

LINEAR EQUATIONS

A **linear equation** can be defined as an equation in which the highest exponent of the equation variable is one. When graphed, the equation is shown as a single line.

Example: $x + 2 = 4$ ← Linear equation: highest exponent of the variable is 1.

A linear equation has only one solution. The solution of a linear equation is equal to the value of the unknown variable that makes the linear equation true.

Example: $x + 2 = 4$ ← The value of the unknown variable that makes the equation
 $x = 4 - 2$ true is 2: $2 + 2 = 4$
 $x = 2$

There are different forms of writing a linear equation. Each form has a different way of solving the linear equation that makes the process easier. There is only one rule that applies to any form of linear equations: whatever you do on one side you need to do on the other side. That is, if you add on the left side, you need to add on the right side. If you multiply on the left side, you need to multiply on the right side, and so on.

1. Equations of the form $x + a = b$ or $x - a = b$.

Linear equations in this form are solved by adding or subtracting the same quantity to both sides with the idea of leaving the variable by itself. That is, isolating the variable.

Examples: $x - 5 = 10$
 $x - 5 + 5 = 10 + 5$ ← Add 5 to both sides to eliminate the number
On the left side of the equation and leave x alone.

$$x = 15$$

$y + 3.2 = 4.1$
 $y + 3.2 - 3.2 = 4.1 - 3.2$ ← Subtract 3.2 to both sides to eliminate the
 $y = 0.9$ number on the left side and isolate y .

2. Equations of the form $ax = b$.

Linear equations in the form of multiplication are solved by dividing both sides of the equation by the number multiplying the variable. When a fraction is multiplying the variable, multiply both sides of the equation by the reciprocal of the fraction attached to the variable.

Examples:

$$3x = 18$$

$$\frac{3}{3}x = \frac{18}{3}$$

$$x = 6$$

← Divide both sides by 3 to leave x by itself.

$$\frac{3}{4}x = \frac{2}{5}$$

$$\frac{4}{3} \cdot \frac{3}{4}x = \frac{2}{5} \cdot \frac{4}{3}$$

$$x = \frac{8}{15}$$

← Multiply both sides by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

3. Equations of the form $ax + b = c$.

This type of linear equation presents a combination of the first two forms of linear equations. The first step in solving this form of linear equations is to eliminate constant (the number unattached to any variable). To do this add or subtract the same quantity to both sides of the equation. Next, eliminate the number attached to the variable by dividing or multiplying, depending on the case, to both sides of the equation. The idea is to leave variables on one side of the equal sign and constants on the other side of the equal sign.

Examples:

$$3y + 7 = 25$$

$$3y + 7 - 7 = 25 - 7$$

$$3y = 18$$

$$\frac{3}{3}y = \frac{18}{3}$$

$$y = 6$$

← Subtract 7 from both sides to remove it from the left side.

← Divide both sides by 3 to leave y alone and solve for y.

$$\frac{8}{5}x + 2 = -1$$

$$\frac{8}{5}x + 2 - 2 = -1 - 2$$

$$\frac{8}{5}x = -3$$

$$\frac{5}{8} \cdot \frac{8}{5}x = -\frac{3}{1} \cdot \frac{5}{8}$$

$$x = -\frac{15}{8}$$

← Follow the steps described above to solve for x.

4. Equations of the form $ax + b = cx + d$.

In this form of linear equations the main goal is to leave the variables on one side and the constants on the other side. To do this, perform all the steps required to solve the previous forms of linear equations step by step. Do not try to do all the steps at once, this can only result in confusion and mistakes.

Be careful!

<i>Example:</i>	$6x - 15 = -8x + 13$	← Start by moving the variable from the right side to the left side.
	$6x + 8x - 15 = -8x + 8x + 13$	← Combine the two variables together.
	$14x - 15 = 13$	← Add 15 to both sides to eliminate the constant on the left side of the equation.
	$14x - 15 + 15 = 13 + 15$	← Divide both sides of the equation by 14.
	$14x = 28$	
	$\frac{14}{14}x = \frac{28}{14}$	← Solve for x .
	$x = 2$	

Distributive Property: $a(b + c) = ab + ac$

The distributive property is used to remove grouping symbols in linear equations. Grouping symbols are terms enclosed in parentheses. To ungroup, distribute the term outside the parentheses to each of the terms inside the parentheses by multiplication.

<i>Example:</i>	$2(x - 3) + 1 = -7$	← Multiply the term outside the parentheses to each of the terms inside the parentheses.
	$2x - 6 + 1 = -7$	
	$2x - 5 = -7$	← Eliminate the constant on the left side of the equation by adding 5 to both sides of the equation.
	$2x - 5 + 5 = -7 + 5$	
	$2x = -2$	
	$\frac{2}{2}x = -\frac{2}{2}$	← Divide both sides by 2 and solve for x .
	$x = -1$	

LINEAR EQUATIONS – EXERCISES

Solve for x:

