

Statements equivalent to Euclid's Parallel (5th) Postulate

In Neutral Geometry (Euclid's Postulates 1 - 4 clarified and made precise) the following statements are equivalent:

- (Euclid's 5th) If two lines are intersected by a transversal in such a way that the sum of the two interior angles on one side is less than 180° , then the two lines meet on that side of the transversal.
- Through a given point not on a given straight line can be drawn exactly one straight line parallel to the given line.
- Two lines parallel to the same line are parallel to each other.
- A line that intersects one of two parallel lines intersects the other also.
- Any two parallel lines have a common perpendicular.
- If parallel lines are cut by a transversal, alternate interior angles are equal.
- Parallel lines are everywhere equidistant from one another.
- The sum of the angles of a triangle is equal to two right angles.
- For any triangle, there exists a similar noncongruent triangle.
- There is no upper limit to the area of a triangle.
- The area of a triangle is half its base times its height.
- The Pythagorean Theorem.
- The converse of the Pythagorean Theorem.
- Opposite sides of a parallelogram are congruent.
- The diagonals of a parallelogram bisect each other.
- If in a quadrilateral a pair of opposite sides are equal and if the angles adjacent to a third side are right angles, then the other two angles are also right angles.
- If in a quadrilateral three angles are right angles, then the fourth angle is also a right angle.
- There exists a circle passing through any three noncollinear points.
- The circumference of any circle of radius r is $2\pi r$.
- The area of any circle of radius r is πr^2 .
- Through a point within an angle less than 60° there can always be drawn a straight line intersecting both sides of the angle.
- A line cannot lie entirely in the interior of an angle.