

CHAPTER 4

LINEAR EQUATIONS IN TWO VARIABLES

Points to Remember :

1. An equation of the form $ax + by + c = 0$, where a, b, c are real numbers, such that a and b are not both zero, is known as **linear equation** in two variables.
2. A linear equation in two variables has infinitely many solutions.
3. The graph of linear equation in two variables is always a straight line.
4. $y = 0$ is the equation of x -axis and $x = 0$ is the equation of y -axis.
5. The graph of $x = a$ is a straight line parallel to the y -axis.
6. The graph of $y = b$ is a straight line parallel to the x -axis.
7. The graph of $y = kx$ passes through the origin.
8. Every point on the graph of a linear equation in two variables is a solution of the linear equation. Also, every solution of the linear equation is a point on the graph of the linear equation.

ILLUSTRATIVE EXAMPLES

Example 1. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case.

$$(i) 4x - 7y = 10 \quad (ii) x + \frac{y}{2} = -5 \quad (iii) x = -4y$$
$$(iv) 4y - 9 = 0 \quad (v) x = 5$$

Solution. (i) $4x - 7y = 10 \Rightarrow 4x - 7y - 10 = 0$
comparing with $ax + by + c = 0$, $a = 4, b = -7, c = -10$

$$(ii) x + \frac{y}{2} = -5 \Rightarrow x + \frac{y}{2} + 5 = 0 = ax + by + c$$

$$\text{on comparing, } a = 1, b = \frac{1}{2}, c = 5$$

$$(iii) x = -4y \Rightarrow x + 4y + 0 = 0 = ax + by + c$$

$$\text{on comparing, } a = 1, b = 4, c = 0$$

$$(iv) 4y - 9 = 0 \Rightarrow 0.x + 4.y - 9 = 0 = ax + by + c$$

$$\text{on comparing, } a = 0, b = 4, c = -9$$

$$(v) x = 5 \Rightarrow x - 5 = 0, \Rightarrow x + 0.y - 5 = 0$$

$$\text{on comparing, } a = 1, b = 0, c = -5$$

Example 2. Which one of the following options is true, and why? $y = 3x + 5$ has (i) a unique solution, (ii) only two solutions, (iii) infinitely many solutions. —NCERT

Solution. Given equation is $y = 3x + 5$
when $x = 0, y = 3(0) + 5 = 5$
 $\therefore (0, 5)$ is one of its solution.

$$\text{when } y = 0, \text{ then } x = \frac{-5}{3}$$

$\therefore \left(\frac{-5}{3}, 0\right)$ is another solution.

when $x = 1$, then $y = 3(1) + 5 = 8$

$\therefore (1, 8)$ is also its solution.

So, it's clear that for every infinitely values we can give to x , we have corresponding value of y .

$\therefore$ This equation has infinitely many solutions.

Example 3. Bhavya and Anisha have a total of Rs. 100. Express this information in the form of an equation.

Solution. Let total amount with Bhavya = Rs x
and total amount with Anisha = Rs. y

according to the question, $x + y = 100$, which is required equation.

Example 4. Give five integer solutions of the equation $3x + y = 8$.

Solution. $3x + y = 8 \Rightarrow y = 8 - 3x$

Now, when $x = 0$, $y = 8 - 3(0) = 8$

when $x = 1$, $y = 8 - 3(1) = 5$

when $x = 2$, $y = 8 - 3(2) = 2$

when $x = -1$, $y = 8 - 3(-1) = 11$

when $x = -2$, $y = 8 - 3(-2) = 14$

Solutions can be represented in the tabular form as follows :

x	0	1	2	-1	-2
y	8	5	2	11	14

Example 5. Write four solutions for each of the following equations :

(i) $2x + y = 7$

(ii) $\pi x + y = 9$

(iii) $x = 4y$

—NCERT

Solution. (i) $2x + y = 7 \Rightarrow y = 7 - 2x$

For $x = 0$, $y = 7 - 2(0) = 7$

For $x = 1$, $y = 7 - 2(1) = 7 - 2 = 5$

For $x = 2$, $y = 7 - 2(2) = 7 - 4 = 3$

$\therefore$ Four solutions of the given equation are $(0, 7)$, $(1, 5)$, $(2, 3)$, $(-1, 9)$.

(ii) Given equation $\pi x + y = 9 \Rightarrow y = 9 - \pi x$

For $x = 0$, $y = 9 - \pi(0) = 9$

For $x = 1$, $y = 9 - \pi(1) = 9 - \pi$

For $x = 2$, $y = 9 - \pi(2) = 9 - 2\pi$

For $x = -1$, $y = 9 - \pi(-1) = 9 + \pi$

$\therefore$ The four solutions of the given equation are $(0, 9)$, $(1, 9 - \pi)$, $(2, 9 - 2\pi)$, $(-1, 9 + \pi)$.

(iii) Given equation $x = 4y \Rightarrow y = \frac{x}{4}$

For, $x = 0$, $y = \frac{0}{4} = 0$

For $x = 4$, $y = \frac{4}{4} = 1$

For $x = -4$, $y = \frac{-4}{4} = -1$

For $x = 8$, $y = \frac{8}{4} = 2$

$\therefore$ The four solutions of a given equation are $(0, 0)$, $(4, 1)$, $(-4, -1)$ and $(8, 2)$.

Example 6. Find a if $x = 3, y = 1$ is a solution of the equation $3x - y = a$.

