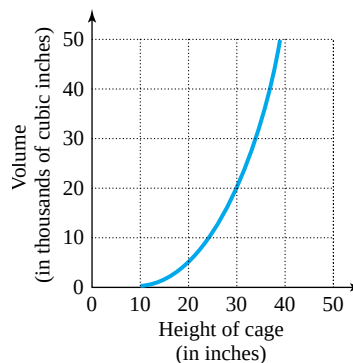


FIGURE FOR EXERCISE 115

116. **Pyramid power.** A powerful crystal pyramid has a square base and a volume of $3y^3 + 12y^2 + 12y$ cubic centimeters. If its height is y centimeters, then what polynomial represents the length of a side of the square base? (The volume of a pyramid with a square base of area a^2 and height h is given by $V = \frac{ha^2}{3}$.)

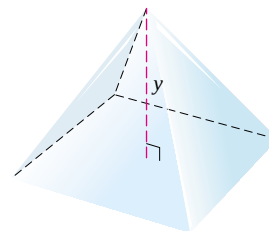


FIGURE FOR EXERCISE 116

GETTING MORE INVOLVED



117. **Cooperative learning.** List the perfect square trinomials corresponding to $(x+1)^2$, $(x+2)^2$, $(x+3)^2$, ..., $(x+12)^2$. Use your list to quiz a classmate. Read a perfect square trinomial at random from your list and ask your classmate to write its factored form. Repeat until both of you have mastered these 12 perfect square trinomials.