1. $8x - 6 = 12$

2. $4x - 5 = 2x + 7$

3. $x - 1 - 3(5 - 2x) = 2(2x - 5)$

4. $\frac{2x - 4}{4} = \frac{4x + 3}{3}$

5. $\frac{2x - a}{b} = \frac{4x - b}{a}$

6. $\frac{3}{x+1} - \frac{4}{5} = \frac{1}{x+1} - 6$

7. $\frac{3}{x-1} + \frac{4}{x+1} = \frac{8}{x+1}$

8. $\frac{1}{ax} + \frac{1}{bx} = \frac{1}{c}$

9. $\frac{3}{x} - \frac{4}{3x} = 10$

10. $\frac{2x+1}{5} + \frac{x-4}{6} = \frac{2}{3}$

11. $3(2x - 1) + x = 5x + 3$

12. $3 - 5(2x - 5) = -2$

13. $-3(5x + 7) - 4 = -10x$

14. $8 + 3(x + 2) = 5(x - 1)$

15. $5(x + 2) - 4(x + 1) = 7$

16. $\frac{1}{2x+1} - \frac{3}{4} = \frac{7}{2x+1}$

$$17. \frac{7}{2x-3} + \frac{4}{2x-3} = 6$$

LINEAR EQUATIONS – ANSWERS TO EXERCISES

$$1. \quad 8x - 6 = 12$$

$$8x - 6 + 6 = 12 + 6$$

$$8x = 18$$

$$x = \frac{9}{4}$$

$$2. \quad 4x - 5 = 2x + 7$$

$$4x - 2x = 7 + 5$$

$$2x = 12$$

$$x = 6$$

$$3. \quad x - 1 - 3(5 - 2x) = 2(2x - 5)$$

$$x - 1 - 15 + 6x = 4x - 10$$

$$7x - 4x = -10 + 1 + 15$$

$$3x = 6$$

$$x = 2$$

$$4. \quad \frac{2x-4}{4} = \frac{4x+3}{3}$$

$$3(2x-4) = 4(4x+3)$$

$$6x - 12 = 16x + 12$$

$$6x - 16x = 12 + 12$$

$$-10x = 24$$

$$x = -\frac{12}{5}$$

$$5. \quad \frac{2x-a}{b} = \frac{4x-b}{a}$$

$$a(2x-a) = b(4x-b)$$

$$2ax - a^2 = 4bx - b^2$$

$$2ax - 4bx = a^2 - b^2$$

$$x(2a - 4b) = a^2 - b^2$$

$$x = \frac{a^2 - b^2}{2a - 4b}$$

$$6. \quad \frac{3}{x+1} - \frac{4}{5} = \frac{1}{x+1} - 6$$

$$5(x+1)\left(\frac{3}{x+1} - \frac{4}{5}\right) = 5(x+1)\left(\frac{1}{x+1} - 6\right)$$

$$15 - 4(x+1) = 5 - 30(x+1)$$

$$15 - 4x - 4 = 5 - 30x - 30$$

$$26x = -36$$

$$x = -\frac{18}{13}$$

$$7. \quad \frac{3}{x-1} + \frac{4}{x+1} = \frac{8}{x+1}$$

$$(x-1)(x+1)\left(\frac{3}{x-1} + \frac{4}{x+1}\right) = (x+1)\left(\frac{8}{x+1}\right)$$

$$3(x+1) + 4(x-1) = 8(x-1)$$

$$3x + 3 + 4x - 4 = 8x - 8$$

$$7x - 8x = 1 - 8$$

$$-x = -7$$

$$x = 7$$

$$8. \quad \frac{1}{ax} + \frac{1}{bx} = \frac{1}{c}$$

$$9. \quad \frac{3}{x} - \frac{4}{3x} = 10$$

$$abcx\left(\frac{1}{ax} + \frac{1}{bx}\right) = abcx\left(\frac{1}{c}\right)$$

$$bc + ac = abx$$

$$\frac{bc + ac}{ab} = x$$

$$x = \frac{c(a+b)}{ab}$$

$$10. \quad \frac{2x+1}{5} + \frac{x-4}{6} = \frac{2}{3}$$

$$30\left(\frac{2x+1}{5} + \frac{x-4}{6}\right) = 30\left(\frac{2}{3}\right)$$

$$6(2x+1) + 5(x-4) = 20$$

$$12x + 6 + 5x - 20 = 20$$

$$17x = 34$$

$$x = 2$$

$$12. \quad 3 - 5(2x - 5) = -2$$

$$3 - 10x + 25 = -2$$

$$-10x = -2 - 3 - 25$$

$$-10x = -30$$

$$x = 3$$

$$14. \quad 8 + 3(x + 2) = 5(x - 1)$$

$$8 + 3x + 6 = 5x - 5$$

$$3x - 5x = -5 - 8 - 6$$

$$-2x = -19$$

$$x = \frac{19}{2}$$

$$16. \quad \frac{1}{2x+1} - \frac{3}{4} = \frac{7}{2x+1}$$

$$3x\left(\frac{3}{x} - \frac{4}{3x}\right) = 3x(10)$$

$$9 - 4 = 30x$$

$$x = \frac{1}{6}$$

$$11. \quad 3(2x - 1) + x = 5x + 3$$

$$6x - 3 + x = 5x + 3$$

$$7x - 5x = 3 + 3$$

$$2x = 6$$

$$x = 3$$

$$13. \quad -3(5x + 7) - 4 = -10x$$

$$-15x - 21 - 4 = -10x$$

$$-15x + 10x = 21 + 4$$

$$-5x = 25$$

$$x = -5$$

$$15. \quad 5(x + 2) - 4(x + 1) = 7$$

$$5x + 10 - 4x - 4 = 7$$

$$x = 7 - 10 + 4$$

$$x = 1$$

$$17. \quad \frac{7}{2x-3} + \frac{4}{2x-3} = 6$$

$$4(2x+1)\left(\frac{1}{2x+1}-\frac{3}{4}\right)=4(2x+1)\left(\frac{7}{2x+1}\right)$$

$$4-3(2x+1)=28$$

$$4-6x-3=28$$

$$-6x-3=24$$

$$x=-\frac{27}{6}$$

$$(2x-3)\left(\frac{7}{2x-3}+\frac{4}{2x-3}\right)=(2x-3)\cdot 6$$

$$7+4=6(2x-3)$$

$$11=12x-18$$

$$11+18=12x$$

$$x=\frac{29}{12}$$