Solution. Given $3x - y = a$

As $x = 3, y = 1$ is a solution of this given equation, it must satisfy it.

$$\therefore 3(3) - 1 = a \Rightarrow a = 9 - 1 \Rightarrow \boxed{a = 8} \text{ Ans.}$$

Example 7. Give the equations of two lines passing through $(2, 14)$. How many more such lines are there, and why? —NCERT

Solution. Equations of two lines passing through $(2, 14)$ can be taken as $2x + y = 18$ or $3x - y = -8$.

Also, equations such as $7x - y = 0, 5x + 2y = 38$ etc. are also satisfied by the co-ordinates of the point $(2, 14)$. So, any line passing through $(2, 14)$ is an example of a linear equation for which $(2, 14)$ is a solution. Thus, there are infinite number of lines through $(2, 14)$.

Example 8. Draw the graph of the following equations :

(i) $x = 3$

(ii) $y = -4$

(iii) $y = x$

(iv) $y = -x$

(v) $2x + 5y = 10$

(vi) $2x - y = 7$

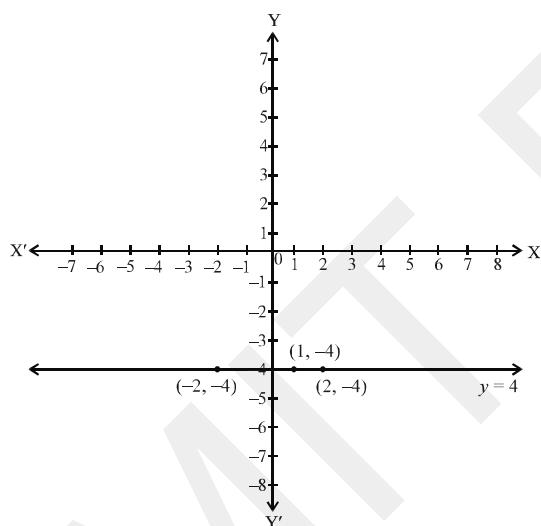
Solution.

(i) $x = 3$

As x is constant, y may take any value.

(ii) $y = -4$

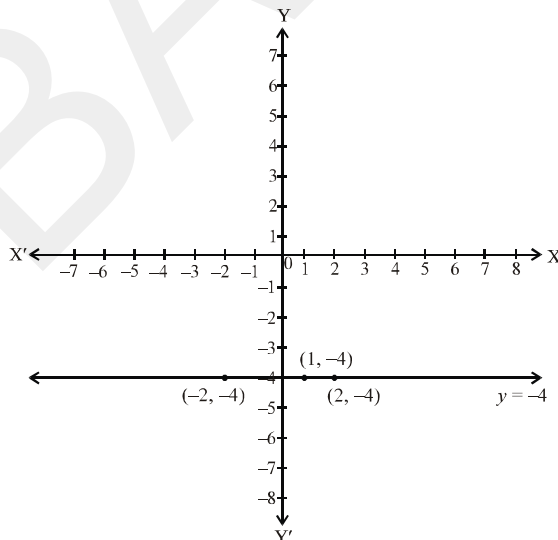
As y is constant, x may take any value.



x	3	3	3	3	3
y	-2	-1	0	1	2

(iii) $y = x$

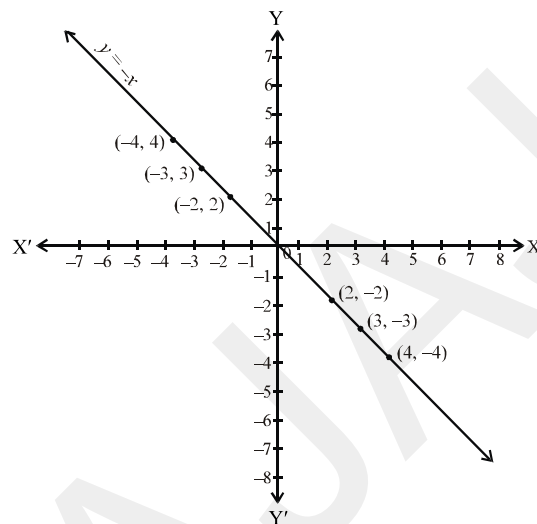
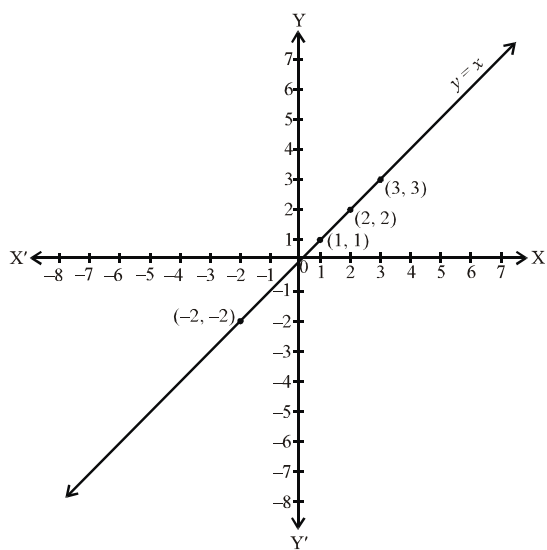
x	2	1	0	-1	-2
y	2	1	0	-1	-2



x	-2	-1	0	1	2
y	-4	-4	-4	-4	-4

(iv) $y = -x$

x	-2	-1	0	1	2
y	2	1	0	-1	-2



(v) $2x + 5y = 10$

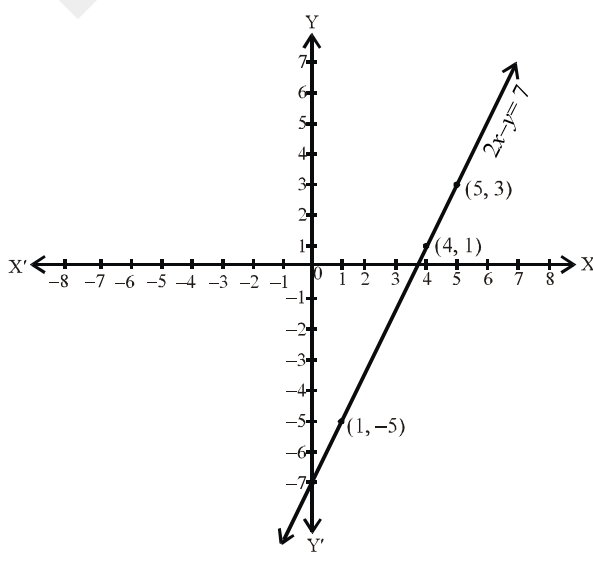
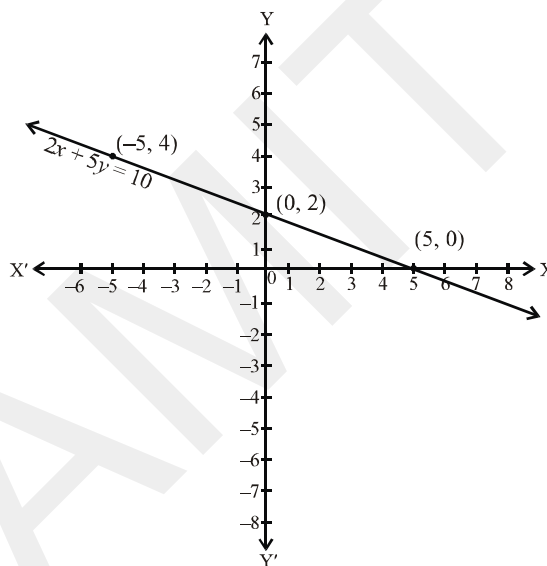
$$y = \frac{10 - 2x}{5}$$

x	0	5	3	-5
y	2	0	0.8	4

(vi) $2x - y = 7$

$$y = 2x - 7$$

x	0	1	4	5
y	-7	-5	1	3



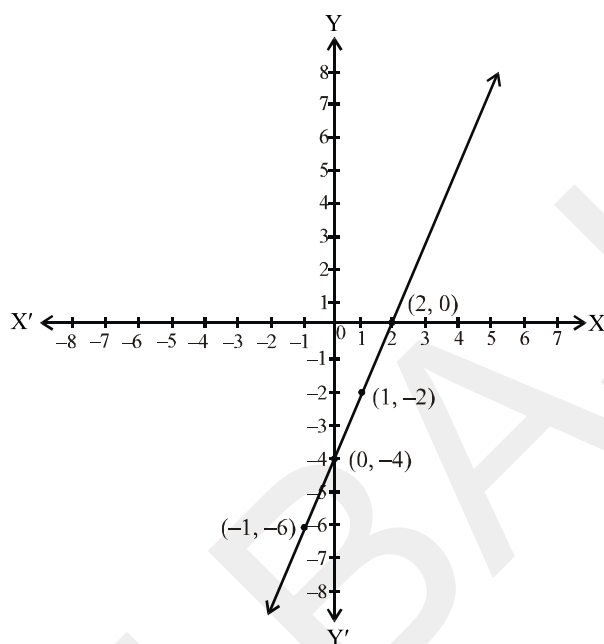
Example 9. From the choices given, choose the equation whose graph is given

(i) $y = 2x$

(ii) $y = 2x + 1$

(iii) $x + y = 0$

(iv) $y = 2x - 4$



Solution. Given points on the graph are (2, 0), (1, -2), (0, -4) and (-1, -6). By trial and error we observe that all these 4 given points satisfy the equation $y = 2x - 4$, so, it is a graph of $y = 2x - 4$.

Example 10. The taxi fare in a city is as follows. For the first km., the fare is Rs. 8 and for the subsequent distance it is Rs. 5 per km. Write a linear equation for this information and draw its graph.

Solution. Let total distance covered is x km.

Let total fare is Rs. y

Since, fare for 1st km is Rs. 8 and for remaining $(x-1)$ km. is Rs. 5 per km.

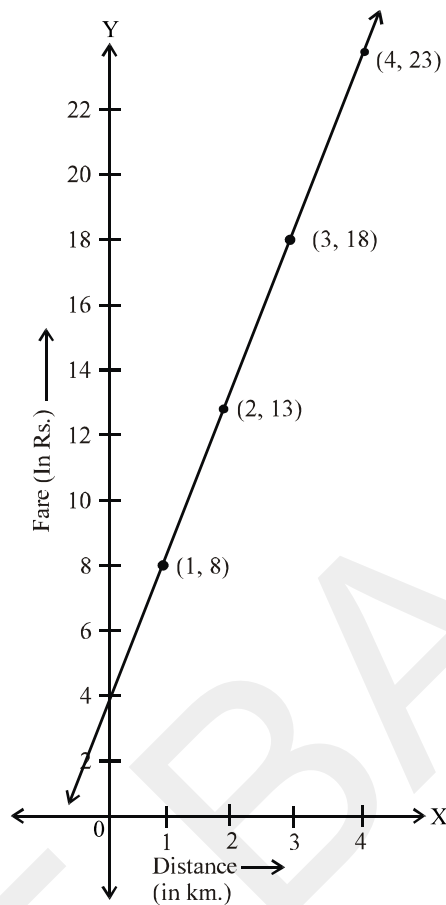
$\therefore$ By given information, we have $8 \times 1 + 5(x-1) = y$

$$\Rightarrow 5x - y = -3 \text{ or } \boxed{y = 5x + 3}$$

Now, equation is $y = 5x + 3$. Giving different values to x , we get corresponding values of y .

Let us represent this in a tabular form.

x	1	2	3	4
y	8	13	18	23



Example 11. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also, read from the graph the work done when the distance travelled by the body is : (i) 2 units (ii) 0 units. —NCERT

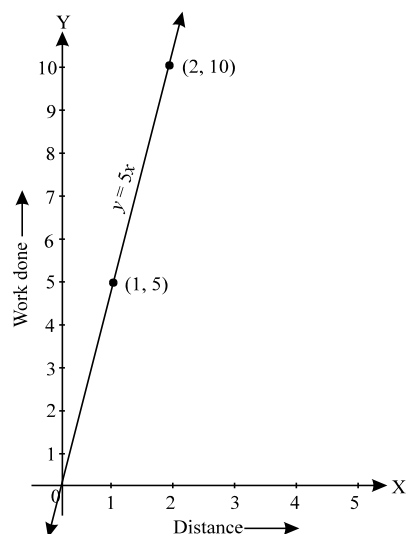
Solution. Let x be the distance and y be the work done. Then, according to the given problem, we have

$$y = 5x. \quad (\because 5 \text{ is the constant force}).$$

Let us not draw the graph of this linear equation in two variables.

Required table is :

x	0	1	2
y	0	5	10



(i) From the graph, we see that, $x = 2$ units distance $\Rightarrow y = 10$ units work done.

(ii) From the graph, we see that, $x = 0$ unit distance $\Rightarrow y = 0$ unit work done.

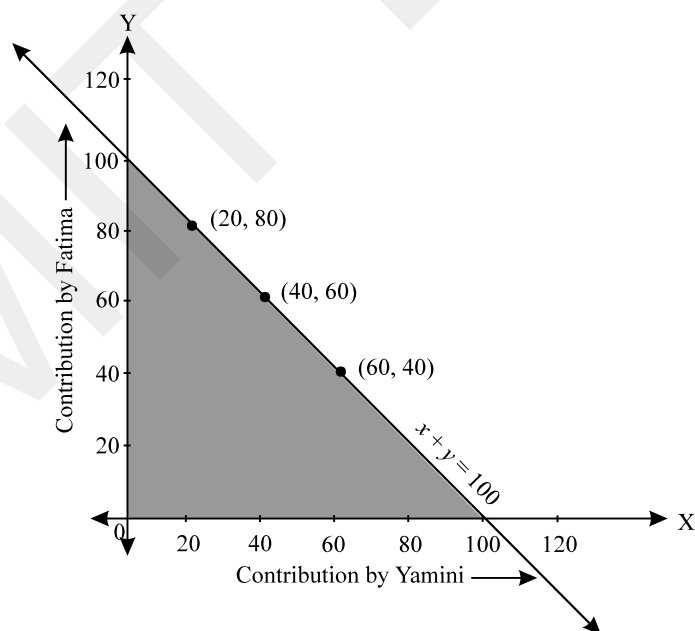
Example 12. Yamini and Fatima, two students of class IX of a school, together contributed Rs. 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this date. (You may take their contributions as Rs. x and Rs. y). Draw the graph of the same.

—NCERT

Solution. Let, Yamini contributed Rs. x and Fatima contributed Rs. y . then, according to the given question, we have, $x + y = 100$

Required table is :

x	20	40	60
y	80	60	40



Every point of the shaded portion including the line $x + y = 100$, x -axis and y -axis in the first quadrant represent the solution set.

Example 13. In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \frac{9}{5}C + 32.$$

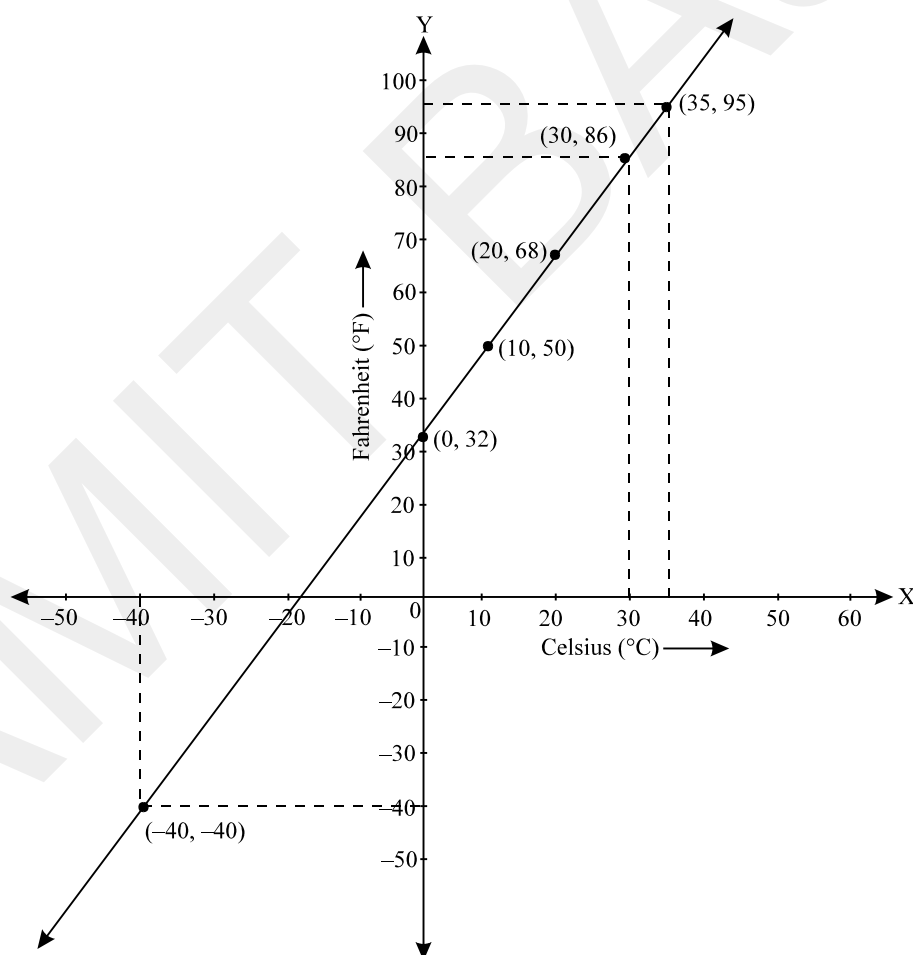
- Draw the graph of the linear equation above using Celsius for x -axis and Fahrenheit for y -axis.
- If the temperature is 30°C , what is the temperature in Fahrenheit?
- If the temperature is 95°F , what is the temperature in Celsius?
- If the temperature is 0°C , what is the temperature in Fahrenheit, and if the temperature is 0°F , what is the temperature in Celsius?
- Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

—NCERT

Solution. $F = \frac{9}{5}C + 32.$

Required table is :

C	10	20	30
F	50	68	86



- (i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.
- (i) The graph of the line $F = \frac{9}{5}C + 32$ is shown in the figure.
- (ii) From the graph : $C = 30^\circ \Rightarrow F = 86^\circ$
- (iii) From the graph : $F = 95^\circ \Rightarrow C = 35^\circ$
- (iv) If $C = 0^\circ \Rightarrow F = 32^\circ$ and, If $F = 0^\circ \Rightarrow C = -17.8^\circ$
- (v) Yes, clearly from the graph, the temperature which is numerically the same is both Fahrenheit and Celcius is $-40^\circ F = -40^\circ C$.

PRACTICE EXERCISE

- Which of the following are linear :
 - $3x(x-2) = 3x^2 + 2x - 7$
 - $(x+1)(x-2) = 10$
 - $x(x+1) = -2x^2 + 7x + 2$
 - $2x^2 + 7x - 3 = 2x(x+1)$
- Write each of the following equation in the form $ax + by + c = 0$
 - $y - 3 = 2(x - 1)$
 - $\frac{x}{3} - y = 5$
 - $\frac{x-3}{2} - y = 6$
 - $2(x-1) - 3(y+1) = 1$
 - $\frac{x}{2} - \frac{y}{3} = 6$
 - $\frac{x+1}{2} - \frac{3y-1}{4} = 3$
- Write four solutions for each of the following :
 - $2x - y = 3$
 - $x = 3y$
- Find out which of the following equations have $x = 2, y = -1$ as a solution :
 - $3x + 2y = 4$
 - $4x - y = 8$
 - $x - \frac{y}{2} = 3$
 - $4x - y = 9$
 - $\frac{x}{2} - y = 2$
 - $7x - 4y = 18$
- Find the value of k if the given point lies on the graph defined by each equation :
 - $3x + ky = 2 ; (1, -1)$
 - $y + kx = 8 ; (-3, 2)$
 - $2x - 3y = k ; (-1, 5)$
 - $2x - k = 4y ; (2, 0)$
- Show that $x = 1, y = -6 ; x = 2, y = -3$ and $x = 3, y = 0$ are all solution of equation $3x - y = 9$.
- The equation of a graph is given by $2x + y = 10$. Indicate which of the following points lies on the graph.
 - $(6, -2)$
 - $(2, 8)$
 - $(2, 6)$
 - $(-3, 16)$
 - $(-6, 18)$
 - $(-6, 22)$
- Draw the graph of line $2x + 5y = 13$. Is the point $(9, -1)$ lies on the line?
- Draw the graph of $y = -2x + 4$. Find co-ordinates of point at which it intersects the axes.
- Draw the graph of $y = 3x + 2$ and $y = 3x - 1$ using the same pair of axes. Are these two lines parallel?
- Draw the graph of $3x + 2y = 7$ and $2x - 3y = 10$ using the same pair of axes. Are these two lines perpendicular?

12. Draw the graph of $3x - y = 5$ and $2x + 3y = 7$ on the same axes. What is their point of intersection?
13. Graph the following equations :
- (i) $2y = -x + 3$ (ii) $\frac{x-1}{2} = \frac{y}{3}$ (iii) $3x - 4y = 12$ (iv) $y = |x|$
14. Amit invests Rs. 100 at the rate of 5% p.a. simple interest. Assuming rate to be same, find graphically the interest he will earn after 5 years?
15. Unit of temperature measurement, Fahrenheit and celsius are related by relation $F = \frac{9}{5}C + 32$.
- (i) Draw graph of linear equation above, using celsius for x -axis and Fahrenheit for y -axis.
(ii) If the temperature is 35°C , what is the temperature in Fahrenheit?
(iii) If the temperature is 86°F , what is the temperature in celsius?
(iv) Is there a temperature which is numerically the same in both units? If yes, find it.

PRACTICE TEST

M.M : 15

Time : $\frac{1}{2}$ hour

General Instructions :

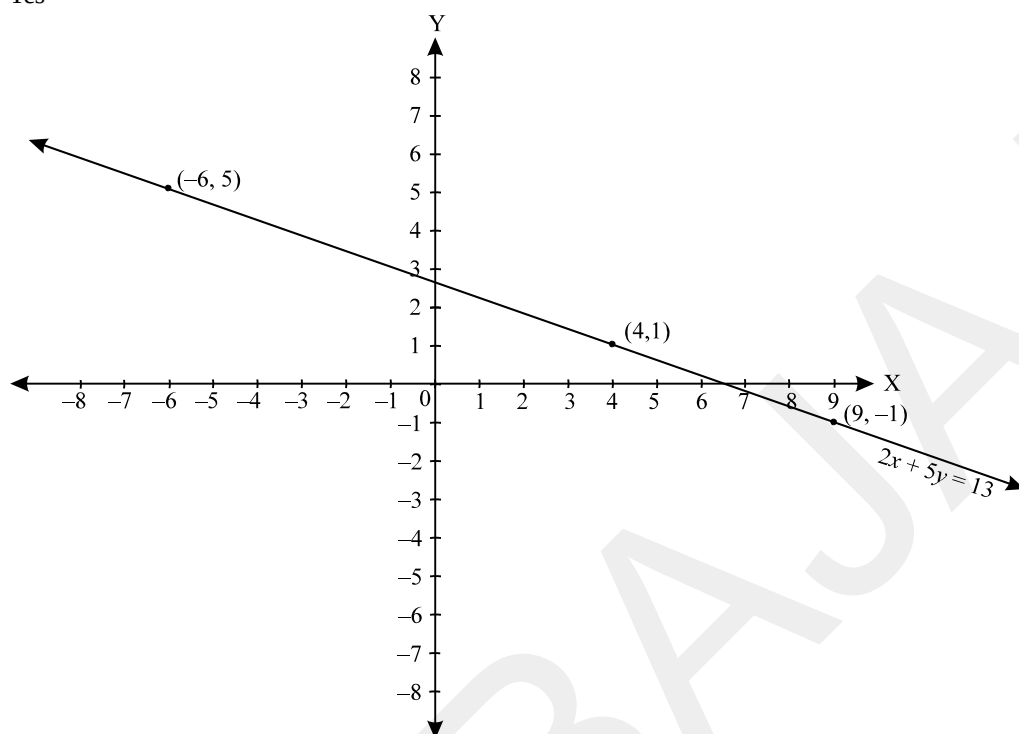
All questions carry 3 marks each.

- Give three integral solutions for equation $5x - y = 9$.
- Find the value of a so that the equation $4x - ay = 7$ have $(2, -3)$ as a solution.
- Give the geometrical representation of $x = -3$ as an equation.
(i) In one variable (ii) In two variables
- Yamini and Fatima, together contributed Rs. 100 towards PM relief fund. Write a linear equation which this data satisfies. Draw the graph of the same.
- Draw the graph of $3x + y = 8$. Is $x = -1, y = 11$, a solution of this equation. Also, shade the portion bounded by this line and both the axes.

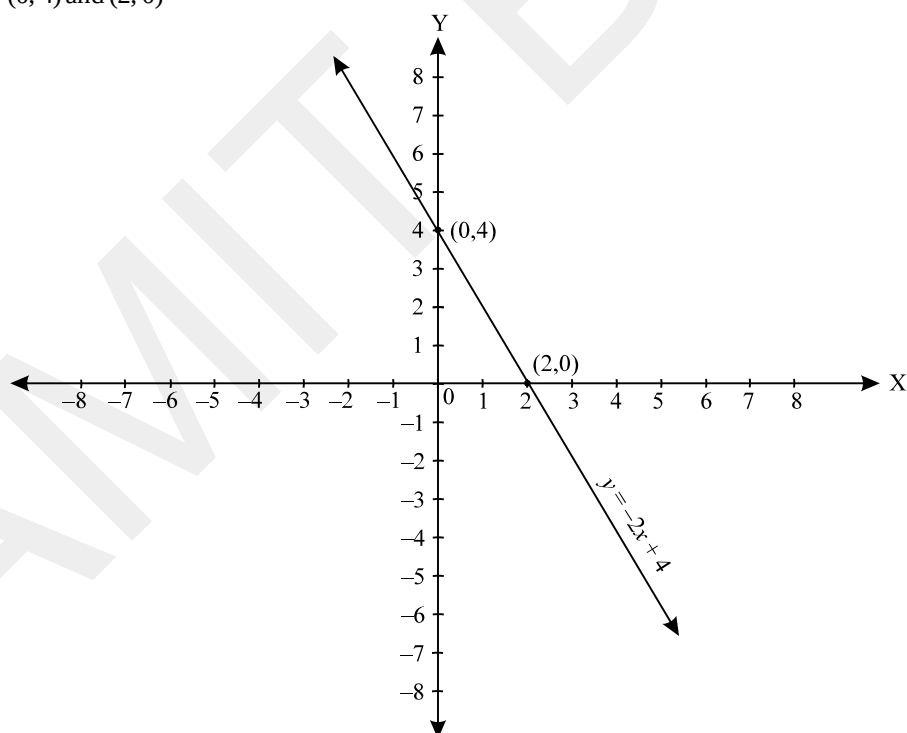
ANSWERS OF PRACTICE EXERCISE

- (i) and (iv)
- (i) $2x - y + 1 = 0$ (ii) $x - 3y - 15 = 0$ (iii) $x - 2y - 15 = 0$ (iv) $2x - 3y - 6 = 0$
(v) $3x - 2y - 36 = 0$ (vi) $2x - 3y - 9 = 0$
- (i) $(0, -3), (1, -1), (2, 1), (-1, -5)$ (ii) $(3, 1), (6, 2), (9, 3), (12, 4)$
- (i), (iv), (v), (vi)
- (i) $k = 1$ (ii) $k = -2$ (iii) $k = -17$ (iv) $k = 4$
- (i), (iii), (iv), (vi)

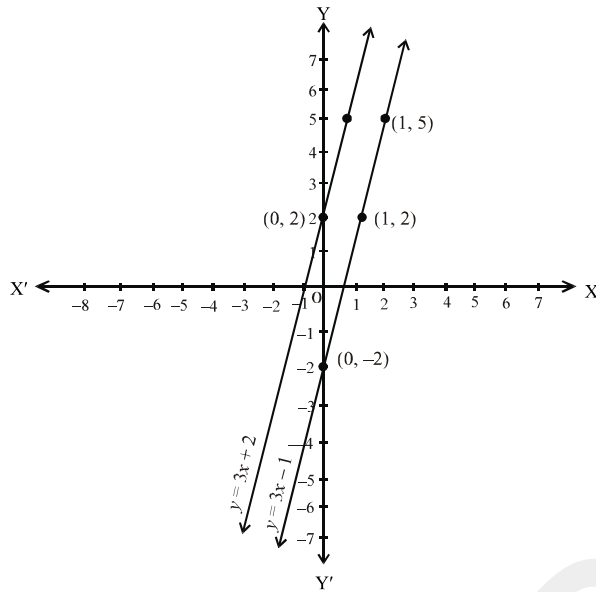
8. Yes



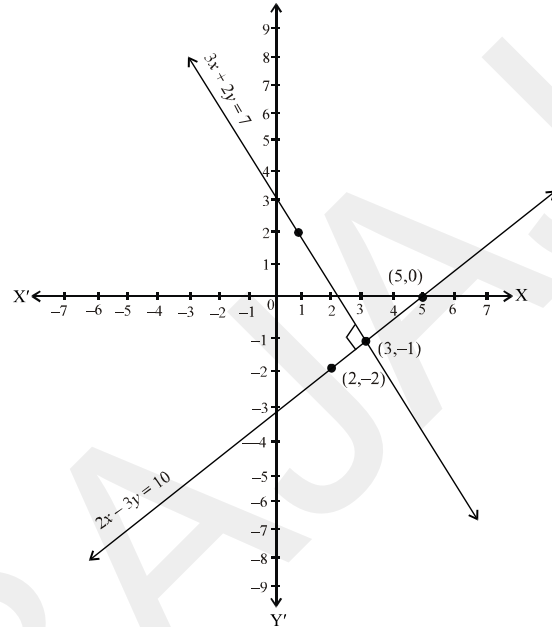
9. $(0, 4)$ and $(2, 0)$



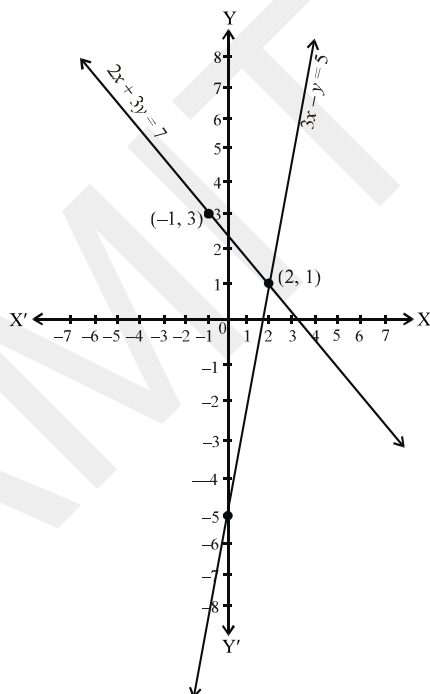
10. Yes



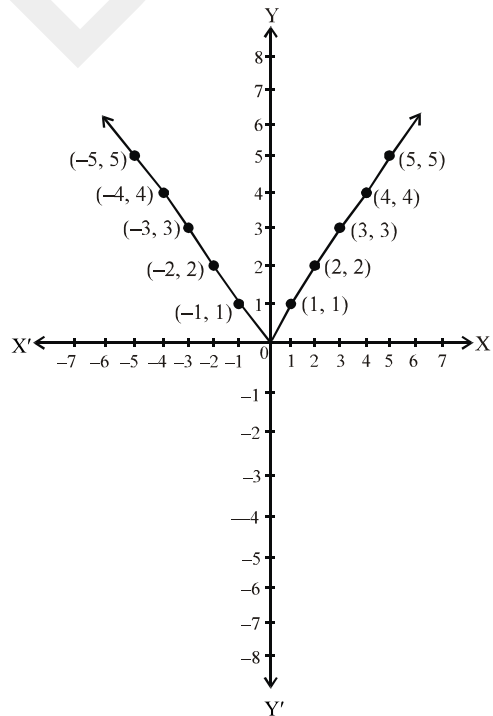
11. Yes



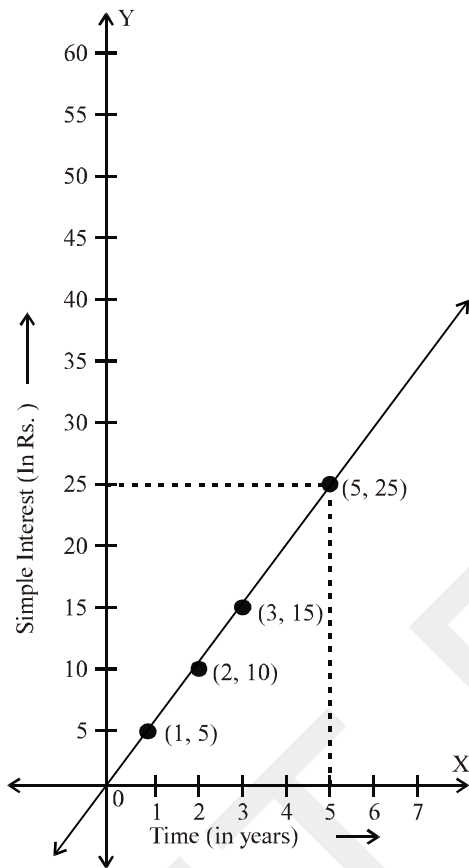
12. (2, 1)



13. (iv) $y = |x|$

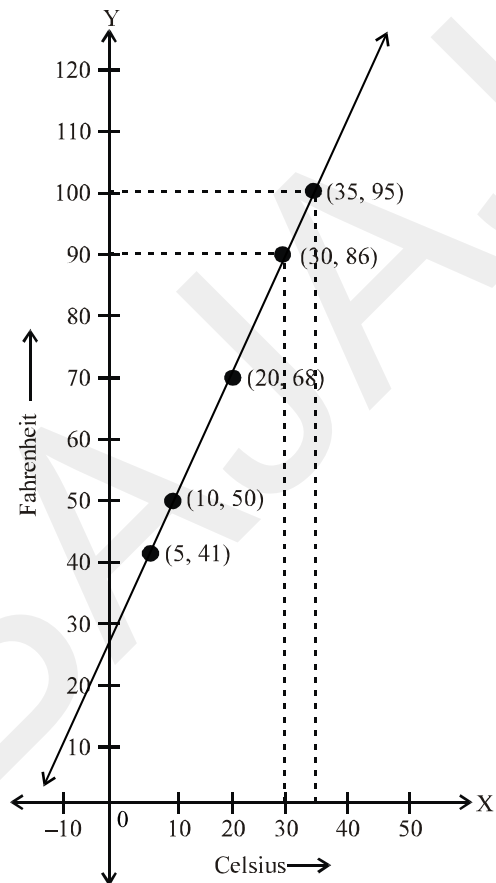


14.



From graph, we observe that after 5 years, he will receive Rs. 25.

15.



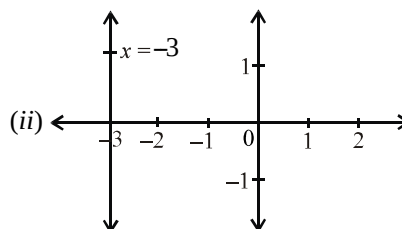
From graph, we have : (ii) 95°F
(iii) 30°C (iv) Yes, -40°

ANSWERS OF PRACTICE TEST

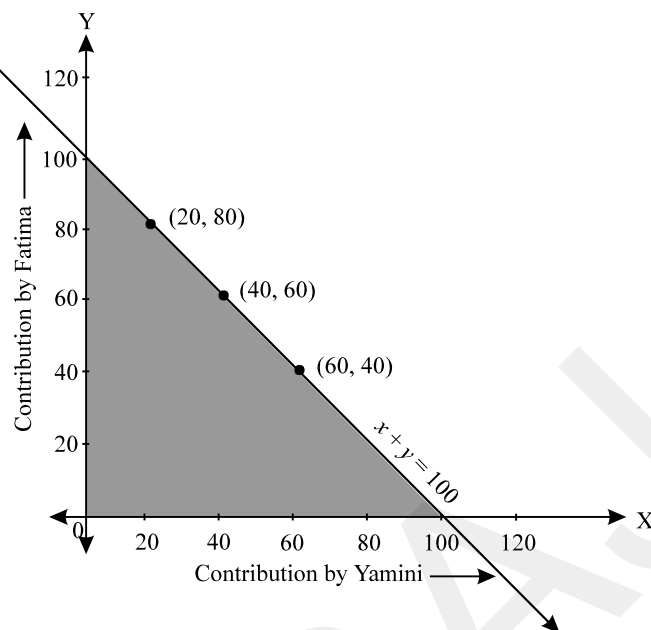
1. $(1, -4)$, $(2, 1)$ and $(3, 6)$

2. $a = -\frac{1}{3}$

3. (i) $x = -3$



4. $x + y = 100$



where, x Rs. = contribution by Yamini and y Rs. = contribution by Fatima.

5. Yes

